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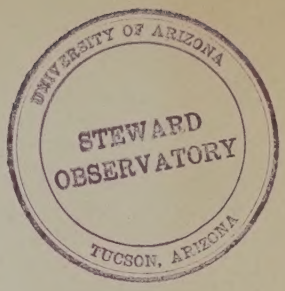


# LICK OBSERVATORY BULLETINS

VOLUME XIV
NUMBERS 402 TO 421
1928-1930

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# CONTENTS

PAGE

NUMBER 402.—A Determination of the Longitude of the Lick Observatory. H. M. Jeffers ..... 1

NUMBER 403.—The Binary Orbit Whose Plane is Perpendicular to the Tangential Plane of Projection. R. T. Crawford ..... 6

NUMBER 404.—Observations of Comets and of the Satellite of Neptune. R. T. Crawford ..... 8

NUMBER 405.—On the Number of Solutions in Leuschner's Direct Method of Determining Parabolic Orbits. C. H. Smiley ..... 10

NUMBER 406.—Elements of Minor Planet Y. O. 26 (1928 DJ) = (841) *Arabella*. Maud W. Makemson ..... 15

NUMBER 407.—The Spectra of Minor Planets. N. T. Bobrovnikoff ..... 18

NUMBER 408.—On the Disintegration of Comets. N. T. Bobrovnikoff ..... 28

NUMBER 409.—Investigation of the Orbit, Mass and Parallax of 99 *Herculis*. Maud W. Makemson ..... 38

NUMBER 410.—A Photometric Study of the Continuous Spectra of Giant and Dwarf Stars. N. W. Storer ..... 41

NUMBER 411.—Elements and Ephemeris of Minor Planet Y.O. 6 (1922 MY). C. J. Krieger and D. C. Duncan ..... 58

NUMBER 412.—The Radial Velocity of the Cepheid Variable $\kappa$ *Pavonis*. T. S. Jacobsen ..... 60

NUMBER 413.—Measures of 296 A Double Stars. R. G. Aitken ..... 62

NUMBER 414.—On the Use of a Half Color Filter for the Determination of Color Indices. R. J. Trumpler ..... 89

NUMBER 415.—Observations of Comet *a* 1929 (Schwassmann-Wachmann). S. Einarsson ..... 93

Photographic Positions of Comets. L. Berman ..... 93

Micrometrical Measures of Double Stars. L. Berman ..... 93

NUMBER 416.—A Determination of Magnitudes, Spectral Types and Color Indices in the Scutum Cloud, with a Statistical Discussion. C. J. Krieger ..... 95

NUMBER 417.—Elements and Ephemeris of Comet *b* 1929 (Neujmin). E. C. Bower and H. C. Willis ..... 130

NUMBER 418.—A Spectrographic Study of $\beta$ *Cephei*. H. S. Mendenhall ..... 133

NUMBER 419.—Elements and Ephemeris of Comet *d* 1929 (Wilk). E. C. Bower and F. L. Whipple ..... 152

NUMBER 420.—Preliminary Results on the Distances, Dimensions and Space Distribution of Open Star Clusters. R. J. Trumpler 154

NUMBER 421.—Preliminary Elements and Ephemeris of the Lowell Observatory Object. E. C. Bower and F. L. Whipple ..... 189

Photographic Positions of the Lowell Observatory Object. W. F. Meyer and F. J. Neubauer ..... 192

# PLATES

PLATE I.—Spectrogram of $\eta$ *Pegasi*, enlarged 7 Times ..... 44

PLATE II.—Sample Set of Microphotometer Curves ..... 44

PLATE III.—Fig. 2.—Enlargement of two-hour exposure of Area A (Scutum Cloud) ..... 102

Fig. 5.—Slitless Spectrogram of Area A. Four-hour Exposure ..... 102

Fig. 6.—Slitless Spectrogram of Area E. Four-hour Exposure ..... 102

PLATE IV.—Fig. (a).—Spectrum of $\beta$ *Cephei* obtained with III-prism violet spectrograph and 36-inch refractor ..... 142

Fig. (b).—Spectrum of $\beta$ *Cephei* obtained with the Mills III-prism spectrograph and 36-inch refractor ..... 142

Fig. (c).—Spectrum of $\beta$ *Cephei* between spectra of comparison star 19 *Cephei* obtained with a slitless quartz spectrograph ..... 142

Fig. (d.).—Microphotometric tracings of the above spectra ..... 142

PLATE V.—Typical Open Clusters ..... 179

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ASTRONOMY

# LICK OBSERVATORY BULLETIN

NUMBER 402

## A DETERMINATION OF THE LONGITUDE OF THE LICK OBSERVATORY

BY

H. M. JEFFERS

Advantage was taken of the Radio Time Signals for the World Longitude Determination in October and November, 1926, to effect a re-determination of the longitude of the Lick Observatory. The work was undertaken, first, to obtain an improved value of the longitude of this place, one consistent with the results of the other observatories cooperating in the International scheme, and second, to examine into the performance of the Meridian Circle as an instrument for the determination of time.

The Repsold Meridian Circle has been amply described in previous *Publications*. The only important change made was the substitution of a self-registering micrometer for the fixed wire arrangement. A spring driven clock-work for driving the transit wire was available but was not used, since it did not seem advisable to add so much unsymmetrically placed weight to the telescope tube. A single transit thread was used. The micrometer produces ten breaks per revolution, with two additional ones to locate the beginning of a revolution. Usually twenty breaks were measured on each star. The value of one revolution of the micrometer is $3^{s}.206$. It conveniently happens that the correction for one-half the width of the micrometer contact strip almost exactly cancels the correction for diurnal aberration.

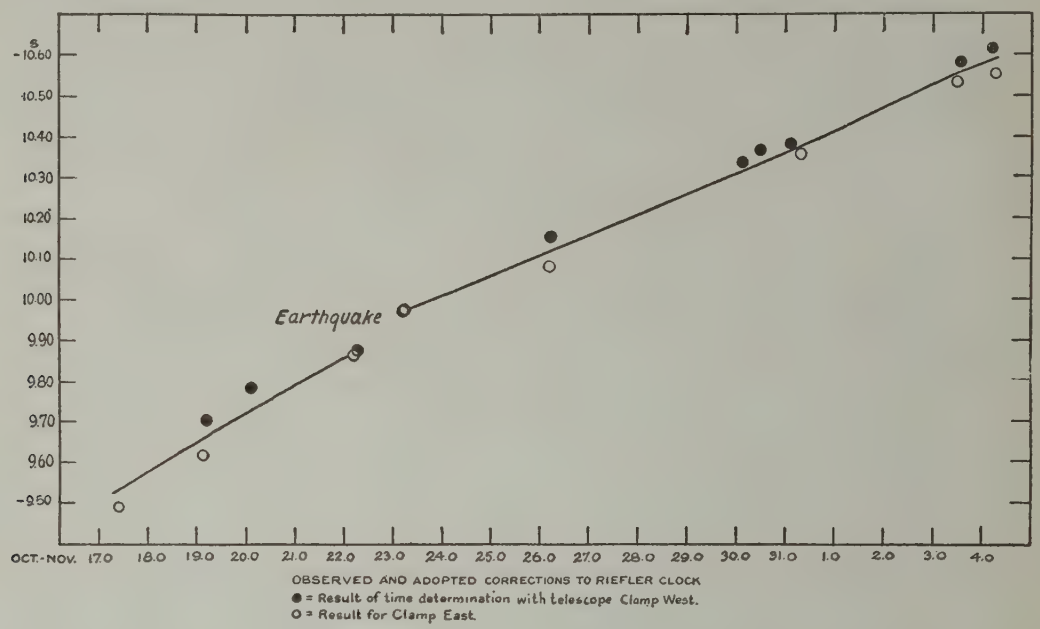
The level error was obtained from nadir observations only, as there was no spirit level to give an independent check. The collimation was obtained from the collimators, and also from the nadir at reversal. Twenty-seven determinations of collimation were made, and the two methods gave results in satisfactory accord. The azimuth was usually obtained by using one circumpolar in combination with several time stars. A time determination consisted of six or seven stars, usually, with a similar set after reversal. On seven nights observations were made in both positions of the instrument, and on four other nights in but one position.

There is decided evidence that the time determinations are affected by a reversing error, such as has been found elsewhere. An inspection of the clock correction diagram will show that observations made with the telescope in the position clamp west indicate the clock to be several hundredths of a second faster, usually, than similar observations made in the position clamp east. The pivots of the instrument have not been investigated for inequality or imperfections of figure, though possibly the source of the error does not lie here. At any rate it is hoped to look into the effect more closely as opportunity allows. The best that can be done now appears to be to take a reasonable average between the results in the two positions. The separate determinations of clock correction were plotted on a large scale, and the curve was then drawn which seemed best to indicate the state of the clock. This procedure leads, of course, to a "smoothed" clock rate. From the curve were read off the values of the clock corrections for the times of the radio signals.

The Riefler Clock was used as the time keeper. Shortly before this work it was taken apart and cleaned, and some leaks in the containing cylinder were stopped. There was no temperature control in the clock room, but in the thick-walled basement, where the clock is mounted, the variations in temperature are quite

slow, with scarcely any diurnal fluctuations. The rate seems to have been fairly good, though the series was not long enough to give an effective test of clock performance. The series, moreover, was interrupted by an earthquake which seems to have changed the correction several hundredths of a second. This earthquake stopped the Hohwü clock, the pendulum of which swings at right angles to that of the Riefler.

For the reception of the short wave signals a "Grebe Short Wave Receiver" was used, Type CR-18. It comprises a regenerative detector and one stage of audio frequency amplification, two tubes. An antenna about fifty feet long was used with it. The quickly changed



OBSERVED AND ADOPTED CORRECTIONS TO RIEFLER CLOCK
● = Result of time determination with telescope Clamp West.
○ = Result for Clamp East.

coils and stability made this receiver very convenient to handle. For the reception of long waves, an amplifier was constructed in our shop, after a design furnished through the courtesy of the Bureau of Standards. This amplifier contained four stages of resistance coupled amplification, followed by an autodyne detector using honeycomb coils, and one stage of low frequency amplification, a total of six tubes. For the input to the amplifier an arrangement of honeycomb coils was tried, but the selectivity was insufficient. In the latter part of October, a United States Navy Type SE-1899 tuner was lent by courtesy of the Naval authorities in San Francisco. With this tuner between antenna and amplifier, the recording of Annapolis (NSS) and Honolulu (NPM) was quite successful. An antenna about 300 feet long was used. Undoubtedly a loop collector would have still further increased the selectivity, but this was not tried. With the arrangement used, Saigon and Bordeaux signals were heard at times with good intensity, but the interference would scarcely have permitted automatic records, had the attempt been made.

The recording apparatus was similar to that described in the U. S. Naval Observatory circular, with the differences that one more stage of low frequency amplification was used, and that a "tone filter" or "tuned by pass" was used at the input to the apparatus. This simple contrivance, made from such material as happened to be at hand, assisted considerably in reducing both static and interference, and it also had the unexpected effect of making the system more stable. Relay chattering, which sometimes causes trouble in such installations, was nearly, if not quite, suppressed. The relay was a standard telegraph type, with the coils wound to a high resistance. The output of either the short or long wave set was fed into the recorder, so that in receiving short wave signals, a total of five tubes was used, while nine tubes were used for long waves. The long waves gave the better records, but the long wave set was not in reliable operation till most of the good observing weather had passed. No attempt was made to use the numerous records of time signals obtained which were not favorably situated with respect to observations for time.

Two lags needed to be considered, that of the transit micrometer relay, which changed the "makes" of the transit micrometer to "breaks" in the chronograph circuit, and that of the radio and recording apparatus. On most of the observing nights, the lag of the recorder alone was measured, by putting the clock relay, in series with about three volts of dry cells, directly into the

recorder. This amounted to about 0$^s$.03, and the variation from this was small. Later a more careful investigation was made, using both a high frequency buzzer and a tube oscillator. These gave a signal which passed through the radio apparatus, and thus measured the entire lag, not that of the recorder alone. The buzzer and tube oscillator were arranged to run continuously, the clock relay being adjusted to suppress the signal in one case, and change the frequency in the other. The buzzer, which gave a rough note in the oscillating receiver, gave a slightly larger lag than the tube oscillator. Various trials showed a negligible difference in lag for short and long wave signals, 0$^s$.045, and 0$^s$.048 respectively. In the reductions the value 0$^s$.046 was used for both. It was scarcely to be hoped that the lag would stay constant for varying conditions of signal strength and apparatus adjustment, but it is believed that the variations were not important. An effort was made to keep the adjustments uniform, and the measures of recorder lag during the reception of signals are quite consistent. The determination of the lag of the transit micrometer relay, which breaks the circuit when there is current through the coils, yielded a value of 0$^s$.011, which was used in the reductions. The lag was measured by putting this relay alternately in and out of the circuit controlling the tube oscillator, and measuring the difference on the chronograph as recorded through the radio apparatus.

Using the reference numbers of the U. S. Naval Observatory, signals observed here were as follows:

| No. | GCT | Meters | |
|---|---|---|---|
| 1 | 02:55—03:00 | 17,145 | Annapolis |
| 4 | 03:20—03:25 | 75 | Bellevue |
| 5 | 03:30—03:35 | 11,500 | Honolulu |
| 6 | 03:40—03:45 | 37 | Honolulu |
| 13 | 10:10—10:15 | 17,145 | Annapolis |
| 15 | 10:20—10:25 | 75 | Bellevue |
| 16 | 10:30—10:35 | 11,500 | Honolulu |
| 17 | 10:40—10:45 | 37 | Honolulu |

All signals were of the Rhythmic type except No. 1, Annapolis, which was the American code. Thirty or forty ticks were measured, in general, distributed over the record. The times were reduced to that of the last tick, using the Naval Observatory values for the rates of the radio signals. The clock times of reception of the last tick, corrected for a transit micrometer relay lag of +0$^s$.011, a radio apparatus lag of −0$^s$.046, and for the errors of the clock, are:

| | No. | |
|---|---|---|
| Oct. 17 | 4 | 20$^h$57$^m$55$^s$.9 |
| | 6 | 21 18 05.5 |
| | 17 | 3 19 13.4 |
| 19 | 6 | 21 26 49.630 |
| 22 | 4 | 21 17 38.770 |
| | No. | |
| Oct. 22 | 6 | 21$^h$38$^m$27$^s$.444 |
| 23 | 4 | 21 21 35.319 |
| 25 | 15 | 4 30 37.459 |
| 26 | 6 | 21 53 57.500 |
| 30 | 6 | 22 09 28.545 |
| | 15 | 4 50 20.155 |
| 31 | 4 | 21 53 07.741 |
| | 6 | 22 13 21.351 |
| Nov. 3 | 4 | 22 04 57.371 |
| | 15 | 5 06 6.390 |
| | 16 | 5 16 15.915 |
| | 17 | 5 26 7.696 |
| 4 | 5 | 22 19 00.971 |
| | 6 | 22 29 51.957 |
| 17 | 1 | 22 35 17.031* |
| | 13 | 5 51 16.508 |
| | 15 | 6 01 18.134 |
| | 16 | 6 06 40.004 |
| | 17 | 6 21 27.683 |
| Nov. 18 | 1 | 22 39 13.638* |

* Clock time as observed, no corrections. These were used only to get the clock rate. The time observation Nov. 17 was with telescope clamp E only, and the clock correction used includes −0$^s$.020 for reversing error.

The times of reception of the signals at Washington and San Diego were obtained from the Naval Observatory Circular, the U. S. Coast and Geodetic Survey Circular gives the times observed at Honolulu, and those observed at Vancouver and Ottawa have been published by the Dominion Observatory at Ottawa. The times as published were corrected for transmission time to reduce them to consistency with the times observed here, assuming a transmission velocity of 180,000 miles per second.

The following table gives the concluded difference in longitude between the Meridian Circle of the Lick Observatory and the various other stations.

| | Signal No. | Washington | San Diego | Honolulu | Ottawa | Vancouver |
|---|---|---|---|---|---|---|
| Oct. 19 | 6 | ..... | ..... | 20$^s$.35 | ..... | 53$^s$.40 |
| 22 | 4 | 19$^s$.19 | 46$^s$.54 | ..... | 43$^s$.01 | ..... |
| 22 | 6 | ..... | ..... | ..... | ..... | .43 |
| 23 | 4 | ..... | .56 | ..... | 42.99 | ..... |
| 25 | 15 | .18 | .55 | ..... | 43.02 | .43 |
| 26 | 6 | ..... | ..... | .34 | ..... | .38 |
| 30 | 6 | ..... | ..... | .32 | ..... | .39 |
| 30 | 15 | .23 | .60 | ..... | .04 | .37 |
| 31 | 4 | .20 | .59 | ..... | .03 | .36 |
| 31 | 6 | .23 | ..... | ..... | ..... | .36 |
| Nov. 3 | 4 | .19 | .57 | ..... | .00 | .39 |
| 3 | 15 | .21 | .58 | ..... | .02 | .36 |
| 3 | 16 | ..... | .58 | .33 | .01 | .36 |
| 3 | 17 | ..... | ..... | ..... | ..... | ..... |
| 4 | 5 | ..... | .57 | .36 | ..... | .43 |
| 4 | 6 | ..... | ..... | .43 | ..... | .42 |
| 17 | 13 | .25 | .61 | ..... | .03 | .34 |
| 17 | 15 | ..... | .58 | ..... | .05 | .31 |
| 17 | 16 | ..... | .55 | ..... | .06 | .35 |
| 17 | 17 | ..... | ..... | ..... | ..... | .32 |

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

# LICK OBSERVATORY BULLETIN

NUMBER 403

## THE BINARY ORBIT WHOSE PLANE IS PERPENDICULAR TO THE TANGENTIAL PLANE OF PROJECTION

BY

R. T. CRAWFORD

This is the special case of which 42 *Comae Berenices* is an example. This type of orbit has been solved very neatly by Laves.[1] He makes use of a simple geometric relation to derive the equations for $a$ and $\omega$. The object of this paper is to derive these elements by another equally simple geometric relation and to put the final equations into a somewhat simpler form. This method is much shorter than that of Henroteau.[2] For the sake of completeness this new work will be combined here with that of Aitken,[3] so as to have the whole theory in one place.

The revolution period can then be read directly from the curve. In general, the elements $e$, the eccentricity, and T, the time of periastron passage, are easily determined.

In the figure let the ellipse represent the true orbit. Let the primary be at the focus S, and let C be the center of the ellipse. Let the Earth be in the direction indicated, and T′ C′ $T_1'$ be the projected orbit. The point C′ is known for T′ C′ must equal half the amplitude of the curve of distances. The point S′ is the projected position of the primary and hence is known.

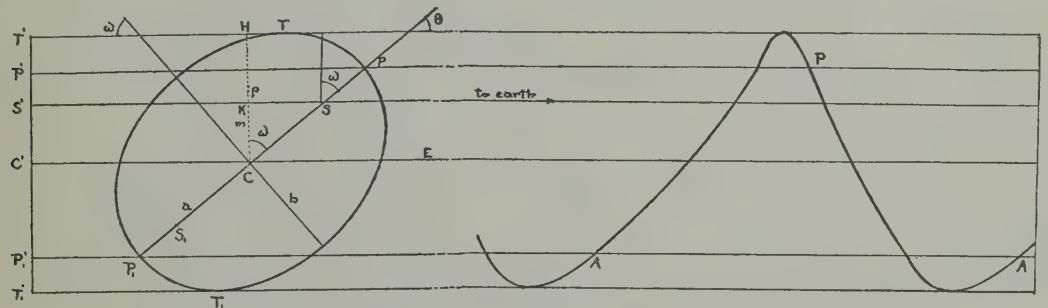



Figure 1. Apparent and True Orbits and Interpolating Curve of Observed Distances.

The element $i=90°$. The element $\Omega$ is known for this is the mean of the observed position angles. The remaining elements must be determined from the curve representing the observed distances. This is obtained by plotting the times as abscissae and the corresponding observed distances as ordinates and drawing the most probable smooth curve to represent the plotted points.

The points on the curve of distances which correspond to the points P and $P_1$ of the true orbit must be separated by exactly half of the revolution period and their distances from the line C′CE must be equal in length and of opposite sign. The point corresponding to periastron, P, must lie on the same side of this line as S′, and on the steeper branch of the curve as the rate of change of distance is greater near periastron than near apastron. In practice these two points are readily found by cutting a rectangular slip of paper to a width

[1] *A. J.* **37**, *97*, 1926.

[2] *Handbuch der Astrophysik* **6**, *338*, 1928.

[3] Aitken: *The Binary Stars*, pp. 95–96.

equal to half that of the period on the adopted scale and sliding it along the curve until the edges, kept perpendicular to the line C′CE, cut equal ordinates (with respect to C′CE) on the curve. Laves cleverly suggests "draw the central axis in the graph and make a tracing of the time graph of distances, turn it through 180° about the central time-axis and then advance the tracing by half the period of the orbit. Of the four points of intersection of graph and tracing it is easy to find that pair which is separated by half the period." In either of these ways the positions of the points P′ and P, are obtained.

Since ratios are not altered in the projection we have

$$e=\frac{C'S'}{C'P'} \tag{1}$$

The remaining elements $a$ and $\omega$ are simply derived. Laves does this by making use of a theorem relating to perpendiculars dropped from the foci of an ellipse on the tangent to the ellipse. Here a relation is used involving the perpendicular from the center of the ellipse on a tangent to the ellipse. In the figure let CH be such a perpendicular and call its length $p$. Let $\omega$ (which is here our element $\omega$) be the angle the tangent line makes with the minor axis. Then, if $a$ and $b$ are the major and minor semi-axes respectively, from the properties of the conic it can be shown that

$$p^2=a^2\cos^2\omega+b^2\sin^2\omega \tag{2}$$

which can be written

$$p^2=a^2-(a^2-b^2)\sin^2\omega=a^2-a^2e^2\sin^2\omega \tag{3}$$

But from the figure

$$CK\equiv m=\overline{CS}\cos\omega=ae\cos\omega$$

Hence
$$\cos\omega=\frac{m}{ae} \tag{4}$$

and

$$\sin^2\omega=1-\frac{m^2}{a^2e^2} \tag{5}$$

Substituting (5) into (3) and solving for $a$ we have

$$a=\sqrt{\frac{p^2-m^2}{1-e^2}} \tag{6}$$

$p$ is C′T′, and $m$ is C′S′ both of which are known.

The order of solution is, then, equations (1), (6), and (4), which give $e$, $a$, and $\omega$ respectively.

By a simple transformation Laves' (2) may be put into the same form as (6), which is slightly simpler. Equation (4) is decidedly simpler than his (3) from which $\omega$ is determined.

In the *special case* $\omega=90°$, the points C′, P′, and S′ coincide (this means that the major axis is pointed toward the Earth) and (1) becomes indeterminate. Further, in this case $p=b$ and $m=0$, Hence from (6)

$$a=\sqrt{\frac{b^2}{1-e^2}} \tag{7}$$

To determine $e$, let $\tau$ be the ratio of the intervals of time from elongation to elongation such that $\tau<1$. Then from the law of areas we have $\tau$ equal to the ratio of the area described by the radius vector in the true orbit in the shorter interval to the area described by the radius vector in the longer interval, or

$$\tau=\frac{\frac{1}{2}\pi ab-abe}{\frac{1}{2}\pi ab+abe}=\frac{\frac{1}{2}\pi-e}{\frac{1}{2}\pi+e}$$

whence
$$e=\frac{\pi}{2}\cdot\frac{1-\tau}{1+\tau} \tag{8}$$

If the intervals are equal then $\tau=1$ and from (8) $e=0$ and the orbit is a circle.

The periastron passage time, T, (in the case of the ellipse) is the middle instant of the shorter interval between elongation times.

Lick Observatory, May 17, 1928.
Issued June 12, 1928.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

# LICK OBSERVATORY BULLETIN

NUMBER 404

## OBSERVATIONS OF COMETS AND OF THE SATELLITE OF NEPTUNE

By

R. T. CRAWFORD

The following positions were obtained from photographs made with the Crossley reflector:

COMET *j* 1927 (SCHWASSMAN-WACHMANN)

| 1928 U. T. | α1928.0 | δ1928.0 | log pΔ α | log pΔ δ | Comparison Stars |
|---|---|---|---|---|---|
| Jan. 13d 4h 36m 0s | 1h 27m 50s.49 | +19° 17′ 08″.1 | 9.472 | 0.500 | Paris ph. +20°; 1h 28m : 144, 154, 158. |
| 18 4 15 0 | 1 29 10.49 | +19 17 29.9 | 9.465 | 0.497 | ibid. : 167, 171, 174, 175. |
| 20 3 46 0 | 1 29 45.99 | +19 18 01.0 | 9.402 | 0.480 | ibid. : 174, 182, 189, 192. |
| 21 4 42 0 | 1 30 05.65 | +19 18 26.2 | 9.549 | 0.530 | ibid. : 182, 189, 192, 199. |
| Feb. 10 4 15 0 | 1 38 17.16 | +19 38 17.8 | 9.617 | 0.569 | Paris ph. +20°; 1 36 : 219, 225, 230, 231. |
| 11 4 56 0 | 1 38 47.59 | +19 39 56.4 | 9.662 | 0.613 | ibid. : 225, 230, 231. |

COMET *d* 1927 (STEARNS)

| 1928 U. T. | α1928.0 | δ1928.0 | log pΔ α | log pΔ δ | Comparison Stars |
|---|---|---|---|---|---|
| Apr. 26d 10h 59m 0s | 17h 38m 34s.34 | +62° 13′ 02″.0 | 9.107n | 0.565n | Vatican ph. +62°; 17h36m : 70, 79, 84. |
| 28 11 7 0 | 17 37 31.84 | +62 38 11.9 | 8.740n | 0.577n | ibid. : 97,102,107,108,112. |
| May 26 9 12 0 | 17 12 31.19 | +67 02 54.8 | 8.603 | 0.641n | Gr. ph. +66°; 5483 : +67°; 5606, 5612, 5616. |
| June 14 7 14 0 | 16 48 38.56 | +68 19 30.5 | 8.816n | 0.657n | Gr. ph. +68°; 5858, 5862, 5873, 5878. |
| 15 7 9 0 | 16 47 22.04 | +68 21 14.5 | 8.809n | 0.657n | ibid. ; 5858, 5870, 5878. |

COMET *a* 1928 (REINMUTH)

| 1928 U. T. | α1928.0 | δ1928.0 | log pΔ α | log pΔ δ | Comparison Stars |
|---|---|---|---|---|---|
| Mar. 15d 6h 37m 0s | 9d 16m 24s.16 | +23° 29′ 13″.5 | 8.992 | 0.328 | Paris ph. +23°; 9h16m : 40, 44, 46. |
| 15 7 36 0* | 9 16 25.11 | +23 29 16.9 | 9.347 | 0.373 | ibid. : 35, 39, 44. |
| 17 8 36 0 | 9 17 14.68 | +23 32 35.4 | 9.539 | 0.449 | ibid. : 44, 56, 57. |
| Apr. 8 4 50 0 | 9 33 36.61 | +23 01 52.1 | 8.509 | 0.333 | Paris ph. +23°; 9 32 : 57, 63, 74. |
| 9 4 54 0 | 9 34 39.05 | +22 57 54.0 | 8.678 | 0.336 | ibid. : 74, 81, 176. |
| 14 6 17 0 | 9 40 12.65 | +22 34 50.4 | 9.399 | 0.409 | Paris ph. +23°; 9 40 : 128, 130, 133, 138. |
| 23 5 00 0 | 9 51 11.71 | +21 42 57.7 | 9.156 | 0.389 | Paris ph. +22°; 9 52 : 153, 156, 159. |
| May 10 5 11 0 | 10 15 04.61 | +19 31 58.1 | 9.395 | 0.474 | Paris ph. +20°; 10 16 : 44, 46, 49, 50. |
| 14 5 19 0 | 10 21 06.20 | +18 56 05.8 | 9.447 | 0.498 | ibid. : 69, 72, 74, 79. |
| June 6 5 30 0 | 10 57 25.24 | +15 03 37.0 | 9.574 | 0.600 | Bordeaux ph. +16°; 10 56 : 134, 143, 145. |
| 7 5 25 0 | 10 59 02.05 | +14 52 45.4 | 9.570 | 0.600 | ibid. : 156, 160, 163, 165. |
| 14 5 30 0† | 11 10 25.99 | +13 35 09.2 | 9.598 | 0.626 | Bordeaux ph. +14°; 11 12 : 36, 37, 38, 42. |
| 15 5 30 30† | 11 12 04.20 | +13 23 49.9 | 9.601 | 0.630 | ibid. : 44, 49, 53. |

* Plate measured and reduced by L. B. Berman.

† Image of the comet is very faint and difficult to measure.

All of the parallax factors for these observations, as well as those for the visual observations which follow, were computed by Miss Rebecca Jones.

The following positions of Comet *d* 1927 (Stearns) were obtained with the 36-inch refractor. Corrections for differential refraction have been applied.

COMET *d* 1927 (STEARNS)

| 1928 U. T. | Star | No. of Comp. | Comet—Star Δα | Comet—Star Δδ | α1928.0 | δ1928.0 | log pΔ α | log pΔ δ |
|---|---|---|---|---|---|---|---|---|
| May 12d 6h 50m 0s | 1 | 3,*10 | + 0s.01 | +1′ 14″.1 | 17h 27m 28s.38 | +65° 10′ 24″.4 | 9.938n | 0.243n |
| 16 7 25 30 | 2 | 6, 10 | −30.72 | −0 19.9 | 17 23 37.72 | +65 47 15.2 | 9.840n | 0.462n |
| 24 6 34 45 | 3 | 10, 10 | + 7.13 | +2 05.0 | 17 15 00.27 | +66 49 06.1 | 9.883n | 0.453n |
| June 9 6 38 0 | 4 | 10, 10 | −18.75 | +3 11.8 | 16 55 05.20 | +68 07 28.5 | 9.600n | 0.622n |
| 16 10 8 12 | 5 | 10, 10 | + 7.73 | −0 16.1 | 16 45 57.26 | +68 22 50.3 | 9.944 | 0.438n |

* After three settings I could not tell which was preceding, the comet or the star. So I consider this as good as ten settings where there is an appreciable difference in right ascension.

MEAN PLACES OF COMPARISON STARS FOR 1928.0

| Star | α | δ | Authority |
|---|---|---|---|
| 1 | 17h 27m 28s.37 | +65° 09′ 10″.3 | A. G. Christiania 2665. |
| 2 | 17 24 8.44 | +65 47 35.1 | Crossley photographic by L. B. Berman. −14½ mag. |
| 3 | 17 14 53.14 | +66 47 01.1 | Greenwich astrographic +66° : 5463. |
| 4 | 16 55 23.95 | +68 04 16.7 | A. G. Christiania 2563. |
| 5 | 16 45 49.53 | +68 23 06.4 | A. G. Christiania 2536. |

## SATELLITE OF NEPTUNE

| 1928 U. T. | *p* | *s* | Wt. |
|---|---|---|---|
| Feb. 13d 6h 35m | 259°.3 | ..... | 2 |
| 13 7 05 | ..... | 9″.34 | 2 |
| 19 7 12 | 244.2 | ..... | 1 |
| 19 7 34 | ..... | 8.57 | 1 |
| Mar. 10 10 33 | 100.5 | 11.23 | 1 |
| 18 7 24 | 320.8 | ..... | 2 |
| 18 7 35 | ..... | 16.83 | 2 |
| Apr. 15 7 44 | 41.1 | 8.22 | 5 |
| 22 6 29 | 330.2 | ..... | 1 |
| .... 6 53 | ..... | 15.94 | 1 |
| May 2 5 00 | 110.7 | 12.09 | 2 |
| 12 4 58 | 180.6 | 10.94 | 2 |
| 16 5 19 | 316.0 | 16.22 | 3 |
| 24 4 51 | 166.1 | 12.83 | 5 |
| June 1 5 01 | 57.3 | 8.37 | 3 |

The weights assigned are my estimates of the relative value of the measures based upon seeing conditions, swaying of the telescope in the wind, and number of settings made. Conditions for the observations weighted 5 were excellent.

All of the satellite observations were made with the 36-inch refractor with a power of 270.

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June 18, 1928.

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UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

# LICK OBSERVATORY BULLETIN

NUMBER 405

## ON THE NUMBER OF SOLUTIONS IN LEUSCHNER'S DIRECT METHOD OF DETERMINING PARABOLIC ORBITS

BY

CHARLES H. SMILEY

The object of this paper is a rigorous discussion of a sextic equation which arises in the problem of determining the geocentric distance of a body from three complete observations, when the body is assumed to be moving in a parabolic path about the Sun. One point of especial interest, from the practical point of view, is the consideration of the possibility of the existence of more than one real positive solution.

This same equation arises in both the Laplacean and Olbers' method of orbit determination, and has been discussed, among others, by Oppolzer, Charlier, Leuschner, and Banachiewicz.

Oppolzer pointed out that three solutions are sometimes possible. The discussion given by him in his "Bahnbestimmung" as improved by Pasquier,[1] is rigorous, but requires the determination of all the real roots of a cubic equation and the substitution of these roots in the sextic. Geometrically, this is equivalent to finding the coordinates of the three real bend-points (if there be three) of the curve,

$$y = f(z) \qquad (1)$$

where $f(z) = 0$ is the sextic under consideration. Oppolzer's discussion as revised by Pasquier may be considered as satisfactory from a purely theoretical point of view, but practically, it is rather tedious in its application.

M. Thadée Banachiewicz[2] has given two graphical treatments of this equation. In one, he uses a logarithmic scale and a method of trial and error. In the other, he uses a family of generalized Cassinian ovals[3] and a straight line. He gives a set of necessary conditions [without proof] for the existence of a multiple solution which show the relative rarety of such a case. However, for cases near the border line, the discussion of his conditions is not complete, in that, as he states, they are offered only as necessary conditions.

Banachiewicz[4] has questioned the validity of Leuschner's criterion for a multiple solution and has offered a fictitious case in proof of his point. The conditions given in Volume VII of the *Publications of the Lick Observatory* would indicate a single solution for this case while, as Banachiewicz points out, there are three.

In his paper, "Second Note on Multiple Solutions in the Determination of Orbits from Three Observations," C. V. L. Charlier[5] shows that, if the sextic has three positive solutions, then the only one which is of any practical value, is the one obtained by elimination between the sextic equation of the parabolic case and the eighth degree equation which arises in the general case.

This conclusion has been criticized by Th. Banachiewicz[6] in an article in which he has shown by means of three fictitious positions in the ecliptic that there exist

[1] Oppolzer-Pasquier, *Traité de la Détermination des Orbites*, 314 ff.

[2] Th. Banachiewicz, *C. R. Paris Acad.*, **161**, 122 ff, 1915.

[3] Th. Banachiewicz, Sur les points d'inflexion des courbes généralisées de Cassini; *Bull. international de l'Acad. Polonaise des Sci. et des Let.*, Série A, 1921, 42 ff.

[4] Th. Banachiewicz, *Circ. de l'Obs. de Cracovie*, 1922, No. 11, 5.

[5] C. V. L. Charlier, On multiple Solutions in the Determination of Orbits from three Observations, *Mon. Not., R.A.S.*, **71**, 120 ff, 1910.
Second Note on multiple Solutions in the Determination of Orbits from three Observations, *loc. cit.*, **71**, 454 ff, 1911.

[6] Th. Banachiewicz, Sur un théorème de Legendre relatif à la détermination des orbites cométaires, *Bull. international de l'Acad. Polonaise des Sci. et des Let.*, Série A, 1924, 13 ff.

cases where three observations would lead to three well determined parabolic orbits, each satisfying the given data of the problem.

In the case of a parabolic orbit Luc Picart[7] determines the geocentric distance of the object from an equation of the first degree which arises in the finding of the common root between Cauchy's cubic equation[8] and the sextic by the method of the greatest divisor.

The roots of this sextic equation are determined by Leuschner[9] as the abscissae of the intersections of a parabola and an asymptotic curve. In general, the graphical solution of this problem as given by Leuschner and Bernstein,[10] is the most satisfactory means of determining the number of roots and their approximate values. However, in those cases where three real positive solutions exist, and two of them are very nearly equal, it is difficult to ascertain by graphical methods whether there are three real and distinct roots; three real roots, two of which are coincident; or one real root and two imaginary roots. In this case, the solution of the cubic equation which is necessary for the application of Oppolzer's criterion for multiple roots, becomes quite difficult. The reason for this will become evident in the discussion that follows.

In his discussion of the possibility of a multiplicity of positive real roots, Leuschner[11] bases his work upon a consideration of points of contact of the parabola and the asymptotic curve mentioned above. It should be borne in mind that by "the number of possible contacts," he means the number of contacts possible when either curve is translated in a direction parallel to the asymptote while the other is kept fixed. This kind of translation would amount to a variation of the parameter $p$ in a particular way. However, the equations (41a) and (41b) are the conditions for simultaneous points of contact. That is, for a given fixed set of values for the parameters, the root common to (41a) and (41b), [if there be one], will be the value of $z$ for which the two curves are in contact. It should be noted that the satisfaction of (42) by a value $z_1$ is necessary, but not sufficient that $z_1$ be a root common to the two equations, (41a) and (41b).

Remembering that a root common to an equation, $f(z)=0$, and the derived equation, $f'(z)=0$, will be a double root of the first equation, we see that any root common to (41a) and (41b) will be a double root of (41a). But (41a), when expanded, is the sextic considered below. This sextic must have an odd number of positive roots and an odd number of negative roots and no zero roots. It cannot have more than four real roots. Hence two or more simultaneous points of contact are impossible.

In view of these considerations, the sufficiency of the theoretical conditions given by Leuschner for a triple solution is questionable. It should be pointed out that this does not affect the rigidity of his graphical method of determining the roots of the sextic.

It was at Professor Leuschner's suggestion that I have undertaken a study of this equation. I have chosen to base my discussion on an algebraical, rather than a geometrical, treatment of the equation. I shall follow his notation, except that the primes will be dropped from the p's and q's, and I shall introduce a new constant

$$k^2=p^2+q^2$$

These three things are to be accomplished:

(i) A determination of the possible number of positive real roots of the sextic.

(ii) The establishment of a practical means of determining the exact number of positive roots in any case.

(iii) A physical interpretation of some of the important constants entering the equation.

## I

The first point is discussed in the customary manner. There is the usual consideration of maxima and minima, etc. The second point is taken care of by Sturm's Functions. These give us the exact number of roots of the sextic, both positive and negative. At first sight, some of the quantities involved seem rather complicated, but they are very easily computed with the aid of a computing machine, Crelle's *Rechentafeln*, or a slide-rule. The particular advantage of this method of discussing the number of roots is that, while the conditions obtained are rigorous, they are not too complicated for practical application. The discussion of the third point is based upon the definitions of the observational constants which appear in the basic equation.

There are three forms in which this equation for determining the geocentric distance of a comet appears in Leuschner's memoir:

$$a^2\rho_o^2-2b\rho_o+G_s^2=\frac{2}{r_o} \tag{2}$$

$$z^2-2pz+p^2+q^2=\frac{h}{\sqrt{z^2-2cz+1}} \tag{3}$$

$$(z^2-2pz+k^2)^2(z^2-2cz+1)-h^2=0 \tag{4}$$

[7] Luc Picart, *Calcul des Orbites et des Ephémérides*, p. 224 ff. Sur la suppression des essais dans le calcul des orbites paraboliques, *Bull. Astron.*, **16**, 412 ff, 1899.

[8] A. Cauchy, Méthodes Nouvelles pour la détermination des Orbites des corps célestes et en particulier des comètes, *Oeuvres complètes*, Série I, Tome X, 196 ff.

[9] A. O. Leuschner, *Publ. Lick Obs.*, **7**, 284 ff.

[10] A. O. Leuschner, B. A. Bernstein, *Bull. Am. Math. Soc.*, **18**, No. 6.

[11] A. O. Leuschner, *Publ. Lick Obs.*, **7**, 284 ff.

The form (2) is the basic equation. It is essentially a statement of the vis-viva integral and hence is an equation in linear velocities. In it, $a^2$ and $b$ are constants easily computed from the observational data. $G_s^2$ is the square of the velocity of the Earth in its orbit at the epoch, and may be obtained from any of the standard ephemerides. $\rho_o$ and $r_o$ are the unknown geocentric and heliocentric distances of the body.

Replacing $r_o$ in (2) by its value from the triangle equation:

$$r_o^2=\rho_o^2+R^2-2\rho_o R\cos\psi \qquad (5)$$

then dividing by $a^2R^2$, and introducing as a new variable,

$$z=\frac{\rho_o}{R}$$

we obtain (3).

$R$ is the known heliocentric distance of the Earth at the epoch, and $c$ is the cosine of the angle $\psi$ subtended at the Earth by the Sun and body.

Equation (4) follows at once after a very few simple algebraic manipulations. It may be expanded, giving:

$$f(z)=z^6-A_1z^5+A_2z^4-A_3z^3+A_4z^2-A_5z+A_6=0 \quad (6)$$

where

$$A_1=4p+2c,\ A_4=k^4+8cpk^2+4p^2+2k^2$$
$$A_2=4p^2+2k^2+8cp+1,\ A_5=2k^2(ck^2+2p)$$
$$A_3=4p+8cp^2+4ck^2+4pk^2,\ A_6=k^4-h^2$$

Let us first examine the constant term,

$$A_6=k^4-h^2=(p^2+q^2)^2-h^2$$
$$=\frac{G_s^4}{a^4R^4}-\frac{4}{a^4R^6}=\frac{R^2G_s^4-4}{a^4R^6}$$
$$=\frac{R^2(R-1)^2-4}{a^4R^6}=\frac{(2-R)^2-4}{a^4R^6}$$
$$=\frac{R(R-4)}{a^4R^6} \qquad (7)$$

Since $R$ is always positive and less than four, this constant term is always negative.

Then $f(0)$ is negative, $f(+\infty)$ is positive and $f(-\infty)$ is positive. The function $f(z)$ is continuous, so the equation of the sixth degree, $f(z)=0$, has always an odd number of positive roots and an odd number of negative roots.

Next let us consider the maxima and minima of the curve

$$y=f(z)\equiv[z^2-2pz+k^2]^2(z^2-2cz+1)-h^2 \qquad (8)$$

$$\frac{dy}{dz}=2[(z-p)^2+q^2]\left\{(z-c)[(z-p)^2+q^2]+2(z-p)[(z-c)^2+s^2]\right\}$$

Setting $\frac{dy}{dz}$ equal to zero, the factor $[(z-p)^2+q^2]$ gives two imaginary roots, provided $q$ is not zero. If $q$ is zero, this factor gives as a double root, $z=p$. Hence the curve, $y=f(z)$, cannot have more than three real bend-points, and the equation, $f(z)=0$, cannot have more than four real roots. Furthermore, since there is always at least one negative root, there cannot be more than three real positive roots.

Again, from the above expression for $\frac{dy}{dz}$, we see that it is negative for all values of $z$ less than both $c$ and $p$. Hence there can be, at most, one root of our equation, $f(z)=0$, which is less than both $c$ and $p$. If there be such a root, it must be negative, and we are not interested in it.

Likewise, for all values of $z$ greater than both $c$ and $p$, $\frac{dy}{dz}$ is positive and there can be, at most, one root of $f(z)=0$ greater than both $c$ and $p$. There will be such a root in case

$$f(d)\equiv[(d-p)^2+q^2]^2[(d-c)^2+s^2]-h^2<0 \qquad (9)$$

where $d$ is the larger (algebraically) of the two quantities, $c$ and $p$.

We might note at this point that if $c$ and $p$ are both negative, there will be one and only one real positive root of $f(z)=0$. [For there is always at least one positive root, and there can be at most one root greater than both $c$ and $p$.]

Now for bend-points, $\frac{dy}{dz}=0$, or $3f_1(z)$

$$\equiv(z-c)[z^2-2pz+k^2]+2(z-p)[z^2-2cz+1]=0$$
$$=3z^3-(5c+4p)z^2+(k^2+6cp+2)z-(ck^2+2p)=0 \qquad (10)$$

And the curve, $y=f(z)$, will have three bend-points or one, according as the discriminant of this cubic is greater than or less than zero. The discriminant is:

$$D=4[13c^2k^4+225c^4p^2+180c^3pk^2+10cpk^4-288c^3p^3$$
$$-120c^2p^2k^2+4p^2k^4+144c^2p^4-16cp^3k^2-3k^6$$
$$-125c^4k^2-100c^3p+160c^2k^2-198c^2p^2-176cpk^2$$
$$-18k^4+264cp^3+124p^2k^2-128p^4+25c^2+94cp$$
$$-11p^2-36k^2-24] \qquad (11)$$

The equation, $f_1(z)=0$, is the cubic which must be solved in order to apply Oppolzer's criterion for three positive solutions of the sextic. From the discussion above, we see that all of the roots of this cubic lie between $z=c$ and $z=p$. And, when there are three,

they must then lie comparatively close together. It is in such a case that the solution of a cubic by the methods of Newton and Horner becomes difficult.

Furthermore, $f_1'(z)$ must vanish between every two successive roots of $f_1(z)=0$.

$$3f_1'(z)\equiv 9z^2-(10c+8p)z+k^2+6cp+2 \qquad (12)$$

Then $f_1'(z)=0$ will have two real roots or two imaginary roots according as

$$(5c+4p)^2-(9k^2+54cp+18)>0 \text{ or } <0$$

or as $$7(p-c)^2-9(2s^2+q^2)>0 \text{ or } <0$$

where $s$ is the sine of $\psi$.

Then if

$$7(p\text{-}c)^2-9(2s^2+q^2)<0, \text{ our sextic} \qquad (13)$$

has but two real roots, one negative and one positive.

And if

$$7(p\text{-}c)^2-9(2s^2+q^2)>0, \text{ it may have} \qquad (14)$$

two or four real roots, of which an odd number are positive.

## II

Sturm's Theorem will be used to determine the exact number of positive roots of the sextic. Sturm's Functions for $f(z)=0$ are:

$$\begin{aligned} f(z)&=z^6-A_1z^5+A_2z^4-A_3z^3+A_4z^2-A_5z+A_6\\ f_1(z)&=z^3-B_1z^2+B_2z-B_3\\ -R(z)&=-(Hz^2+Jz+K)\\ -S(z)&=-(Mz+N)\\ +T&=+T \end{aligned} \qquad (15)$$

where the $A$'s are as defined above, and the remaining constants are defined by the following equations:

$$\begin{aligned} B_1&=\frac{5c+4p}{3} & D_i&=A_i-B_i\\ B_2&=\frac{k^2+6cp+2}{3} & E_2&=D_2-D_1B_1\\ B_3&=\frac{ck^2+2p}{3} & F_3&=E_3-E_2B_1 \end{aligned} \qquad (16)$$

$$\begin{aligned} H&=A_4-D_1B_3-E_2B_2-F_3B_1\\ J&=E_2B_3+F_3B_2-A_5\\ K&=A_6-F_3B_3\\ M&=H^2B_2+HB_1J+J^2-HK\\ N&=-H^2B_3+HB_1K+JK\\ T&=M^2K-MNJ+N^2H \end{aligned}$$

The signs of these functions are given for

| $z=$ | $-\infty$ | $0$ | $+\infty$ |
|---|---|---|---|
| by | $+$ | $-$ | $+$ |
| | $-$ | $-B_3$ | $+$ |
| | $-H$ | $-K$ | $-H$ |
| | $+M$ | $-N$ | $-M$ |
| | $+T$ | $+T$ | $+T$ |

Then since four variations of sign are all that are possible, $V_{-\infty}=4$ and $V_{+\infty}=0$ for four real roots of $f(z)=0$. And a necessary and sufficient condition that the sextic, $f(z)=0$, have four real distinct roots is that the following three inequalities be satisfied:

$$H<0;\ M<0;\ T>0; \qquad \text{(i)} \quad (17)$$

And if these inequalities are satisfied, the further condition which must be satisfied if three of the four roots are to be positive is that either

$$\left.\begin{aligned} &B_3<0 \text{ and not both } K \text{ and } N \text{ are negative; (ii) or}\\ &B_3>0 \text{ and } K<0 \text{ and } N>0. \qquad \text{(iii)} \end{aligned}\right\} (18)$$

If and only if the conditions (i) and (ii) or the conditions (i) and (iii) are satisfied, our sextic equation, $f(z)=0$ has exactly three real distinct positive solutions. If $T=0$, the sextic has coincident real roots. If $T=0$, and the necessary and sufficient conditions for three distinct positive solutions are not satisfied, then the sextic has one and only one positive solution.

Let us now apply this criterion to the case of Comet 1910a. The numerical data with which we start, have been taken from Miss Levy's papers.[12]

| | |
|---|---|
| $\log c = 9.997151$ | $\log p = 9.24434$ |
| $\log h = 8.92332$ | $q^2 = 0.01176$ |

| | | |
|---|---|---|
| $c=0.99346$ | $A_1=+2.68900$ | $B_1=1.88979$ |
| $p=0.17552$ | $A_2=+2.60336$ | $B_2=1.02960$ |
| $h=0.08382$ | $A_3=+1.14600$ | $B_3=0.13111$ |
| $k^2=0.04257$ | $A_4=+0.26955$ | $D_1=0.79921$ |
| $q^2=0.01176$ | $A_5=+0.03349$ | $D_2=1.57376$ |
| $p^2=0.03081$ | $A_6=-0.00522$ | $D_3=1.01489$ |

| | | |
|---|---|---|
| $E_2=0.06342$ | $H=-0.03692$ | $M=-0.000\ 1526$ |
| $E_3=0.19202$ | $J=+0.04913$ | $N=+0.000\ 1238$ |
| $F_3=0.07217$ | $K=-0.01468$ | $T=+2052\times10^{-14}$ |

Noting the signs of these quantities, we see that our conditions (i) and (iii) are satisfied. Hence our equation in this case will have three real positive roots.

[12] Sophia H. Levy, *Publ. Lick Obs.*, **7**, 418. See also *Lick Obs. Bull.*, **6**, 136 ff, 1910.

It should be noted that the work is to be carried to a certain prearranged number of *significant figures*, rather than to a specified number of decimal places.

## III

Let us now consider the physical meaning of the quantity, $b$, which appears in equation (1), as it is defined in Leuschner's development of this equation.

$$b=[a_x X_0'+a_y Y_0'+a_z Z_0'] \cos \delta_0 \text{ (by definition)} \quad (19).$$

But since

$$\begin{aligned} x_0' &= a_x \sigma_0 - X_0' \\ y_0' &= a_y \sigma_0 - Y_0' \\ z_0' &= a_z \sigma_0 - Z_0' \end{aligned} \quad (20),$$

it follows that

$$\begin{aligned} a_x &= \frac{1}{\sigma_0}(x_0'+X_0') \\ a_y &= \frac{1}{\sigma_0}(y_0'+Y_0') \\ a_z &= \frac{1}{\sigma_0}(z_0'+Z_0') \end{aligned} \quad (21).$$

And

$$\begin{aligned} b &= [(x_0'+X_0')X_0'+(y_0'+Y_0')Y_0'+(z_0'+Z_0')Z_0'] \frac{\cos \delta_0}{\sigma_0} \\ &= \frac{1}{\rho_0}[G_s^2+x_0'X_0'+y_0'Y_0'+z_0'Z_0'] \\ &= \frac{1}{\rho_0}\left[G_s^2+\frac{x_0'X_0'+y_0'Y_0'+z_0'Z_0'}{G_0 G_s} \cdot G_0 G_s\right] \\ &= \frac{G_s}{\rho_0}[G_s+G_0 \cos \phi] \end{aligned} \quad (22),$$

where $\phi$ is the angle between the two positive directions of motion of Earth and comet.

Then $b$ will be negative, when and only when

$$\cos \phi < \frac{-G_s}{G_0} \quad \text{or} \quad G_0 \cos \phi < -G_s \quad (23)$$

This means that the component of the comet's velocity in the direction of the instantaneous line of motion of the Earth is negative and numerically greater than the velocity of the Earth in its orbit.

Furthermore, since

$$\left.\begin{aligned} &\cos \phi > -1 \\ &\text{we shall have } -\frac{G_s}{G_0} > -1 \\ &\text{or} \quad G_s < G_0 \end{aligned}\right\} \text{ when } b \text{ is negative.} \quad (24)$$

And if the second body is moving in a parabola about the Sun, as is assumed,

$$r_0 < \frac{2}{G_s^2} \text{ and hence } z_0 < 3. \quad (25)$$

Again, let us consider the physical meaning of $a^2$.

$$\begin{aligned} a^2 &= [a_x^2+a_y^2+a_z^2] \cos^2 \delta_0 \text{ (by definition)} \\ &= \frac{\cos^2 \delta_0}{\sigma_0^2}[(x_0'+X_0')^2+(y_0'+Y_0')^2+(z_0'+Z_0')^2] \\ &= \frac{1}{\rho_0^2}[G_0^2+G_s^2+2G_0 G_s \cos \phi] \end{aligned} \quad (26)$$

I wish to acknowledge my indebtedness to Professor Leuschner for the inspiration and guidance he has given me in this and other problems. I am indebted also to Professor R. H. Sciobereti for friendly criticism and advice.

Berkeley Astronomical Department,
August 31, 1927.
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UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

# LICK OBSERVATORY BULLETIN

NUMBER 406

## ELEMENTS OF MINOR PLANET Y. O. 26 [1928DJ] = (841) ARABELLA

BY

MAUD WORCESTER MAKEMSON

Minor Planet Y. O. 26 was discovered 1928 February 24 by Van Biesbroeck at the Yerkes Observatory and was followed photographically until April 22, when it became too faint for further observation. The planet had passed opposition on February 20 and reached the stationary point west on April 1.

The orbit given below for Y. O. 26 establishes its identity with (841) *Arabella* which had not been observed since the discovery opposition, 1916.

Planet 1916 AL, later known as (841) *Arabella*, was found by Wolf at Heidelberg on October 1, 1916, and was observed photographically by Thiele at Bergedorf on October 16, 17, 27 and November 26.[1] Elements derived by Berberich from the positions of 1916 October 1, 27 and November 26 represented the observation of October 17 with residuals of $+3''.1$ in $\lambda$ and $+1''.6$ in $\beta$. It will be shown that his value for the mean motion ($1050''.896$) is about $3''$ too large. The uncertainty of this element and of the eccentricity is probably due to the position of the planet in the orbit ($M = 258°$ on October 1). Berberich's orbit is still used by the Recheninstitut for the computation of ephemerides.[2]

In March 1918, Wolf made an unsuccessful search for (841) *Arabella* on two successive nights. An ephemeris based on the elements of Y. O. 26 gives a correction of $-5^m.1$, $+44'$ to the predicted position of March 12[3] and indicates that the planet may yet be found on the Bruce plate No. 8 of March 12.[4]

In the perihelion opposition of January 1921 ($M = 354°$) Reinmuth was unable to find *Arabella* in the predicted position, twenty days after opposition. The reason may lie in the proximity of the planet to the Milky Way, near *Gemini*, to the rapid motion, and to the reputed poor quality of the plate.[5] The elements of Y. O. 26 give a correction of $-4^m.6$, $+6'$ to the ephemeris place for January 26, 1921.[6]

During February 1928, (841) *Arabella* came into opposition in the same part of the orbit as in 1918. Reinmuth looked for the planet on February 24 without success. The very same night, Wolf found a new object which received the provisional designation 1928 DJ and proved to be the planet that Van Biesbroeck had photographed a few hours earlier, Y. O. 26. No further positions of 1928 DJ were measured at Heidelberg. On February 25 Reinmuth found an object which he suspected to be (841) *Arabella*, but the identity was later disproved and the new planet given the provisional designation of 1928 $DB_1$.[7] The correction to the ephemeris of (841) *Arabella* for 1928 February 24[8] given by the elements of Y. O. 26 is $-20^m.2$, $+2°10'$.

Successively improved elements and ephemerides were computed for Y. O. 26 by Leuschner's Short Method from increasing arcs, as an aid in following the planet. Upon receipt of the April positions, the middle date was changed to March 21.0 and the orbit differentially corrected on the basis of three normal places, based on the positions for February 26 and 27; March 18, 21 and 23; and April 22 (2 observations).

[1] *A. N.* **203**, 233, 1917; **217**, 287–288, 1922.
[2] *A. N.* **205**, 201, 1918; *Kl. Pl.* 1918–28; *V. R. I.* **45**, 1926.
[3] *Kl. Pl.* 1918.
[4] *A. N.* **206**, 120, 1918.
[5] *B. Z.* 1921, 5.
[6] *Kl. Pl.* 1921.
[7] *B. Z.* 1928, 26.
[8] *Kl. Pl.* 1928.

OBSERVATIONS OF Y. O. 26, BY VAN BIESBROECK, YERKES OBSERVATORY, FOR 1928.0

| | U. T. | α | δ | | | |
|---|---|---|---|---|---|---|
| (1) | Feb. 24.22523 | $10^h\ 00^m\ 46^s.90$ | +15° 16′ 32″.7 | 8.58n | 0.603 | photo. |
| (2) | Feb. 25.26022 | 9 59 40.61 | 19 34.7 | 8.85+ | 0.604 | " |
| (3) | Feb. 26.23216 | 58 39.92 | 22 23.7 | 7.18 | 0.601 | visual |
| (4) | Feb. 27.19381 | 57 40.05 | 25 04.9 | 8.977n | 0.604 | " |
| (5) | Mar. 1.36007 | 54 28.17 | 33 05.7 | 9.545 | 0.665 | photo. |
| (6) | Mar. 15.21212 | 43 08.02 | 51 18.2 | 9.054 | 0.600 | visual |
| (7) | Mar. 17.22302 | 41 55.27 | 51 27.7 | 9.222 | 0.607 | photo. |
| (8) | Mar. 18.10243 | 41 28.20 | 51 18.2 | 9.201n | 0.606 | " |
| (9) | Mar. 21.21522 | 39 58.78 | 49 50.7 | 9.252 | 0.610 | " |
| (10) | Mar. 23.14258 | 39 13.55 | 48 10.5 | 7.72n | 0.637 | " |
| (11) | Apr. 10.13819 | 38 20.77 | 05 38.9 | 9.057 | 0.611 | " |
| (12) | Apr. 11.09452 | 38 36.05 | +15 02 6.4 | 6.98n | 0.606 | " |
| (13) | Apr. 22.11854 | 43 25.31 | +14 13 45.3 | 9.150 | 0.626 | " |
| (14) | Apr. 22.15257 | 9 43 26.43 | +14 13 36.5 | 9.355 | 0.640 | " |

ELEMENTS, 1928.0

Epoch 1928 March 21.0, U.T.

M = 39° 41′ 56″.2
ω = 119 05 11.9
Ω = 354 29 39.3
i = 3 48 37.3
e = 0.0695637
μ = 1047″.653
log a = 0.353193

The observations are represented by these elements as follows:

| | Δα | Δδ | |
|---|---|---|---|
| Feb. 24.22 | − 2″.1 | +14″.5* | photo. |
| Feb. 25.26 | − 7.0 | + 2.8 | " |
| Feb. 26.23 | 0.0 | + 1.6 | visual |
| Feb. 27.19 | + 0.8 | + 0.7 | " |
| Mar. 1.36 | + 1.2 | − 1.9 | photo. |
| Mar. 15.21 | − 3.0 | + 0.3 | visual |
| Mar. 17.22 | −22.4 | + 0.2 | photo. |
| Mar. 18.10 | + 2.2 | + 0.1 | " |
| Mar. 21.21 | − 0.5 | + 0.6 | " |
| Mar. 23.14 | − 1.2 | + 2.4 | " |
| Apr. 10.13 | − 1.1 | + 0.1 | " |
| Apr. 11.09 | + 1.3 | − 2.5 | " |
| Apr. 22.11 | + 1.9 | − 1.6 | " |
| Apr. 22.15 | + 1.6 | + 0.6 | " |

* The discovery observation was too far from the center of the plate to be reliable.

These residuals admit of slight improvement which, however, is being deferred until the plates for 1918 and 1921 shall have been examined.

COMPARISON OF ELEMENTS FOR 1916.0

| Epoch | (841) Berberich 1916 Oct. 1.5 G.M.T. | Y. O. 26 Makemson 1916 Oct. 1.5 G.M.T. |
|---|---|---|
| $M$ | 258° 01′ 14″.6 | 260° 55′ 48″.5 |
| $\omega$ | 120 57 55.2 | 119 05 11.4 |
| $\Omega$ | 354 43 34.3 | 354 19 36.7 |
| $i$ | 3 49 30.2 | 3 48 31.7 |
| $\varphi$ | 3 42 16.8 | 3 58 45.5 |
| $\mu$ | 1050″.896 | 1047″.653 |
| log $a$ | 0.352298 | 0.353193 |

The elements of Y. O. 26 represent the 1916 observations of (841), without perturbations, as follows:

| 1916 | October 16 | October 17 |
|---|---|---|
| Δα | +15$^s$ | +15$^s$ |
| Δδ | −1′.9 | −1′.9 |

The following approximate ephemerides for the 1918 and 1921 oppositions are published in the hope that the planet may be found on plates taken during these periods. Such observations, if accurate, will be utilized in improving the orbit with the consideration of general perturbations by *Jupiter* from the tables by Strömberg for planets of mean motion 1050″. It is of interest to note that the period of (841) *Arabella* is almost in the exact commensurability 2/7 with the period of Jupiter.

EPHEMERIS FOR 1918.0

| 1918 G.M.T. | α | δ | log Δ |
|---|---|---|---|
| March 8.5 | $11^h\ 52^m.9$ | +0° 55′ | 0.073 |
| March 10.5 | 11 50.9 | +1 03 | 0.072 |
| March 12.5 | 11 48.9 | +1 11 | 0.072 |
| March 14.5 | 11 46.9 | +1 19 | 0.071 |

EPHEMERIS FOR 1921.0

| 1921 G.M.T. | α | δ | log Δ |
|---|---|---|---|
| January 1.5 | $7^h\ 26^m.8$ | +28° 41′ | 0.051 |
| January 9.5 | 7 17.2 | +28 53 | 0.049 |
| January 17.5 | 7 07.6 | +28 56 | 0.053 |
| January 25.5 | 6 59.2 | +28 48 | 0.064 |

There is a difference of over two magnitudes between the predicted brightness of (841) and the observed brightness of Y. O. 26 in 1928.

Predicted opposition magnitudes for (841) based on Wolf's value of 13.3 in 1916 have been published in *Kleine Planeten* as follows:

| Year | Magnitude | Mean anomaly |
|---|---|---|
| 1918 | 13.0 | 53° |
| 1919 | 13.6 | 202 |
| 1921 | 12.9 | 354 |
| 1922 | 13.6 | 146 |
| 1923 | 13.1 | 294 |
| 1925 | 13.3 | 90 |
| 1926 | 13.5 | 238 |
| 1928 | 12.9 | 33 |

These magnitudes correspond to $g = 11.0$.

Van Biesbroeck estimated the magnitude of Y. O. 26 near opposition as 15 on February 24, 1928, while Wolf assigned a value of 16 on the same date. Van Biesbroeck's estimate of 15 corresponds to $g=13.1$ for Y. O. 26 while Wolf's value would give $g=14.1$, with a mean value $g=13.6$. The difference of 2.6 magnitudes of the values of $g$ for (841) and Y. O. 26 is more than might be expected, unless Wolf's estimate of 13.3 in 1916 is subject to a correction, which possibly may be determined, if magnitude estimates, so far not published, can be obtained for the Bergedorf observations in 1916.

The magnitude of Y. O. 26 on April 22, 1928, was estimated as 17.5 by Van Biesbroeck, corresponding to $g=14.7$. The change of 1.1 magnitude from February 24 to April 22 may be due chiefly to phase angle.

Berkeley Astronomical Department,
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UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

# LICK OBSERVATORY BULLETIN

NUMBER 407

## THE SPECTRA OF MINOR PLANETS

BY

N. T. Bobrovnikoff

It has been tacitly assumed that minor planets should have a purely reflected solar spectrum and the only attempt to obtain information on the subject was made by H. C. Vogel[1] some fifty-five years ago. He observed *Vesta* visually on a single night and could recognize Hβ and two bands at λλ 577 and 518 μμ. His inference was that *Vesta* possesses an atmosphere. He also observed *Flora*, but in this case he could not even see the colors in the spectrum. In spite of the considerable interest of Vogel's results they have been entirely overlooked although *Vesta* in opposition is accessible to many spectrographs of moderate power.

However, the difference in the albedo of asteroids shows that they cannot all reflect solar light in the same way. Furthermore, many asteroids show remarkable periodic and non-periodic variations in light which fact suggests an irregular reflection from their surface. It may be surmised that the reflectivity of asteroids depends in some way on the wave-length of the incident solar light. Consequently we may expect to gain some information on the physical properties of asteroids from their spectra.

On the other hand it is becoming increasingly clear that there is no essential difference between comets and asteroids. Recently a number of objects of intermediate class have been discovered. It seems now that asteroids may be comets which have lost their gaseous envelopes ("dead comets"), or fragments of a single giant comet. In view of these considerations a research on the spectra of asteroids was desirable.

### Photographic Region

The writer has undertaken a study of the spectra of twelve minor planets whose visual magnitude is brighter than 10.0. The instruments used were the 36-inch Lick refractor in conjunction with the one-prism spectrograph. One light prism of 63° angle with a 12-inch camera was employed. This gives the dispersion of 11.4 mm between Hβ and Hε. The whole spectrum is in tolerably good focus between these lines. Plates used for this purpose were Eastman 40, developed under as nearly uniform conditions as was possible, to ensure their photometric value. All necessary precautions as to the temperature of developer, time of development, etc. were taken. The developer used was D-61a recommended by the Eastman Kodak Company.

Although the spectra of asteroids did not show any new lines or bands as compared with the lunar spectrum, it soon became evident that they differed from it in the distribution of intensity. It is difficult to compare the spectrum of an extended object like the Moon with the spectrum of a star-like asteroid especially in the ultra-violet region where the spectrum of the Moon is always brighter owing to the use of a visual refractor. Therefore several stars of the G0 type and of approximately the same absolute magnitude as the Sun (M 4.85) have been selected.[2] It can be safely assumed that they have the same distribution of intensity in the continuous spectrum as the Sun. These stars are given in Table 1.

TABLE 1

| Star | 1900.0 α | 1900.0 δ | m | M |
|---|---|---|---|---|
| λ Serpentis | $15^h\ 41^m.6$ | + 7° 40′ | 4.4 | 4.4 |
| 15 Sagittae | 19 59.6 | +16 46 | 5.9 | 5.0 |
| $\omega_2$Scorpii | 16 01.5 | −20 36 | 4.6 | |

[1] *Untersuchungen über die Spectra der Planeten*, Leipzig, 1874, p. 23.

[2] Absolute magnitudes were taken from *Mt. Wilson Contr.*, No. 199, 1921.

The last named star was used only as a check for the atmospheric absorption since no information as to its absolute magnitude could be secured.

The spectra of the stars were measured in the Hartmann microphotometer with a wedge in four places at approximately λλ 4700, 4300, 4000, and 3945 A. The photographic densities at these places are denoted respectively as *a*, *b*, *c*, *d*, in Table 2. The exact places at which these measurements were to be taken were determined by using a micrometer screw which successively brought into the field of view different portions of the spectrum. Place *a* was chosen near the maximum of intensity of the spectrum. Then the densities at *b*, *c*, *d*, can be considered as functions of *a*. Figure 1 shows these densities as ordinates while density *a* was taken as abscissa. As is well known the densities between 55 and 30 can be estimated within several tenths of a unit, but on both sides of this interval uncertainty increases rapidly. Consequently the lower parts of the curves are of small value.

It will be seen that all points lie on rather smooth curves and the distance of individual points from the mean curves is of the order of several tenths of a unit, which is within the limits of error of measurement. The spectra were obtained under widely different conditions as to the seeing and transparency and this gives assurance that the ordinary changes in the atmosphere were not of much significance in the work.

TABLE 2

DENSITIES IN THE SPECTRA OF 15 SAGITTAE AND λ SERPENTIS

| Star | Exposure 1928 | *a* | *b* | *c* | *d* |
|---|---|---|---|---|---|
| 15 Sagittae | July 28 | 61.5 | 58.8 | 49.8 | 39.6 |
| | 5 | 56.8 | 55.0 | 42.0 | 29.0 |
| | Aug. 2 | 55.7 | 54.5 | 39.9 | 27.0 |
| | July 5 | 54.5 | 50.8 | 35.5 | 24.3 |
| | 28 | 54.3 | 50.3 | 36.0 | 26.2 |
| | 5 | 53.5 | 49.2 | 35.0 | 21.2 |
| | Aug. 2 | 51.2 | 45.9 | 29.5 | 17.6 |
| λ Serpentis | 4 | 51.1 | 46.4 | 29.5 | 17.9 |
| | June 3 | 45.6 | 37.0 | 22.0 | 13.0 |
| 15 Sagittae | Aug. 2 | 40.0 | 31.1 | 18.1 | 10.0 |
| | 4 | 38.0 | 29.0 | 18.0 | 9.6 |
| | 2 | 36.6 | 28.3 | 17.6 | 9.1 |
| | 5 | 33.6 | 26.3 | 16.0 | ..... |

TABLE 3

| Asteroid | Mid.-Exp. U. T. | Duration of Exposure | Magn. | a | b | c | d | Δb | Δc | Δd |
|---|---|---|---|---|---|---|---|---|---|---|
| (1) Ceres | Aug. 16.425, 1928 | $2^h\ 05^m$ | $7^m8$ | 56.7 | 50.8 | 45.8 | 32.2 | − 4.3 | + 3.8 | + 2.2 |
| | 24.422 | 2 00 | | 55.0 | 51.2 | 37.8 | 21.0 | − 2.0 | + 0.1 | − 3.7 |
| | 25.384 | 2 00 | | 54.5 | 51.7 | 39.5 | 28.0 | 0.0 | + 3.2 | + 3.7 |
| (2) Pallas | 2.452 | 2 30 | 8.8 | 38.2 | 28.0 | ..... | ..... | − 1.3 | ..... | ..... |
| | 5.413 | 3 40 | | 42.8 | 38.4 | 28.8 | ..... | + 5.4 | + 9.4 | ..... |
| | Sept. 3.273 | 3 55 | | 55.8 | 51.0 | 41.0 | 32.0 | − 3.0 | + 1.0 | + 4.0 |
| (3) Juno | Feb. 27.474 | 2 42 | 8.7 | 39.1 | 25.0 | ..... | ..... | − 5.0 | ..... | ..... |
| (4) Vesta | Aug. 13.477 | 1 15 | 6.9 | 56.1 | 50.7 | 28.7 | ..... | − 3.6 | −12.0 | ..... |
| | 24.519 | 1 00 | | 58.2 | 56.6 | 51.0 | 40.5 | + 3.0 | + 6.2 | + 6.4 |
| | 25.460 | 0 45 | | 52.6 | 47.0 | 23.6 | 20e | − 1.3 | − 9.0 | 0e |
| | .485 | 0 20 | | 54.0 | 46.2 | 28.5 | 19e | − 4.5 | − 2.5 | − 4e |
| | Sept. 11.427 | 0 20 | | 52.5 | 46.7 | 33.1 | 24.7 | − 2.0 | + 1.2 | + 4.8 |
| | .444 | 0 20 | | 53.7 | 44.0 | 27.4 | 19.0 | − 6.2 | − 7.0 | − 3.0 |
| | .467 | 0 20 | | 52.0 | 46.1 | 34.5 | 28.0 | − 1.6 | + 3.5 | + 8.7 |
| | .488 | 0 20 | | 50.7 | 45.9 | 31.5 | 23.4 | + 0.3 | + 2.7 | + 5.9 |
| | .506 | 0 20 | | 50.7 | 44.2 | 33.3 | 28.4 | − 1.3 | + 4.4 | +11.0 |
| | .522 | 0 20 | | 48.9 | 40.7 | 28.7 | 20.7 | − 1.5 | + 2.5 | + 5.2 |
| | 22.368 | 0 24 | | 54.4 | 52.6 | 38.2 | 26.0 | + 1.1 | + 2.2 | + 2.2 |
| | .399 | 0 24 | | 55.5 | 52.5 | 40.6 | 37.6 | − 1.0 | + 2.1 | +10.8 |
| | .424 | 0 24 | | 55.9 | 53.9 | 41.6 | 33.8 | − 0.4 | + 1.6 | + 6.0 |
| | .451 | 0 24 | | 54.9 | 52.2 | 43.7 | 37.0 | − 0.3 | + 6.2 | +11.5 |
| | .479 | 0 24 | | 54.7 | 51.1 | 40.4 | 31.1 | − 0.9 | + 3.7 | + 6.8 |
| | .507 | 0 24 | | 54.6 | 46.6 | 34.9 | 24.5 | − 5.0 | − 0.3 | + 1.4 |
| | .533 | 0 24 | | 52.2 | 45.8 | 23.6 | 8e | − 1.7 | − 7.1 | −12e |
| | 24.265 | 0 24 | | 47.0 | 31.7 | ..... | ..... | − 8.0 | ..... | ..... |
| | .310 | 0 24 | | 51.1 | 47.0 | 10e | ..... | + 1.0 | −17e | ..... |
| | .344 | 0 24 | | 50.6 | 39.2 | ..... | ..... | − 6.0 | ..... | ..... |
| (6) Hebe | Nov. 2.378, 1927 | 1 20 | 7.2 | 53.2 | 50.7 | 36.3 | ..... | + 2.9 | + 4.3 | ..... |
| | 4.354 | 1 00 | | 50.5 | 29.0 | ..... | ..... | −15.8 | ..... | ..... |
| (7) Iris | July 5.413, 1928 | 2 10 | 8.6 | 40.3 | 31.8 | 13e | ..... | + 0.6 | − 6e | ..... |
| | Aug. 4.321 | 3 35 | | 43.6 | 33.9 | 18e | ..... | − 0.5 | − 3e | ..... |
| | 17.269 | 3 50 | | 48.0 | 38.5 | 30.0 | 22e | − 2.0 | + 5.1 | + 7e |
| (8) Flora | June 22.426 | 3 00 | 9.1 | 32.5 | 22.0 | ..... | ..... | − 5.1 | ..... | ..... |
| (9) Metis | Nov. 25.308, 1927 | 1 46 | 8.1 | 51.0 | 47.4 | 24.2 | 12e | + 1.6 | − 5.1 | − 8e |
| (12) Victoria | May 13.413, 1928 | 3 00 | 8.8 | 46.7 | 32.6 | 13e | ..... | − 5.9 | −10e | ..... |
| (28) Bellona | Mar. 15.471 | 2 30 | 9.3 | ..... | ..... | ..... | ..... | ..... | ..... | ..... |
| (40) Harmonia | May 23.456 | 2 05 | 9.3 | ..... | ..... | ..... | ..... | ..... | ..... | ..... |
| (79) Eurynome | Oct. 14.426 | 3 30 | 9.4 | 37.1 | 27.1 | 10e | ..... | − 1.2 | − 7e | ..... |

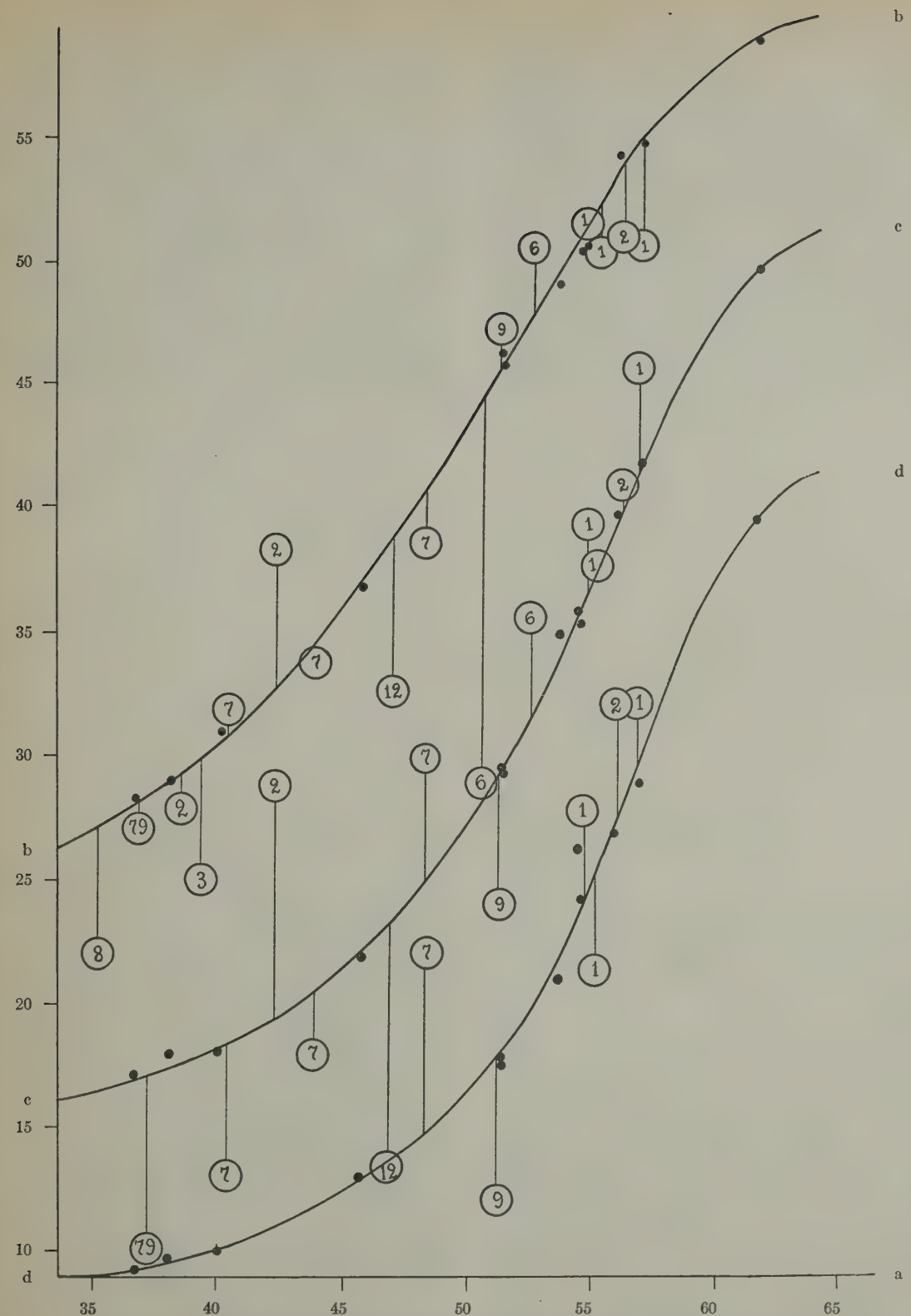


Fig. 1. Intensity curves from the spectra of G0 type stars (filled circles). Intensities in the same places in the spectra of asteroids (open circles). The asteroid's number is given in the circle. Departures in the density of the spectra of asteroids from the G0 type stars are given by vertical lines.

The shape of the curves of Figure 1 can be easily accounted for. They have flat lower ends, nearly straight middle parts, and flat tops. All this can be deduced from the consideration of the curves of photographic density for different wave-lengths as functions of the logarithm of time.[3] The curves naturally flatten out as we proceed from the lower wave-lengths to λ 4700, and would become a straight line if *b*, *c*, *d* become *a*.

The spectra of asteroids have been measured in the same way and densities *a*, *b*, *c*, *d*, are given in Table 3. The quantities Δb, Δc, Δd, denote the departures from the mean curves b, c, d, constructed from the spectra of the comparison stars. These departures even in the regions of lower density are far too large to be explained by the errors of measurement. The letter *e* affixed to some figures means that the densities were too low or otherwise inaccessible for actual measurement, and that they were estimated. The magnitudes are given for the opposition. They have been taken from the *Oppositions-Ephemeriden*.

Figure 1 has been drawn from the data of Tables 2 and 3. The number of each asteroid is given in a circle. The departures Δb, Δc, Δd, are represented by the vertical lines joining each circle to its corresponding curve. The densities obtained from the same spectrogram lie on the same ordinate. *Vesta* is omitted from Figure 1 on account of abundance of data for this asteroid. Positive residuals do not mean of course that the densities in the spectra of asteroids are higher than the density of the solar spectrum in that place. Densities *b*, *c*, *d*, have been compared with density *a*, and in the case of too high densities for *b*, *c*, *d*, we probably have to deal with a decrease of density *a*. It would mean a selective reflection from the surface of asteroids. As is seen from Table 3 and Fig. 1 the departures Δb, Δc, Δd, are mostly negative, that is, asteroids reflect little violet light as compared with blue.

The departures Δb, Δc, Δd, for *Vesta* for Sept. 11 are represented in Figure 2. There is a sharp minimum for the violet part of the spectrum at Sept. 11.444. Another minimum of the same kind occurs on Sept. 22.533. Still another minimum of the same depth can be seen at about Sept. 24.265. The interval between the first two observations is $11^d089$, between the last two $1^d732$. It seems plausible that such variation in the spectrum is caused by the rotation of *Vesta* and that both minima may be due to the presentation toward the earth of a spot which reflects especially little violet light. In such a case if we adopt the period of $0^d2465 = 5^h55^m$ we shall have $0^d2465 \times 45 = 11^d092$, and $0^d2465 \times 7 = 1^d726$. The period found from the first interval satisfies the second within 2 per cent of its value. Such good agreement is probably accidental but it gives nevertheless some indication as to the existence of spots on *Vesta* and of its rotation. The figures cannot be very precise as the times of minima were determined approximately and no account was taken of the change in the phase-angle of the asteroid.

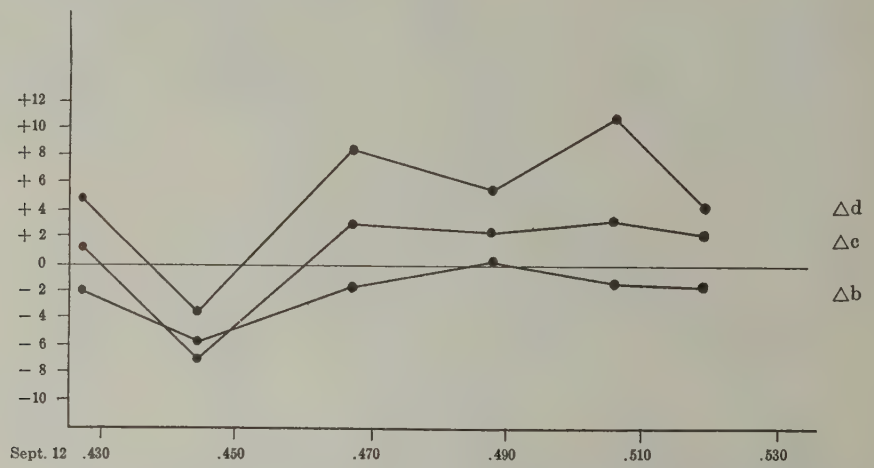


Fig. 2. Variations in the intensity in the spectrum of Vesta on Sept. 11, 1928. Residuals from the curves b, c, d of Fig. 1 in the units of the Photometric Wedge.

There is another indication that this period of rotation of *Vesta* is not far from the truth. *Vesta* was studied by W. H. Harrington[4] in 1883, who found remarkable changes in its brightness. He was not able to assign any period because of irregular secondary variations. Taking from the three long series of his observations on Apr. $13^d14^h57^m$, Apr. $16^d12^h26^m$ and Apr. $17^d12^h11^m$, Sidereal Time, when there were unquestionable maxima, we find from the first interval

[3] See, for instance, Ross, *The Physics of the Developed Image*, 1924, p. 73.

[4] *Amer. Journ. of Sci.* **26**, 462, 1883.

the value of period $5^h46^m$ and from the second $5^h55^m$, Mean Time, both in sufficiently good agreement with the spectro-photometric period, taking into account the roughness of measurement.

Unfortunately, it was usually impossible to observe directly the variation of light of asteroids during the exposure. The comparison stars were too far away and the asteroids were too close to the cross-wire in the finder. Attempts in this direction were made on Sept. 22 and Sept. 24 while exposing on *Vesta*. The comparison star which happened to be in the field of the finder was BD−3°239, magnitude[5] 6.97. The star was of the same color and the comparisons were made between exposures with the necessary photometric precautions. The values were then interpolated for the time of mid-exposure. The results are given in Table 4.

TABLE 4

| Mid-Exposure U. T. | Hour-angle | Magnitude | Phot. Dens. *a* |
|---|---|---|---|
| Sept. 22.368 | $0^h\ 48^m$ East | 6.8 | 54.4 |
| .399 | 1 28 | 6.8 | 55.5 |
| .424 | 2 08 | 6.9 | 55.9 |
| .451 | 2 48 | 7.0 | 54.9 |
| .479 | 3 28 | 7.0 | 54.7 |
| .507 | 4 08 | 6.9 | 54.6 |
| .533 | 4 46 | 6.75 | 52.2 |

The plates were taken from the same box and developed simultaneously under exactly the same conditions. The seeing, temperature, etc. during the exposure remained the same, so that the variations in the intensity of the spectrum must be due only to the changes in the light of *Vesta*, or to the change in the position of the telescope. As only one comparison star has been used the magnitudes given cannot be very accurate, but there seems to be no doubt in the fact of variation.

It appears that the spectrum in the photographic region was changing in the opposite direction from the changes in the visual brightness. It is especially pronounced when we consider two adjacent exposures when the position of the telescope did not change much. For instance compare exposures at .399 and .424, and especially at .507 and .533. During the last exposure the brightening up of *Vesta* was noticeable on the slit and dawn was beginning to break. In spite of this the spectrogram is surprisingly weak. Apparently, during the exposure, the rotation of *Vesta* brought new regions into view which reflected more orange-yellow light than photographic blue-violet. Analogous changes were noticed on Sept. 24, but on that night the seeing was poor.

The variations in the spectra of other asteroids like *Ceres*, *Pallas*, *Hebe*, and *Iris* are also apparent from Table 3 and Figure 1.

[5] *Harvard Annals* **54**, 21.

## Comparison with the Spectrum of the Moon and of Saturn

There was some doubt at the beginning of the investigation as to the value of the spectro-photometric results obtained with a visual refractor and under varying atmospheric conditions. The smooth curves of Figure 1 serve as a check against giving too much weight to the difference either in the position of the telescope or in the state of the atmosphere.

Another check was afforded by the Moon and *Saturn*. As is well known R. W. Wood[6] has found that some portions of the Moon are affected differently when monochromatic filters are used. Thus the area between craters *Aristarchus* and *Schiaparelli* ("Wood's Spot") appears very distinctly on the ultra-violet photographs and is hardly visible in the yellow light. A number of spectrograms have been secured by the writer, different details on the Moon being alternately placed on the slit. The general results are in complete accordance with Wood's photographs. When a *mare* was especially dark in the violet according to Wood it gave an especially weak violet spectrum and *vice versa*. It is particularly true of Wood's Spot as compared with *Aristarchus*. The continent between *Tycho* and *Mare Nectaris* gave also a very weak violet spectrum. On the other hand *Mare Imbrium* came out considerably brighter, and *Mare Tranquilitatis* of the same intensity in the violet as *Aristarchus*. All these results can be deduced as well from Wood's photographs.

The spectrograms were obtained in such a way that their maxima of intensity were of the same density. The time of exposure being very different for different parts of the Moon, it was necessary to take a large number of spectrograms of each selected area and match them with each other. As it is impossible to compare directly the lunar spectrograms with those of asteroids and, furthermore, since it was exceedingly difficult to guide the telescope on the same point of the lunar surface except on very bright craters, it has been considered sufficient to secure only qualitative agreement of results and not to undertake a precise photometric study.

*Saturn* has also been the subject of investigations by Wood and others.[7] A remarkable feature of the photographs taken through different filters is that the ball of *Saturn* is much fainter than the rings in the violet, and brighter in the visual rays. Another interesting feature is that while the inner and outer rings on both sides of Cassini's division are very different in brightness in the visual region they are of equal intensity in the violet. Six spectrograms of *Saturn* were

[6] *Astroph. Journ.* **36**, 75, 1912. Also Miethe and Seegert, *Astr. Nachr.* **188**, 239 and 371, 1911.

[7] Wood, *Astroph. Journ.* **43**, 314, 1916; Wright, *Publ. Astr. Soc. Pac.* **39**, 231, 1927.

taken on Sept. 4 and three on Sept. 24. The slit of the spectrograph was placed along the major axis of the rings and was made long enough to include them on both sides of the ball. Both of the above mentioned peculiarities have been confirmed by the spectrograms. It may be noted here that while near Hβ the difference in the photographic density between *Saturn* and the rings is small it increases rapidly toward the violet and is striking in the region of λ4000. Apparently the rings reflect much of the violet and ultra-violet light, and may contain dust or even gaseous molecules, enhancing the reflection in that part of the spectrum. This explanation is quite possible theoretically as has been pointed out by Schoenberg.[8]

### Visual Region

For the visual region Eastman 33 plates were stained with the three dyes (pinacyanol, pinaverdol, homocol) according to Wallace's formula. The spectrum extends from λ4500 to λ6900. A six-inch camera was used. This gives a dispersion of 6.4 mm between Hβ and Ne λ6717. The camera and collimator were set for λ6000. Only the spectra of the two brightest asteroids, *Ceres* and *Vesta*, could be obtained. For comparison the spectra of 15 *Sagittae* and *Uranus* were taken on the same nights. Table 5 gives the spectrograms obtained.

TABLE 5

| Asteroid | Mid-Exp. U. T. 1928 | Duration |
|---|---|---|
| Ceres | Oct. 15.227 | $2^h$ $20^m$ |
| Vesta | 8.295 | 0 15 |
| | .310 | 25 |
| | .334 | 40 |
| | 15.323 | 6 |
| | .329 | 12 |
| | .342 | 25 |

### Discussion of Results

There is no doubt that minor planets show variations in their continuous spectrum. This can be reasonably explained as an effect of rotation and of different reflectivity of the portions of their surface. This could be established for all asteroids the spectra of which were obtained twice or more. Very probably the same is true for the rest. This result might be anticipated as all asteroids discussed in this paper, with the exception of *Eurynome*, have been found variable in brightness.[9]

We know that for most asteroids the periods of the variation in brightness and presumably of rotation, do not exceed a few hours. An exposure of 3 to 4 hours, therefore, should give the average reflectivity of the asteroid's surface. Only in the case of *Vesta* could the specific reflectivity of different portions of the surface be traced and thus the period of rotation estimated.

The low reflectivity of asteroids in the violet and ultra-violet is well illustrated by the microphotometric records. Figure 3 gives some of the tracings obtained by means of the Moll microphotometer of the Lick Observatory. In comparing the intensity curves of the spectra of asteroids with the G0 type stars one point should be kept in view. Owing to the different laws for the building up of photographic density in the blue and violet region (that is the inequality of γ for different parts of the spectrum[10]), the shape of the intensity curves depends somewhat on the exposure. The region of λ4700 is the first to reach the maximum of intensity after which its density does not increase (the flat top of the characteristic curve). The part between λ4300 and λ4000 continues to increase in density, and the intensity curve flattens out. It would be erroneous, therefore, to compare the tracings of the stars and asteroids of different maximum density.

Figures 3a, 3g and 3k show the intensity curves of 15 *Sagittae* and λ *Serpentis*. At moderate maximum density (between 55 and 40) the curve of the continuous spectrum descends approximately in a straight line to G and then forms a characteristic hump from G to H. This hump is less pronounced with smaller maximum density, D, but is still well noticeable even at D 37, which is below the maximum intensity in the spectra of all asteroids but *Flora*. This may be seen in Figures 3a (D 51), 3g (D 46) and 3k (D 37). The spectrum of the Moon, Figure 3b, has approximately the same distribution of intensity as that of the G0 stars of the same D.

If we turn now to asteroids we see that many of them show a remarkably faint violet and ultra-violet part of the spectrum. Most of them show the characteristic hump of the G0 stars much weakened or altogether absent, even when the maximum intensity is such that we might expect a well developed hump. The spectrum of *Metis*, Figure 3d (D 51), shows a perfectly straight line from the maximum intensity to H and K. Comparison with the spectrum of λ *Serpentis*, 3a, and of the Moon, 3b, illustrates the point. Still more pronounced is this effect in Hebe, 3h (D 51), which does not show any continuous spectrum to the violet of G.

The spectra of asteroids *Victoria*, 3i (D 47), and *Iris*, 3j (D 44) show practically nothing of the violet spectrum, although we might expect it to be strong. *Flora*, 3m (D 33), is much stronger in the violet than either of these two asteroids, but is still below the G0 stars. The spectra of *Juno*, *Bellona* and *Harmonia* are too faint for measurement, but it seems that all of them are weak in the violet.

[8] "Über die Strahlung der Planeten," *Ergebnisse der Ex. Naturwiss.* **5**, 1926.

[9] Miss Harwood, *Harv. Circ.* No. 269, 1924.

[10] Ross, *Op. cit.* p. 71 ff.

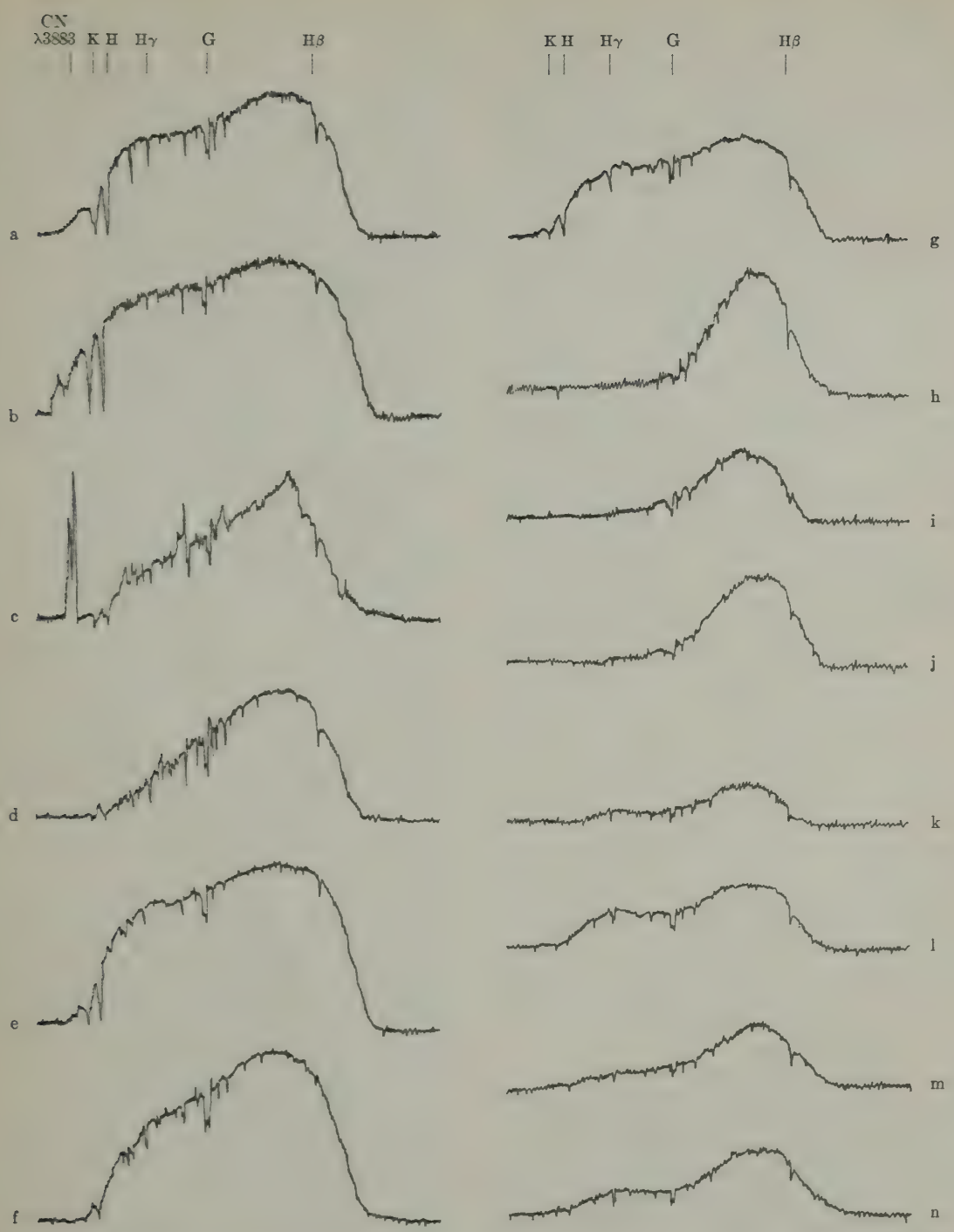


Fig. 3. Microphotometric tracings of the spectra of G0 type stars, the Moon, Halley's comet and asteroids. (a) 15 Sagittae; (b) Moon; (c) Halley's comet; (d) Metis; (e) and (f) Vesta; (g) λ Serpentis; (h) Hebe; (i) Victoria; (j) Iris; (k) 15 Sagittae; (l) Pallas; (m) Flora; (n) Eurynome.

On the other hand, *Pallas*, 3l (D 43) and *Eurynome*, 3n (D 37) have in their spectra a well pronounced violet part approaching to the G0 stars. It seems that their spectra underwent just the opposite modification, the relative intensity of the violet part being greater than in the G0 stars. In fact, their intensity curves show two distinct maxima at λ4700 and λ4100.

The spectra of *Ceres*, *Pallas*, *Vesta*, *Hebe* and *Iris* show variations in accordance with Table 3. For illustration, two tracings of the spectrum of *Vesta* are given in Figure 3. On 3f (D 52), Sept. 22.533, the violet part is much weaker than on 3e (D 55), Sept. 22.451, although the difference in D is small. Curve 3f approaches to 3d of *Metis*, and curve 3e to that of the Moon, 3b.

The faint violet and ultra-violet part of the spectrum of some asteroids cannot be due to their low position in the sky. The spectra of *Hebe*, *Metis*, and *Victoria* were obtained when they were high in the sky. *Iris* and *Flora* were low. Both groups show the same effect. *Flora* has a considerably stronger violet part than *Iris*, although they were almost in the same field of view of the finder. On the other hand, *Ceres*, being very low in the sky, showed nevertheless a quite strong violet part in its spectrum. The spectrum of $\omega_2$ *Scorpii* did not differ from the spectra of 15 *Sagittae* and λ *Serpentis*, although the difference in altitude was as much as 40°. The interference of the Moon's light was avoided so far as it was possible, as this should enhance the violet part of the spectrum. Perhaps the strong violet part in the spectrum of *Pallas* is due to this factor, as all its spectrograms were obtained when the Moon was nearly full.

The faintness of the violet part of the spectra of asteroids reminds one of the similar feature in comets. Figure 3c gives the intensity curve of the spectrum of Halley's comet on Apr. 23.510, U. T., 1910, obtained by W. H. Wright with a spectrograph attached to the 12-inch refractor. It gives exactly the same dispersion as the spectra of asteroids and can be compared with them. The run of the continuous spectrum of bright comets with a sharp nucleus like Halley's,—1910 I, 1911 IV, etc.—is very peculiar and shows a very weak violet and ultra-violet part.[11]

If in the spectrum of Halley's comet, Figure 3c, the continuous spectrum associated with the Swan band λ4737 is excluded, the rest of the continuous spectrum will give a straight line without a hump between G and H. In this it strikingly resembles the spectrum of *Metis*, 3d. If Halley's comet did not have its gaseous envelopes its spectrum would not differ from the spectrum of *Metis*. Comet 1911 IV could be well matched with asteroids *Hebe*, *Victoria*, and *Iris*. In other words, the reflectivity of the cometary nuclei closely resembles that of asteroids.

[11] See Bobrovnikoff, "On the Spectra of Comets," *Astroph. Jour.* **66**, 439, 1927.

In view of the cosmological implications this conclusion must be received with caution. As the matter stands now we seem to have a long looked for link between asteroids and comets. The nucleus of a comet consists of an assemblage of small bodies, each of which reflects solar light in its own way. The continuous spectrum of the cometary nucleus is as changeable as any other characteristic of the comet, which is the natural consequence of the perpetual change in the distribution of the components of the nucleus. When the comets are far from the Sun and their nuclei are concealed in the gaseous envelopes their continuous spectrum is totally different from the solar spectrum. When, near perihelion, the nucleus becomes sharp and bright, it shows the solar Fraunhofer lines and its continuous spectrum is predominantly due to reflection from the solid components of the nucleus. It seems safe to assume that the modification of the solar spectrum resulting in the weak violet and ultra-violet part is due to the specific reflectivity of the meteors of which the nucleus is composed. The continuous cometary spectrum of the solar type represents therefore an average result of the reflection of solar light from a very large number of bodies. This average result is characterized by the faintness of the violet and ultra-violet part of the spectrum.

On the other hand, we have seen that asteroids reflect solar light very unevenly both in intensity and in quality. The light of most of them is remarkably poor in the violet rays. This is also an average result as the long exposures do not allow the isolation of the effects of small portions of their surface. In cases like *Ceres* and *Vesta*, where such separation of effects has to some degree been possible, we have obtained what might be expected: the variation in intensity of the different parts of the spectrum.

It may be said therefore that asteroids in their ability to reflect solar light show the characteristics of the constituents of cometary nuclei. Some of them reflect more of the violet and ultra-violet light, some less, but on the average all of them very little. In other words, if a giant comet fell to pieces, the fragments of its nucleus would reflect solar light very much like individual asteroids.

If an occasional asteroid were not a single body but consisted of several pieces resembling a comet in all but the extensive atmosphere, we could never tell the difference. It is possible that some of the asteroids may show in their spectra the traces of cometary bands. The search for these quasi-comets is difficult on account of the number of minor planets and their inaccessibility to all but the most powerful instruments.

It is difficult to say to what agency the weakening of the violet part in the spectrum of comets and asteroids should be attributed. Wood has found that among other substances iron and especially sulphur reflect little

of violet and ultra-violet light. Both iron and sulphur are common in meteorites. In the iron meteorites pyrrhotite, FeS, is frequent. It forms nodulae and tends to be especially abundant toward the periphery.[12] It is conceivable that the low reflectivity of cometary nuclei in the violet is due to the presence of iron and sulphur. These considerations lend weight to the view that asteroids diminishing in size merge into meteorites inaccessible to observation as long as they are outside the Earth's atmosphere.[13]

The results arrived at in this investigation seem to fill up the gap that separated comets and asteroids in their physical properties. It seems possible that comets losing their gaseous material and breaking up into fragments give rise to asteroids, as has been suggested by Leuschner.[14] It should be added here that the present study was undertaken by the writer in full conviction that Leuschner's ideas on the common origin of comets and asteroids were wrong.

The variation in the spectra of asteroids renders possible the determination of their periods of rotation. Visual or ordinary photographic observations give a summary effect which tends to equalize the reflection from the different portions of the surface. If we separate the spectral regions by using appropriate filters we may hope to obtain larger variation in light and the period of rotation may be established more easily. Especially large amplitudes of variation may be expected in the ultra-violet and the efforts of observers should be applied in this direction.

Figure 4 gives the intensity curves of the spectra of *Uranus* (a), 15 *Sagittae* (b), *Vesta* (c), and *Ceres* (d), in the visual region. *Uranus* shows its heavy bands at λλ5430 and 6190A. As compared with the spectrum of 15 *Sagittae* the spectrum of *Uranus* has quite a different distribution of intensity. It has a well pronounced maximum of intensity in the green, at about λ5000. This is of course natural to expect taking into account the greenish light of the planet, but it is not in accordance with the general view that the color of *Uranus* is solely due to the presence of heavy bands in the orange and red part of the spectrum.[15]

The distributions of intensity in the spectra of *Vesta* and *Ceres* differ from each other as well as from 15 *Sagittae*. The star has its maximum intensity under the conditions of observation near λ6100, while *Vesta* has its maximum at about λ5300, and *Ceres* near λ4800. In other words *Ceres* is considerably bluer than *Vesta*. This is in accordance with the aspect of the asteroids as *Vesta* is conspicuously bright, while *Ceres* is dull. The geometric albedo of *Vesta* is 0.48 and of *Ceres* only 0.10. Therefore *Vesta* reflects more of the orange light to which the eye is more sensitive than to the blue. This can be seen from Figures 4c and 4d.

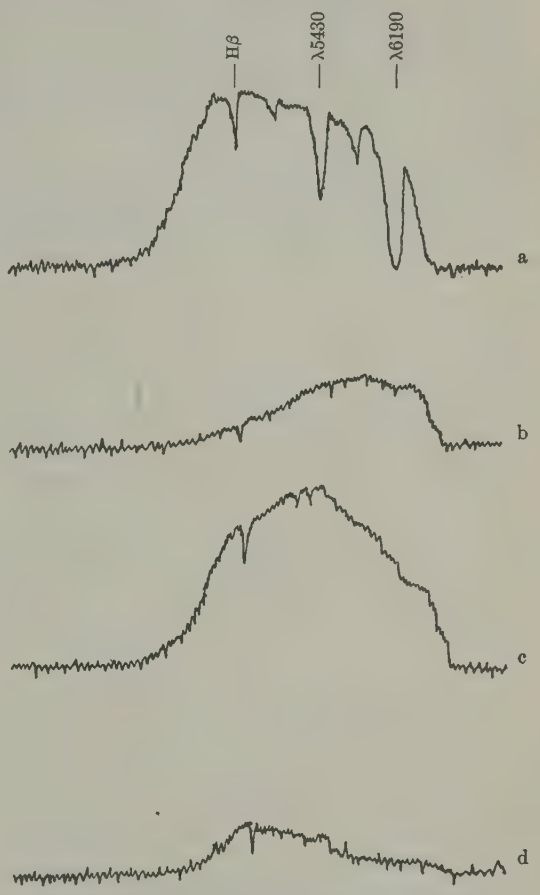


Fig. 4. Microphotometric tracings of the spectrum of (a) Uranus; (b) 15 Sagittae; (c) Vesta; (d) Ceres. Visual region.

The hydrogen lines in the spectra of asteroids are not modified to any perceptible degree as compared with the spectrum of 15 *Sagittae*. No indication of the existence of bands at λλ5770 and 5180, which were suspected by Vogel in the spectrum of *Vesta*, has been found. No considerable variation in its spectrum in the visual region has been observed.

[12] *Farrington*, Meteorites, 1916, p. 140.

[13] See, for instance, L. Bell, "The Physical Interpretation of Albedo," *Astroph. Jour.* **45**, 1, 1917.

[14] "The Pons-Winnecke Comet", *Publ. Astr. Soc. Pac.* **39**, 275, 1927.

[15] See Tikhoff, "La couleur des planètes Neptune et Uranus," *Bull. Obs. Poulkovo* **8**, No. 5, 1922.

## Summary

1. Spectra of twelve minor planets have been investigated. No bright lines or bands have been found and the spectra are of the solar type showing that the light of asteroids is wholly due to reflection.

2. The continuous spectrum of asteroids has been found to vary in intensity. The violet and ultra-violet part of the spectrum of many asteroids has been found very weak in comparison with the same part of the spectrum in the G0 type stars.

3. The run of the continuous spectrum of many asteroids is very peculiar and closely resembles that of comets near perihelion, obtained under comparable instrumental conditions.

4. The rotation period of *Vesta* has been found to be $5^h55^m$ from the variation of its continuous spectrum, in good agreement with the result from visual observations.

5. The spectra of *Ceres* and *Vesta* in the visual region do not show any absorption bands like those of the major planets. In the distribution of intensity they differ from each other and from the G0 type stars.

6. The spectra of *Saturn*, *Uranus*, and of the different portions of the Moon have been studied in connection with the spectra of asteroids.

7. The possibility of the study of the physical properties of asteroids by means of monochromatic filters has been pointed out.


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ASTRONOMY

# LICK OBSERVATORY BULLETIN

NUMBER 408

## ON THE DISINTEGRATION OF COMETS

BY

N. T. Bobrovnikoff

Since the discovery of the connection between comets and meteors many excellent investigations have been made on the stability of comets and their disintegration into meteors. Very little, however, has been done on the astrophysical side of the problem. It seems possible that the distribution of brightness among the periodic comets may give us some insight into the process of disintegration and settle the question whether comets really lose their gaseous material and, in the course of time, disperse into meteors. The purpose of this paper is to study the disintegration of comets on the basis of the correlation between their average brightness and their orbital elements.

Recent investigations by S. V. Orlow, Kritzinger, Vsechsviatsky, and others have established the fact that the brightness, $I$, of a comet varies according to the law:

$$(1) \quad I=\frac{I_o}{\rho^2 r^n}$$

Where $I_o$ is a constant, $\rho$ is the geocentric, and $r$ is the heliocentric, distance of the comet. The exponent $n$ differs from comet to comet, and does not remain constant even for the same comet for different values of $r$. If we put $\rho=r=1$, then $I_o=I$, that is, the constant $I_o$ represents the brightness of the comet at unit distance from the Earth and from the Sun. We may call it *"the absolute brightness."* Very little is known concerning the influence of the phase on the brightness of comets. Even in the cases where this influence could be suspected[1] the correction for the phase could be calcuated on certain assumptions as to the albedo and physical constitution of the comet. The phase-angle under the conditions $\rho=r=1$ is 60°, and the correction for the phase would be considerable. This introduces some uncertainty, the more so as the change in the phase-angle of a comet during the whole period of observation is usually small.[2]

Expressing the brightness of the comet in magnitudes we obtain the following formula correlating the observed, $H$, and the absolute, $H_o$, magnitude of a comet:

$$(2) \quad H_o=H-5\log\rho-2.5n\log r$$

A series of observations furnishes the values of $H$ for different $\rho$'s and $r$'s, from which the average value of $H_o$ and of $n$ can be determined by the method of least squares. It is customary to denote the total, or integrated, magnitude of the comet by $H$ and the magnitude of its nucleus by $h$.

A comprehensive study of the brightness of comets was carried out by Holetschek in his "Untersuchungen über die Grösse und Helligkeit der Kometen und ihrer Schweife" (five volumes, 1896–1917). He investigated about 200 comets and arranged them according to the perihelion distance $q$. His conclusion was that the smaller $q$ is, the brighter is the comet. On the other hand, while for some short period comets ($P<100$ years) there could be suspected a gradual decrease in brightness, for none could an increase in brightness be shown in its subsequent apparitions.

[1] This influence is more noticeable on the brightness of the nuclei. According to S. V. Orlow the phase correction for the nucleus of Halley's comet in 1910 ranged from $0^m2$ to $1^m9$ (*Bull. Ac. St. Petersb.* **7**, 255, 1913).

[2] A more correct definition of the absolute brightness would be the brightness which the comet would have if viewed from the center of the Sun at $r=1$. A similar conception was introduced by A. Klose in his study of the brightness of asteroids (*A. N.* **231**, 313, 1927). The expression *"reduzierte Helligkeit"* used by Holetschek and others for *"absolute brightness"* is ambiguous.

Holetschek operated with the law of brightness $n=2$, which he himself found unsatisfactory. Recently two large catalogues of absolute magnitudes of comets have been published by Vsechsviatsky.[3] They include also the absolute magnitudes of comets found by Holetschek and corrected on the basis of $n=4$. After this correction had been applied there could not be found any dependence of the absolute brightness on $q$. Vsechsviatsky came to the conclusion that $n$ is on the average very close to 4 and that the average absolute brightness of comets increases with the increase of the major semi-axis, $a$, which would mean that the comets with smaller $a$ are farther on their way to disintegration. This result is not very definite as the averages were taken over wide ranges of $a$ and no consideration was given to the eccentricity, $e$. It is, however, evident that two comets with the same $a$ but different $e$ will be subject in different degree to the disintegrating influence of solar radiation.

In formula (1) the part of the comet's light due to reflection is proportional to $\frac{1}{\rho^2 r^2}$; the remaining factor $\frac{1}{r^{n-2}}$ is therefore due to the luminosity of the comet excited by the action of the sun. According to Vsechsviatsky $n-2$ on the average is very close to 2. Since the fluorescence of comets is accompanied by the continuous loss of matter in ejection from the nucleus, resulting in the phenomena of coma and tail, we may assume that the dissipation factor producing the secular decrease in brightness of comets also varies as $\frac{1}{r^2}$. As we shall see later, the results do not change much whether we assume $n-2=1$, 2, or 3. The diminution of a comet's brightness may be any complicated function of the loss of its matter. What we can observe is the change in its brightness, and this seems to be in proportion to $\frac{1}{r^2}$.

If we denote the decrease of the comet's brightness during one revolution as $J$, we shall have then for the element of time, $dt$:

$$(3) \quad dJ = \frac{A\,dt}{r^2}$$

where $A$ is the constant of dissipation. Taking into account the relation

$$(4) \quad r^2\frac{dv}{dt} = k\sqrt{a(1-e^2)}$$

we obtain for one revolution of the comet:

$$(5) \quad J = A\int_0^P \frac{dt}{r^2} = \frac{A}{k\sqrt{a(1-e^2)}}\int_0^{2\pi} dv = \frac{2\pi A}{k\sqrt{a(1-e^2)}}$$

[3] *Russian Astr. Jour.* **2**, 68, 1925 and **5**, 40, 1928.

If the comet came into existence $B$ years ago, then up to the time of observation it had performed $\frac{B}{P} = \frac{B}{a^{3/2}}$ revolutions. The constant $B$ is the number of years elapsed from the time the comet became a member of the solar system up to the time of observation. The total decrease in brightness, $J'$, is therefore

$$(6) \quad J' = \frac{2\pi AB}{k}\,\frac{1}{a^2\sqrt{1-e^2}}$$

assuming a uniform rate of disintegration. Expressing this in magnitudes, we have for the loss in brightness, or for the increase $H'$ in the magnitude, of the comet:

$$(7) \quad H' = C - 2.5\,(2\log a + \log\sqrt{1-e^2}\,) = C - 2.5\,\Phi\,(a, e)$$

where $C = 2.5\log\left(\frac{2\pi}{k}AB\right)$. The absolute magnitude $H_o$ of a comet observed at the present time equals the original magnitude $H_o'$ plus the increase $H'$ in magnitude:

$$(8) \quad H_o = H_o' + H'.$$

We know nothing about the original magnitude of the comets, that is with what brightness they have started their career. As a working hypothesis we shall assume that they all were equally bright at the beginning, and that the present disparity in their brightness is a result of different rates of disintegration. This is probably true statistically for a sufficiently large number of comets. We also assume here that the constant $B$ is the same for all comets, that is they all came into existence at the same time. This is true if we picture the birth of comets from the primeval nebula, simultaneously with the planets. If, on the other hand, they have been captured by the Sun much later, this should have occurred in a comparatively brief time so that all comets must be of approximately the same age. Leuschner has demonstrated that the preponderance of parabolic orbits is due chiefly to conditions of observing. With the increase of time during which comets are under observation, the percentage of parabolic orbits diminishes. It is probable that all parabolic comets if observed sufficiently long would prove to be elliptic. The researches by Strömgren, Fabry, Fayet, and others showed quite conclusively that the influx of comets from other stellar systems ceased long ago. Our solar system has long been completely isolated from all possible sources of comets. As for the constant formation of comets in our own solar system, this seems highly improbable, at least after the formation of the planets.

For a statistical study 94 periodic comets have been selected. Their absolute magnitudes were taken chiefly from Vsechsviatsky's catalogues. The data for the comets of 1911 were taken from an investigation by

Vorontzov-Veliaminov[4] and in a few cases Kritzinger's[5] data were adopted. By periodic comets are meant here those for which any period could be assigned as preferable to a parabolic orbit. In a few cases this was several million years. This of course introduced uncertainty into formula (7), but as we have to deal there with logarithms and the possible error is not of a systematic character, it cannot affect the results fundamentally. Since $q$ is usually well known we can in (7) replace $1-e$ by $\frac{q}{a}$; furthermore, $1+e$ is close to 2, and $q$ is on the average 0.7, so that formula (7) becomes:

$$(9) \quad H' = C' - 2.5 \log P$$

where $C'$ is a constant differing slightly from $C$. The error in $\Phi$ $(a, e)$ is therefore approximately equal to the error in log $P$. To take one of the worst cases, we have for comet 1769 two orbits: Bessel's with $P=2138$ years and $e=0.99924901$, and Lexell's with $P=48\frac{2}{3}$ years and $e=0.9980036$. The value of $\Phi$ $(a, e)$ in the first case is 3.02 and in the second 2.38. Bessel's orbit has finally been adopted as the more accurate one.

The elements have been taken from various catalogues, chiefly from Galle and Crommelin. In most cases they were definitive or semi-definitive. The absolute magnitudes of Halley's and Westphal's comets were taken for their last apparition. For short periodic comets if there were several values of $H_o$ the mean was taken. For Encke's comet $H_o$ of 1924 was adopted as this comet has shown a definite decrease in brightness since the end of the eighteenth century. The data are given in Table 1.

[4] *A. N.* **225**, 147, 1925. [5] *A. N.* **199**, 121, 1914.

TABLE 1

| No. | Comet | | Name | $\Phi(a, e)$ | $H_o$ | log $P$ | log $q$ | $e$ | Remarks |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1924 | III | Encke | +0.41 | $+10^m.0$ | +0.52 | −0.47 | 0.84630 | |
| 2 | 1766 | II | Helfenzrieder | .66 | 6.8 | .69 | .40 | .82380 | |
| 3 | 1902 | II | Grigg | .76 | 9 | .70 | .13 | .74463 | |
| 4 | 1819 | IV | Blanpain | .79 | 8.5 | .70 | .05 | .69875 | |
| 5 | 1770 | II | Lexell | .79 | 7.5 | .75 | .17 | .78684 | Leverrier's orbit. |
| 6 | 1873 | VII | Coggia | .86 | 12 | .79 | .11 | .77062 | |
| 7 | 1909 | II | Pons-Winnecke | .88 | 11.4 | .77 | .01 | .70190 | 1892, $11^m.0$; 1909, $9^m.7$; 1915, $12^m.1$; 1921, $12^m.7$. |
| 8 | 1899 | IV | Tempel 2 | .88 | 9.5 | .72 | +0.14 | .54213 | 1899, $9^m.4$; 1904, $9^m.0$; 1920, $10^m$. |
| 9 | 1844 | I | de Vico | .88 | 8.0 | .74 | .07 | .61765 | |
| 10 | 1916 | II | Neujmin | .89 | 11.4 | .73 | .13 | .56558 | |
| 11 | 1908 | III | Tempel 3 | .89 | 12.2 | .75 | .06 | .63773 | |
| 12 | 1900 | III | Giacobini | .91 | 10.5 | .81 | −0.03 | .73312 | 1900, $10^m.8$; 1913, $10^m.2$. |
| 13 | 1906 | V | Finlay | .93 | 10.4 | .82 | .02 | .72405 | 1906, $9^m.8$; 1919, $11^m.0$. |
| 14 | 1743 | I | Grischow | .94 | 9 | .82 | .06 | .72131 | |
| 15 | 1909 | III | Perrine | .95 | 14.3 | .81 | +0.07 | .66170 | |
| 16 | 1910 | IV | d'Arrest | .98 | 14.3 | .81 | .10 | .63692 | |
| 17 | 1916 | I | Taylor | .99 | 8.8 | .80 | .19 | .54625 | |
| 18 | 1909 | IV | Daniel | 1.01 | 6.7 | .81 | .14 | .49971 | |
| 19 | 1906 | IV | Kopff | 1.02 | 9.5 | .82 | .23 | .51653 | 1906, $10^m.0$; 1919, $8^m.9$. |
| 20 | 1905 | II | Borrelly | 1.02 | 10.4 | .84 | .14 | .61521 | 1905, 10.8; 1918, $10^m.0$. |
| 21 | 1912 | I | Wolf | 1.03 | 9.1 | .83 | .20 | .55797 | 1918, $5^m.0$ (app. error) |
| 22 | 1918 | III | Schorr | 1.04 | 11.0 | .83 | .27 | .47224 | |
| 23 | 1919 | IV | Schaumasse | 1.06 | 11.0 | .91 | .07 | .70934 | 1919, $10^m.9$; 1911, $11^m.1$. |
| 24 | 1906 | III | Holmes | 1.08 | 11.8 | .84 | .33 | .41314 | |
| 25 | 1910 | V | Faye | 1.08 | 9.6 | .87 | .22 | .56560 | |
| 26 | 1906 | VI | Metcalf | 1.10 | 7.9 | .89 | .21 | .58401 | |
| 27 | 1783 | ..... | Pigott | 1.21 | 6 | 1.00 | .17 | .67840 | |
| 28 | 1912 | IV | Tuttle | 1.26 | 8.0 | 1.13 | .01 | .81840 | |
| 29 | 1913 | III | Neujmin | 1.46 | 9.9 | 1.24 | .18 | .77333 | |
| 30 | 1866 | I | Tempel | 1.66 | 9 | 1.52 | −0.01 | .90542 | Perseids' Comet |
| 31 | 1883 | II | Ross | 1.70 | 7.0 | 1.81 | .55 | .98084 | |
| 32 | 1919 | III | Brorsen | 1.74 | 9.6 | 1.86 | .33 | .97189 | |
| 33 | 1827 | II | Pons-Gambart | 1.76 | 7 | 1.66 | .09 | .93722 | |
| 34 | 1910 | II | Halley | 1.89 | 6.5 | 1.83 | .23 | .97105 | 1835, $5^m.2$ |
| 35 | 1913 | VI | Westphal | 1.93 | 9.3 | 1.79 | +0.10 | .91982 | 1852, $5^m.0$ |
| 36 | 1917 | I | Mellish | 1.95 | 7.4 | 2.16 | −0.72 | .99311 | |
| 37 | 1862 | III | Tuttle | 2.23 | 4.0 | 2.09 | .02 | .96127 | Leonids' Comet |
| 38 | 1742 | ..... | ..... | 2.31 | 3.7 | 2.22 | .12 | .97460 | |
| 39 | 1907 | II | Grigg-Mellish | 2.35 | 9.7 | 2.22 | .04 | .96922 | Probably identical with No. 38 |
| 40 | 1857 | III | Klinkerfues | 2.48 | 9.0 | 3.84 | .43 | .99900 | |
| 41 | 1905 | III | Giacobini | 2.65 | 11.4 | 2.47 | +0.05 | .97496 | |
| 42 | 1861 | II | Tebbutt | 2.72 | 3.9 | 2.61 | −0.08 | .98538 | |
| 43 | 1861 | I | Thatcher | 2.74 | 4.9 | 2.62 | .04 | .98346 | |

TABLE 1 (*Continued*)

| No. | Comet | | Name | $\Phi(a, e)$ | $H_o$ | log $P$ | log $q$ | $e$ | Remarks |
|---|---|---|---|---|---|---|---|---|---|
| 44 | 1846 | VII | Brorsen | +2.75 | +8m1 | +2.70 | −0.20 | 0.98994 | |
| 45 | 1793 | II | Perny | 2.86 | 5.5 | 2.62 | +0.17 | .97342 | |
| 46 | 1860 | III | | 2.92 | 5.3 | 3.04 | −0.53 | .99724 | |
| 47 | 1906 | VII | Thiele | 2.95 | 6.9 | 2.76 | +0.08 | .98262 | |
| 48 | 1853 | II | Schweizer | 3.02 | 6 | 2.89 | −0.04 | .98931 | |
| 49 | 1769 | ..... | Messier | 3.02 | 3.2 | 3.32 | .91 | .99925 | |
| 50 | 1763 | ..... | Messier | 3.06 | 9.5 | 3.06 | .30 | .99543 | |
| 51 | 1811 | II | Pons | 3.13 | 5.2 | 2.88 | +0.20 | .98092 | |
| 52 | 1894 | II | Gale | 3.13 | 6.3 | 2.98 | −0.01 | .98990 | |
| 53 | 1827 | III | Pons | 3.14 | 6.2 | 3.42 | .86 | .99927 | |
| 54 | 1807 | ..... | | 3.29 | 2.5 | 3.23 | .19 | .99549 | |
| 55 | 1911 | V | Brooks | 3.33 | 5.1 | 3.33 | .31 | .99704 | |
| 56 | 1857 | V | Klinkerfues | 3.37 | 4.9 | 3.39 | .25 | .99691 | |
| 57 | 1909 | I | Borrelly | 3.42 | 10.9 | 3.31 | .07 | .99476 | |
| 58 | 1858 | VI | Donati | 3.43 | 1.0 | 3.40 | .24 | .99686 | |
| 59 | 1881 | III | Tebbutt | 3.47 | 2.8 | 3.39 | .11 | .99574 | |
| 60 | 1847 | I | Hind | 3.50 | 6.8 | 4.03 | 1.37 | .99991 | |
| 61 | 1826 | V | Pons | 3.63 | 7.0 | 4.44 | 1.57 | .99997 | |
| 62 | 1811 | I | Flaguergues | 3.64 | 0.6 | 3.47 | +0.02 | .99571 | |
| 63 | 1893 | IV | Brooks | 3.65 | 6.7 | 3.54 | −0.09 | .99649 | |
| 64 | 1825 | I | Gambart | 3.72 | 5.6 | 3.59 | .05 | .99639 | |
| 65 | 1864 | II | Tempel | 3.73 | 6 | 3.60 | .04 | .99635 | |
| 66 | 1871 | I | Winnecke | 3.77 | 6.3 | 3.72 | .18 | .99781 | |
| 67 | 1826 | IV | Pons | 3.91 | 6.5 | 3.80 | .17 | .99749 | |
| 68 | 1822 | IV | Pons | 3.91 | 2.2 | 3.74 | +0.06 | .99630 | |
| 69 | 1780 | II | Olbers-Montaigne | 3.94 | 6 | 3.97 | −1.02 | .99908 | |
| 70 | 1858 | VII | Tuttle | 3.98 | 5.9 | 3.78 | +0.15 | .99600 | |
| 71 | 1911 | VI | Quénisset | 4.07 | 6.6 | 3.96 | −0.10 | .99821 | |
| 72 | 1855 | IV | Bruhns | 4.19 | 8.0 | 3.98 | +0.09 | .99726 | |
| 73 | 1874 | III | Coggia | 4.20 | 4.7 | 4.14 | −0.17 | .99882 | |
| 74 | 1901 | I | Viscara | 4.43 | 5.9 | 4.59 | .61 | .99979 | |
| 75 | 1877 | II | Winnecke | 4.43 | 5.7 | 4.29 | .02 | .99870 | |
| 76 | 1892 | I | Swift | 4.54 | 3.3 | 4.39 | +0.01 | .99878 | |
| 77 | 1903 | I | Giacobini | 4.59 | 6.8 | 4.63 | −0.39 | .99967 | |
| 78 | 1850 | I | Petersen | 4.62 | 6.0 | 4.46 | +0.03 | .99855 | |
| 79 | 1873 | V | Henry | 4.68 | 6.4 | 4.73 | −0.42 | .99973 | |
| 80 | 1893 | II | Rordame | 4.71 | 6.6 | 4.65 | .17 | .99946 | |
| 81 | 1819 | II | | 4.74 | 5.0 | 4.82 | .47 | .99979 | |
| 82 | 1845 | II | de Vico | 4.89 | 7.5 | 4.69 | +0.10 | .99906 | |
| 83 | 1905 | VI | Brooks | 5.01 | 5.1 | 4.80 | .11 | .99919 | |
| 84 | 1911 | II | Kiess | 5.09 | 7.7 | 5.02 | −0.16 | .99969 | |
| 85 | 1844 | II | Mauvais | 5.12 | 4.9 | 5.00 | .07 | .99960 | |
| 86 | 1882 | I | Wells | 5.16 | 4.1 | 5.61 | 1.12 | .99999 | |
| 87 | 1910 | I | | 5.19 | 6.9 | 5.50 | .89 | .99997 | |
| 88 | 1845 | III | Colla | 5.44 | 6.0 | 5.49 | .40 | .99991 | |
| 89 | 1898 | X | Brooks | 5.61 | 9.1 | 5.20 | .12 | .99880 | |
| 90 | 1902 | III | Perrine-Borrelly | 5.61 | 5.8 | 6.15 | .40 | .99999 | |
| 91 | 1779 | ..... | Bode | 5.67 | 5.8 | 5.49 | .15 | .99984 | |
| 92 | 1849 | I | Petersen | 5.72 | 6.0 | 5.58 | .02 | .99982 | |
| 93 | 1826 | II | Pons | 6.44 | 2.4 | 6.14 | +0.30 | .99984 | |
| 94 | 1914 | V | Delavan | 7.67 | 1.2 | 7.38 | .04 | .99998 | |

By formulae (7) and (8) the absolute magnitude of a comet is equal to $-2.5\ \Phi\ (a, e)+$ const. If all comets came into existence in a short time and were equally bright at the beginning, the values of $H_o$ must be definitely correlated with the values of $\Phi\ (a, e)$. The problem is to determine the degree of this correlation. We introduce here the following notation

$$(10)\quad \begin{cases} -\Phi\ (a, e) = X \\ H_o \quad\quad = Y \end{cases}$$

and shall consider $Y$ as a function of $X$. On Fig. 1 $Y$ is plotted against $X$. It is clear from the diagram that the correlation between $X$ and $Y$ is beyond any doubt, notwithstanding much scattering. Large $Y$'s correspond generally to small $X$'s (algebraically) and *vice versa*. The scattering may be due to several causes. First, the adopted value $n=4$ in (1) is true only on the average and may considerably depart from it in individual cases. Second, it is probably not even

approximately true that all comets were equally bright in the beginning. Third, the period of revolution is not known with sufficient accuracy in many cases, and the present orbits may depart very considerably from the original, due to the action of planets and to other factors. Such comets as 1807, 1811 I, 1858 VI are much more luminous than the average. On the other hand, short periodic comets Perrine and d'Arrest are just as much below the average. The problem is still more complicated by the uncertainty in the estimate of the brightness of comets, especially in the eighteenth and the first half of the nineteenth century, and by the possibility that the rate of disintegration may depend not only on $a$ and $e$, but also on $i$ and other elements. Under these circumstances the deviations from the average on Fig. 1 cannot receive any simple interpretation. Three comets with exceedingly small perihelion distances, 1843 I, 1680 and 1882 II, do not conform to the general run of brightness at all. It is probable that such a close approach to the Sun results in the partial evaporation of their nuclei, a process entirely different from the normal fluorescence of comets. Accordingly they have not been included in the present investigation.

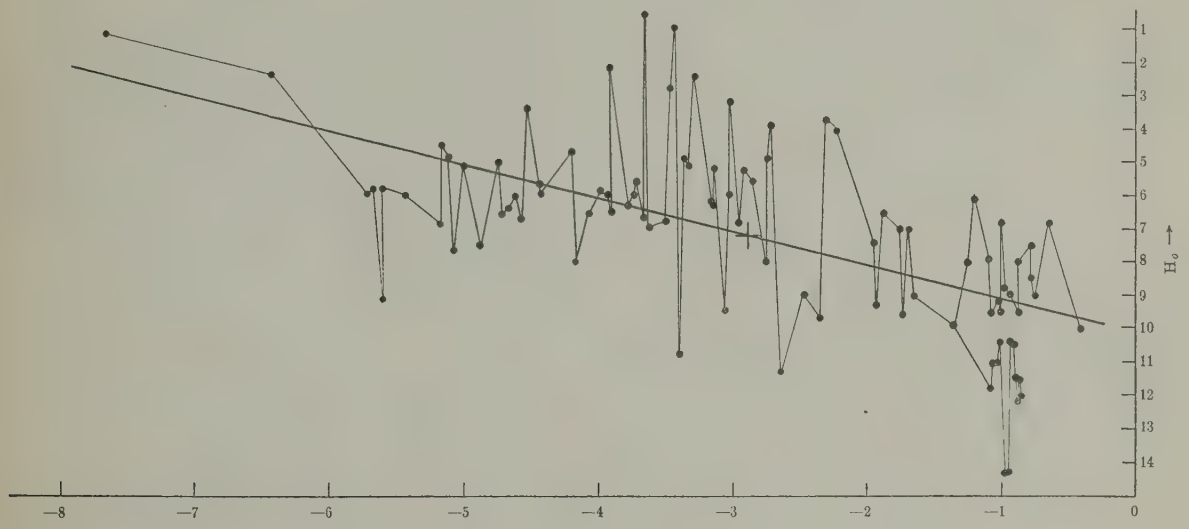


Fig. 1

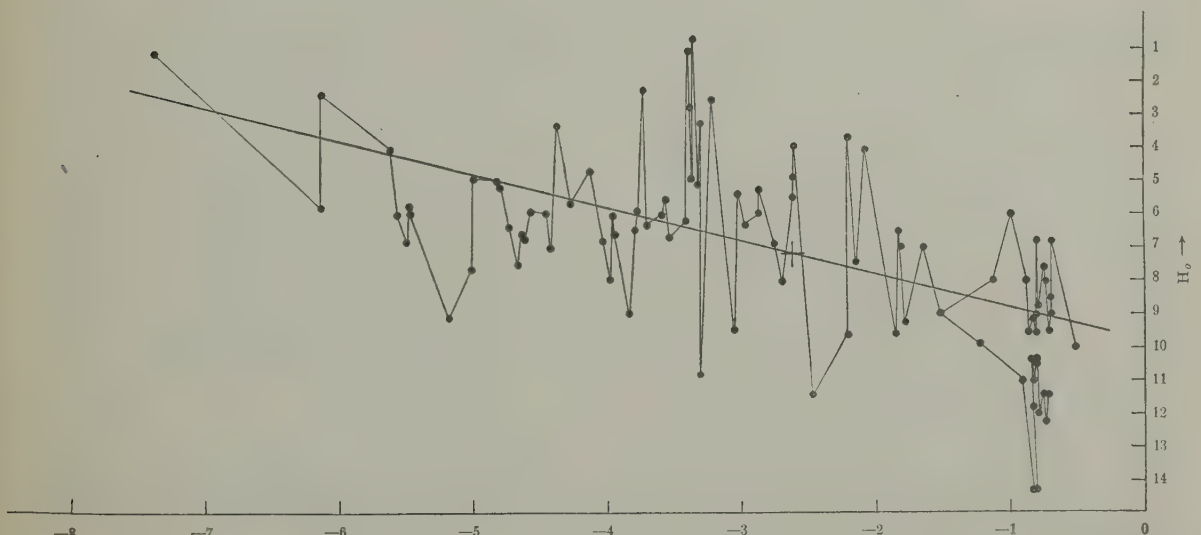


Fig. 2

Figs. 1 and 2. Scatter Diagrams for Data of Table 1. Crosses denote mean values of $X$ and $Y$. The ordinates are $H_o$. The abscissae on Fig 1 are values of $\Phi(a, e) = 2 \log a + \log \sqrt{1-e^2}$; Those on Fig. 2 values of $\log P$. Lines of regression of $Y$ on $X$ are represented by heavy straight lines.

It is evident from Fig. 1 that the regression does not noticeably depart from linear. Applying to this case the methods of mathematical statistics,[6] and denoting with bars the mean values, we have:

$$(11)\quad \begin{cases} x = X - \overline{X} \\ y = Y - \overline{Y} \end{cases}$$

The numerical values of the auxiliary quantities are as follows:

$$(12)\quad \begin{cases} n = 94 \\ \overline{X} = -2.89 \\ \overline{Y} = +7.16 \\ \Sigma x^2 = 258.36 \\ \Sigma y^2 = 736.49 \\ \Sigma xy = +264.97 \end{cases}$$

The standard deviations are:

$$(13)\quad \begin{cases} \sigma_x = 1.658 \\ \sigma_y = 2.799 \end{cases}$$

The coefficient of correlation, $r = \dfrac{\Sigma xy}{n\sigma_x\sigma_y}$, is found to be

$$(14)\quad r = 0.608 \pm 0.043,$$

that is, the probable error is about seven per cent of the value of $r$. These figures show a marked degree of correlation. The probability that this is a chance result is very small.

The equation of the line of regression of $Y$ on $X$ is:

$$(15)\quad Y - \overline{Y} = r\frac{\sigma_y}{\sigma_x}(X - \overline{X}).$$

Substituting the numerical values from (12), (13), and (14), we obtain:

$$(16)\quad \begin{cases} Y = 10.1 + X, \text{ or} \\ H_o = 10\overset{m}{.}1 - 2\log a - \log\sqrt{1-e^2}, \end{cases}$$

which gives the absolute magnitude of a comet as a function of elements $a$ and $e$. The line of regression is shown on Fig. 1 as heavy straight line. The standard error of estimate of $Y$, $s_y = \sigma_y\sqrt{1-r^2}$, comes out $1\overset{m}{.}76$, that is, we cannot, with any considerable degree of reliability, predict $H_o$ from the elements of the orbit. However, if we have a sufficiently large number of cases, we can give a very reliable mean value of $H_o$.

As has been mentioned above, formula (16) may be considerably simplified and $H_o$ may be represented as a function of log $P$ only (9). The values of $H_o$ are plotted against log $P$ in Fig. 2. The general run of the curve is quite similar to that of Fig. 1 and we should expect the correlation to be nearly the same. A direct solution has been made for $X = -\log P$ and $Y = H_o$, with the results:

$$(17)\quad \begin{cases} n = 94 \\ \overline{X} = -2.62 \\ \overline{Y} = +7.16 \\ \Sigma x^2 = 277.22 \\ \Sigma y^2 = 736.49 \\ \Sigma xy = +276.70 \\ \sigma_x = 1.718 \\ \sigma_y = 2.799 \\ r = 0.600 \pm 0.044, \end{cases}$$

that is, the correlation is sensibly the same as before. The equation of the line of regression is:

$$(18)\quad H_o = 9\overset{m}{.}8 - \log P,$$

quite similar to (16).

We have put $n = 4$ in (1) as this proves to be its mean value. It can be shown that the results will not differ much if we take for $n$ any number not too large, as log $P$ will be the decisive factor in determining the value of $H_o$. The correlation between $H_o$ and log $P$ will be essentially the same through a considerable range of $n$. We shall show this on two easily integrable cases with $n = 3$ and $n = 5$.

(a) $n = 3;\ n - 2 = 1.$

$$(19)\quad J = \int_0^P \frac{A\,dt}{r} = A\int_0^{2\pi} \frac{r\,dv}{k\sqrt{a(1-e^2)}}$$

$$= \frac{A\sqrt{a(1-e^2)}}{k}\int_0^{2\pi} \frac{dv}{1+e\cos v}$$

the numerical value of the integral is

$$(20)\quad \int_0^{2\pi} \frac{dv}{1+e\cos v} = \frac{n\pi}{\sqrt{1-e^2}}$$

Substituting (20) in (19) we obtain:

$$(21)\quad J = A'\sqrt{a} = A'\sqrt[3]{P}, \text{ and } J' = \frac{A'B}{P^{\frac{2}{3}}}$$

The decrease in brightness $H'$ in magnitudes will be:

$$(22)\quad H' = 2.5\log(A'B) - \tfrac{5}{3}\log P,$$

that is $H_o$ should be correlated with log $P$.

(b) $n = 5;\ n - 2 = 3$

$$(23)\quad J = \int_0^P \frac{A\,dt}{r^3} = \frac{A}{k\sqrt{a(1-e^2)}}\int_0^{2\pi} \frac{(1+e\cos v)\,dv}{a(1-e^2)}$$

$$= \frac{2\pi A'}{P(1-e^2)^{\frac{3}{2}}}$$

$$J' = \frac{2\pi A'B}{P^2(1-e^2)^{\frac{3}{2}}}$$

[6] Rietz, *Handbook of Mathematical Statistics*, 1924, p. 120 ff.

The denominator $P^2(1-e^2)^{\frac{3}{2}}$ may be represented as $P\ q^{\frac{3}{2}}(1+e)^{\frac{3}{2}}$. The decrease in brightness in magnitude will be:

$$(24)\quad H' = \text{Const.} - 2.5 \log P - \tfrac{3}{2} \log [q(1+e)]$$

The expression $q(1+e)$ is approximately constant and close to unity, so that $H_o$ will again be correlated with $\log P$. Thus we see that the value of $H_o$ will depend primarily on $\log P$, and therefore our assumption of $n=4$ is not essential for obtaining a correlation between $H_o$ and $\log P$.

The numerical relations (16) and (18) can be considered as purely empirical. They have been suggested by theory, but the role of theory was only to show what kind of function of $a$ and $e$ should be adopted as most probably correlated with $H_o$. As soon as that had been settled, the remaining part of the problem was to find the regression line which fits the observed facts best. It is doubtful whether there can be found any line of regression which would fit better than a straight line. At any rate the improvement would be small and void of any significance if we take into account the possibility of errors discussed above. The sources of error are too numerous to draw any detailed conclusions from figures 1 and 2.

There is, however, one peculiarity of the diagrams which may have some meaning. The decrease of the brightness of comets may be due to two causes: to the regular dissipation of their matter in space producing slow secular changes, and to their sudden disrupture, due to the tidal action of the Sun and planets, or to the internal forces. It seems probable that many short-periodic comets had been of long period, before they were captured by Jupiter and perhaps by other planets.[7] Frequent disrupture of comets is also an established fact. We may therefore expect among the short period comets two groups. One of them is formed by brighter comets comparatively recently made short-period comets. Their average brightness corresponds to a much longer period. The other group consists of abnormally faint comets, which have long been in the confines of the planetary system and have advanced farther to their ultimate decomposition. Fig. 1 shows some evidence for the existence of these two groups. It seems that the line of regression near $X=-2$ splits into two distinct branches. The upper branch descends very slowly to the axis of $X$. It corresponds to the short period comets recently captured by planets. The lower branch shows older short period comets. The space between these two branches is comparatively empty.

The relative abundance of gases around the cometary nuclei may also serve as a criterion of disintegration. The comets with smaller dissipation function $-\Phi(a, e)=X$, should possess more of these gases than the short period comets. The abundance of cometary gases may be estimated by the difference between the total magnitude of the comet, $H_o$, and the magnitude of its nucleus, $h_o$. The difference should be more pronounced in the comets with longer period. Table 2 and Fig. 3 give the available information for nine comets.

TABLE 2

| No. | Comet | | Name | $h_o$ | $H_o$ | $\Phi(a, e)$ | $h_o-H_o$ |
|---|---|---|---|---|---|---|---|
| 8 | 1899 | IV | Tempel 2 | $10^m.7$ | $9^m.4$ | 0.88 | $1^m.3$ |
| 20 | 1918 | IV | Borrelly | 11.3 | 10.0 | 1.02 | 1.3 |
| 29 | 1913 | III | Neujmin | 10.8 | 9.9 | 1.46 | 0.9 |
| 34 | 1910 | II | Halley | 8.2 | 6.5 | 1.89 | 1.7 |
| 55 | 1911 | V | Brooks | 8.1 | 5.1 | 3.33 | 3.0 |
| 59 | 1881 | III | Tebbutt | 5.7 | 2.8 | 3.47 | 2.9 |
| 80 | 1893 | II | Rordame | 10.6 | 6.6 | 4.71 | 4.0 |
| 84 | 1911 | II | Kiess | 11.2 | 7.7 | 5.09 | 3.5 |
| 89 | 1898 | X | Brooks | 11.8 | 9.1 | 5.61 | 2.7 |

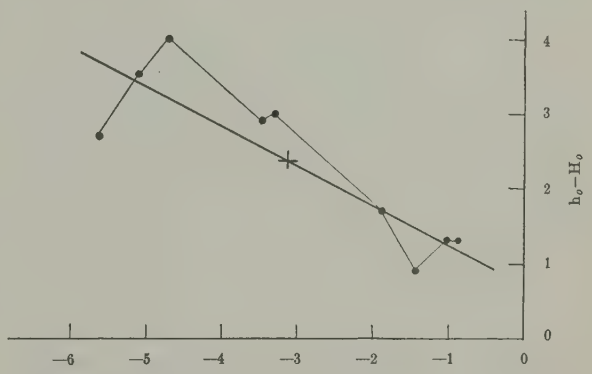


Fig. 3. Scatter Diagram for Data of Table 2. Abscissae $\Phi(a, e)$; ordinates $h_o-H_o$.

There seems to be no doubt that the proportion of light due to the nucleus alone is much greater in the short period comets, that is they are more impoverished in gases than the comets of very long period. Since there are only nine objects, a statistical investigation cannot give very reliable results. For the sake of completeness we give the numerical values of the quantities involved. Putting $-\Phi(a, e)=X$, and $h_o-H_o=Y$, we have:

$$(25)\quad \begin{cases} n=9 \\ \overline{X}=-3.05 \\ \overline{Y}=+2.36 \\ \Sigma x^2=26.495 \\ \Sigma y^2=9.624 \\ \Sigma xy=-13.893 \\ r=-0.871\pm0.054 \end{cases}$$

Such high degree of correlation between $\Phi(a, e)$ and $h_o-H_o$ can hardly be due to chance even with only nine objects. The equation of the line of regression is:

$$(26)\quad Y=-0.53\ X+0.68.$$

[7] See H. N. Russell, *A. J.* **33**, 49, 1920.

The formation of the tail evidently stands in connection with the abundance of gases in the comet. The length of the tail is something evasive, due to arbitrariness of its estimation. Still Holetschek brought weighty evidence that the maximum length of the tail depends on the absolute brightness of comets: the brighter the comet, the longer is its tail. Kritzinger (*loc. cit.*) has even established an empirical formula for the value of $H_o$ at which we might expect the development of a tail.

Vsechsviatsky has shown (*loc. cit.*) that the average absolute brightness of comets increases with the increase of $i$. This he interpreted in the sense that the comets with smaller $i$ remain longer under the influence of planets with a greater probability of disintegration. Although these results must be influenced somewhat by the conditions of visibility, established by Holetschek, they cannot be wholly accounted for on that ground.

The results arrived at in this investigation are of considerable cosmological interest. The process of disintegration is not confined to the short period comets but all comets are subject to it. The average absolute brightness of comets is definitely correlated with their orbital elements $a$ and $e$, or $P$. The objection that the original elements of comets may be different from those of the epoch of osculation can hardly have much weight in a statistical investigation. Furthermore, if we consider only the comets with $P<1000$, the elements of which are known better, the degree of correlation would not change much. In fact, the deviations from the regression line are much greater after $P>1000$, which may be partially explained by possible errors in $P$. There seem to be no strong selective factors influencing the sampling of material, such as the conditions of visibility. The 94 comets of the present investigation may be considered as a fair sample of comets.

In their obedience to the law of disintegration comets form a system all members of which follow the same career with the same ultimate fate. The matter in comets must be in a very peculiar state of excitation which does not occur often in the universe. Yet all comets have fundamentally the same spectrum, regardless of their orbital elements. These facts constitute strong evidence for the common origin of comets.

The system of comets is closed, that is we have no influx of fresh comets coming from the depths of the universe. If we had any, it would affect the correlation between their orbital elements and the average absolute brightness. Indeed, such correlation would be incomprehensible.

On the other hand, this correlation shows that the disintegration of comets is an irreversible process. There is no building up of new comets from the meteoric and gaseous matter diffused in space. Sir William Herschel pictured comets gathering matter while far from the Sun, literally sweeping the heavens. This might compensate or even reverse their disintegration while near perihelion. Some of the modern investigators adopted a similar view, but in no case has any proof or even plausible explanation of this process been offered.[8] Even if such a thing were possible this would not improve the situation at all. The life of a comet is a function of its initial mass, and the accretion of dust and gases during its passage in the interplanetary space would only increase the rate of dissipation of its matter near perihelion, not affecting the fundamental process of disintegration at all.

Comets are gradually losing their power to form a tail and their gaseous envelopes disappear in the course of time. They are ultimate sources of meteors in our solar system, and may be also responsible for the presence in it of larger bodies, like asteroids. Indeed a connection between comets and asteroids has long been suspected and Leuschner has recently brought considerable evidence to support this view.[9]

If all comets have a common origin, they could not come into existence simultaneously with the planets. The possible age of comets seems to be inconsistent with the age of the solar system.[10] According to Vsechsviatsky[11] the rate of decomposition of Encke's comet is about $1^m$ in a century, that is about $0^m.033$ per revolution. From formula (5) the ratio of the decrease in brightness per revolution for two comets is:

$$(27)\quad \frac{J_1}{J_2} = \frac{\sqrt{a_2(1-e)_2^2}}{\sqrt{a_1(1-e)_1^2}} = \sqrt{\frac{q_2}{q_1}}\sqrt{\frac{(1+e_2)}{(1+e_1)}}$$

The last factor is very close to unity. As the bright long period comets have usually small $q$ their rate of decrease in brightness must be greater than that of Encke's comet. Taking a well marked group of comets (Fig. 2) with $H_o=6^m$ and log $P=3.5$, we find that the decrease in their brightness must be at least $1^m$ in 30 revolutions. We can assume that the original average brightness of comets was magnitude 1. This gives the decrease in brightness of $5^m$ since the time when they became members of the solar system. From that time to the moment of observation they had performed about 150 revolutions which gives 760,000 years as their probable age. This is less than one thousandth of the age of the Earth.

Of course the estimate of the age of comets depends on what we assume as their average initial magnitude

[8] See e.g. E. Thomson, *Sci.* **31**, 772, 1910; T. C. Chamberlin, *Two Solar Families*, 1928.

[9] *Publ. A. S. P.* **39**, 275, 1927.

[10] This circumstance has already been noticed by other investigators. See e.g. Belot, *C. R.* **184**, 937, 1927.

[11] *Russ. Astr. Jour.* **4**, 298, 1927.

and other assumptions such as the uniform rate of disintegration. However, it is improbable that comets ever were brighter than the first magnitude on the average, as their instability increases very rapidly with the increase of their volume. It is significant that among 94 comets of this investigation only one was brighter than $1^m$ (Comet 1811 I, $H_o=0.6$). One million years, therefore, is probably a liberal estimate of the age of comets.

If Encke's comet decomposes at the rate of $1^m$ per century, then its life must be very short. In a thousand years it will sink to $H_o=20^m$ and will be hardly accessible to observation. Probably all short period comets became such comparatively recently. The original short period comets disappeared long ago. There is, however, one more consideration that makes the estimate of the age of comets at 1,000,000 years seem too high, and the rest of their life too short.

In many natural processes where we have to deal with the disintegration of matter, such as radioactivity, the rate of the loss of matter is proportional to the amount of matter present. We have then the equation:

$$(28) \quad dm = -Km\Psi(a, e)dt,$$

where $m$ is the mass at the moment $t$, $\Psi$ is a function different from $\Phi$, and $K$ is a constant. Integrating this equation we obtain:

$$(29) \quad m=K_1 m_o \epsilon^{-\int_{t_o}^{t}\Psi(a,e)\,dt}$$

where $m$ is the original mass corresponding to the moment $t_o$. It is probable, therefore, that the mass, and presumably the brightness, of comets decreases exponentially. The rate of decomposition of brighter comets must be much greater than that of the fainter, and it must decrease very rapidly, approaching zero.

We have then evidence, on one hand, that comets are true members of our solar system; they do not come from interstellar space. On the other hand, their age seems to be incompatible with the age of the major planets; no comet could have survived ten billion years. It is possible to reconcile these results by postulating the capture of comets by the Sun a comparatively short time ago, not exceeding one million years. We may think that the Sun appropriated them during one of its passages through diffuse clouds of obscuring matter. Such a theory has been developed by F. Nölke[12] on entirely different grounds. His investigation is based on the mathematical study of the problem and is in complete accordance with the results arrived at in this paper. Nölke's estimate of the span of time between the adoption of comets by the solar system and the present is from 50,000 to 100,000 years.

[12] Neue Erklärung des Ursprungs der Kometen, *Abh. d. Nat. Ver.* Bremen, **20**, 29, 1909.

The Sun is travelling through space with the velocity of about 20 km/sec. This means it covers the distance of one light-year in 15,000 years. It came from the region of *Orion* where agglomerations of meteoric matter are suspected.[13] A million years ago our solar system was some 70 light years away from its present position in the general direction of *Orion*. These considerations give a certain weight to the capture theory of comets. Nölke goes so far as to suggest the possible explanation of the Earth's glacial periods by the action of the diffuse matter through which the Sun once passed. We may note here that there are indications that meteorites were less abundant in the early geological epochs. The conspicuous absence of meteorites from the strata older than one million years has been interpreted by some investigators (notably by Sir Robert Ball) as an argument for their terrestrial origin. It may with equal right be regarded as an indication of the absence of meteorites from the solar system before the quartenary epoch.[14]

As is well known, the question of the origin of comets turns on the character of their orbits. The absence of decidedly hyperbolic orbits has been regarded as a sure sign of the solar origin of comets. However, we must not forget that all investigations on the probability of elliptic and hyperbolic orbits are based on assumptions, chiefly on the value of the distribution of velocities function.[15] With different assumptions the results are different. Although the a priori probability of hyperbolic comets is greater than that of the elliptic, the actual probability is reversed. The original hyperbolic comets went around the Sun and disappeared in space. Only those comets which happened to move in ellipses from the beginning could follow the Sun in its travel through space. Strömgren has shown that there are now no, or very few, comets which move in hyperbolic orbits. It does not follow from this that there were no such comets, say, one million years ago, as has been admitted by Strömgren himself.[16] Furthermore, application of Strömgren's method leads to somewhat ambiguous results. According to Van Biesbroeck[17] Delavan's Comet (1914 V) had an original orbit with the yearly parallax at aphelion of $1''.2$. For some of the comets investigated by Strömgren the aphelion distance must be still greater. We have to deal here with stellar distances, and with the possibility of perturbations from the nearest stars. Under these conditions the assertion that these comets have always been members of our solar system has little meaning.

[13] Shapley and Miss Payne, *Harv. Circ.* No. 317.

[14] For the development of this idea see R. Schwinner, Gerlands *Beitr. z. Geophysik*, **16**, 195, 1927 and *Forschungen und Fortschritte*, **3**, 165, 1927.

[15] See, Oppenheim, Kometen, in *Enc. Mat. Wiss.* **6**, 2A, p. 926.

[16] *Über den Ursprung der Kometen*, 1914, p. 62.

[17] *Publ. Yerkes Obs.* **5**, Part 2, 1927.

The distribution of perihelia (or aphelia) of comets shows some regularities which cannot be accounted for, if they have been in the solar system since its origin. Comets show a decided preference for the Milky Way as the fundamental plane. Out of more recent and thorough investigations on the subject should be mentioned those by W. H. Pickering,[18] K. Jantzen,[19] and Oppenheim.[20] Oppenheim applied to the problem the ellipsoid method, developed earlier by Svedstrup. In the distribution of perihelia the short period comets are quite similar to asteroids. They have the fundamental plane coinciding with the plane of the ecliptic, and the action of *Jupiter* is clearly noticeable in the grouping of perihelia toward the perihelion of *Jupiter*. For the long period comets ($P>1000$ years) the principal plane of the ellipsoid nearly coincides with the plane of the Milky Way and the direction of the minor axis coincides with the vertex of stellar motion. Oppenheim's view is that all comets have been acquired by the solar system in its motion through space. The short period comets have been so thoroughly assimilated by the solar system as to show the properties of planetary orbits. The process of assimilation has not proceeded so far with the long period comets.

It seems from the foregoing that comets are vestiges of a nebula through which our solar system passed comparatively recently. Whether this hypothesis can account for the absence of the hyperbolic orbits and the peculiarities in the distribution of perihelia it is impossible to say without a detailed investigation. It must be kept in view that all deductions from a statistical study based on a comparatively small number of objects have limited value. They show the trend of events rather than give a complete and definite answer.

[18] *Harv. An.*, **61**, Part 3, 1911.

[19] *Statistik der Kometenbahnelemente*, 1912.

[20] *Festschrift für Seeliger*, 1924, p. 131.

## SUMMARY


1. The absolute brightness of 94 periodic comets has been investigated. There is a definite correlation between the absolute magnitude of a comet and the value of the dissipative function $\Phi(a, e)$, derived theoretically.

2. An empirical formula for the absolute magnitude, $H_o$, of a comet has been derived as follows:

$$H_o = 10^m1 - 2 \log a - \log \sqrt{1-e^2},$$

or very approximately:

$$H_o = 9^m0 - \log P.$$

The coefficient of correlation in the first case is

$$r = 0.608 \pm 0.043,$$

and in the second:

$$r = 0.600 \pm 0.044.$$

3. The abundance of gases in the head and the tail of comets shows correlation with the same function $\Phi(a, e)$.

4. The upper limit of the age of comets is estimated from their rate of disintegration at one million years. This seems to be inconsistent with the view that the comets came into existence simultaneously with the planets and have always been members of the solar system.

5. As possible explanation of these facts a capture theory of comets is suggested, which is in accordance with some modern investigations on the origin of comets.



The writer wishes to express his obligation to Professors A. O. Leuschner of the University of California and E. B. Wilson of Harvard for their interest in this paper and valuable criticisms and suggestions.

Mt. Hamilton, January 17, 1929.

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ASTRONOMY

# LICK OBSERVATORY BULLETIN

NUMBER 409

## INVESTIGATION OF THE ORBIT, MASS AND PARALLAX OF 99 HERCULIS

BY

MAUD W. MAKEMSON

For determining the orbit of the visual binary 99 *Herculis* there were available, through the kindness of Dr. R. G. Aitken, 82 observations from 1859 to 1926 extending over slightly more than a period of revolution. A discontinuity in the measurements occurred during the critical years immediately preceding and following periastron passage in 1887. Discounting two discordant positions by O. Struve in 1883 and 1888 there were left only observations by Burnham in the interval between 1878 and 1892, those prior to 1888 being made with the 18½-inch Dearborn refractor, the later ones with the 36-inch at Lick Observatory.

From the graphs of the position angles and the distances an approximate value of 56.5 years was obtained for the period. To fix the orbit on both sides of periastron, Burnham's measurements of 1880.18 and 1890.45 were chosen as the first and second observations on which to base an orbit, while the third was a normal place derived from a number of positions at apastron by Aitken, Comstock and Fox.

After some preliminary computation an orbit was deduced on the assumption of a 56.3 year period as follows:

Positions for the equinox of 1900.0:

| | | | |
|---|---|---|---|
| (1) | 1880.18 | 30°.6 | 0″.91 |
| (2) | 1895.50 | 308.0 | 1.04 |
| (3) | 1912.50 | 341.0 | 1.56 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *a* | 0″.759 | *A* | 197°.07 | a | 0″.877 | A | 165°.89 |
| *b* | 0.780 | *B* | 305.45 | b | 0.645 | B | 262.21 |
| | | | | c | 0.682 | | |

*a* and *b* are conjugate diameters in the apparent orbit and *A* and *B* their corresponding directions. a is the diameter through the principal star and the center of the apparent ellipse, b its conjugate, and A and B the corresponding directions; c is the apparant distance of the principal star from the center.

| | | |
|---|---|---|
| Elements: | *P* | 56.3 years |
| | *T* | 1887.67 |
| | *e* | .777 |
| | *a* | 1″.08 |
| | *ω* | 75°.55 |
| | *i* | 37.22 |
| | Ω | 93.76 |

As a comparison between this orbit and all the observed position angles gave a minus sign at the beginning and a plus sign at the end, the possibility of obtaining a more satisfactory representation by a slight reduction in the period was inferred. Additional small corrections were applied to the first and second places in order to give way to the tendency of the residuals to retard the motion before periastron passage and to accelerate it afterwards, resulting in the following fundamental data:

Relative positions, equinox of 1900.0:

| | | | |
|---|---|---|---|
| (1) | 1880.18 | 28°.6 | 0″.91 |
| (2) | 1895.50 | 309.0 | 1.04 |
| (3) | 1912.50 | 341.0 | 1.56 |

The new values for the conjugate diameters and the elements were:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *a* | 0″.750 | *A* | 195°.94 | a | 0″.872 | A | 165.57 |
| *b* | 0.768 | *B* | 305.62 | b | 0.626 | B | 262.09 |
| | | | | c | 0.787 | | |

Final elements:

| | |
|---|---|
| *P* | 56.0 years |
| *T* | 1887.69 |
| *e* | .787 |
| *a* | 1″.03 |
| *ω* | 71°.61 |
| *i* | 34.20 |
| Ω | 97.46 |

In the following table are given the observed position angles and distances, the residuals from the second

orbit, the number of nights on which the measurements were made, and the observer:

Abbreviations:

| | | | |
|---|---|---|---|
| OΣ | Otto Struve | Doo | Doolittle |
| β | Burnham | VBs | van Biesbroeck |
| Com | Comstock | Lo | Lohse |
| Bar | Barnard | Jon | Jonckheere |
| Ho | Hough | Lv | Leavenworth |
| Mo | Moulton | Prz | Przybyllok |
| A | Aitken | Ber O | Berlin Observatory |
| Gr O | Greenwich Obs. | Bos | van den Bos |
| Hu | Hussey | Pal | Paloque |

| Date | $\theta_o$ | $\rho_o$ | (o−c) $\Delta\theta$ | (o−c) $\Delta\rho$ | $n$ | |
|---|---|---|---|---|---|---|
| 1859.63 | 347°.1 | 1″.71 | + 1°.6 | +″.15 | 2 | Dawes |
| 1865.16 | 353.9 | 1.90 | − .2 | +.40 | 3 | OΣ |
| 1875.06 | 14.0 | 1.32 | − 1.1 | +.11 | 2 | OΣ |
| 1878.46 | 24.4 | 0.99 | + 2.2 | −.03 | 3 | β |
| 1880.18 | 29.9 | 0.91 | + 1.3 | 00 | 3 | β |
| 1881.43 | 29.4 | 0.51 | − 5.0 | −.31 | 1 | β |
| 1883.65 | 77.6 | 1.17 | +28.7 | +.54 | 2 | OΣ |
| 1888.54 | 77.4 | 1.05 | −158.7 | +.79 | 1 | OΣ |
| 1889.50 | 281.2 | 0.65 | +14.6 | +.23 | 1 | β |
| 1890.45 | 285.1 | 0.56 | + 5.2 | 00 | 3 | β |
| 1891.56 | 294.4 | 0.72 | + 3.9 | +.02 | 3 | β |
| 1892.40 | 299.2 | 0.70 | + 3.2 | −.08 | 3 | β |
| 1894.73 | 305.7 | 0.88 | − .6 | −.10 | 1 | Com |
| 1895.30 | 307.0 | 0.84 | − 1.3 | −.18 | 1 | See |
| 1895.46 | 309.5 | 1.04 | + .6 | 00 | 6 | Bar |
| 1895.62 | 297.2 | 0.89 | −12.2 | −.16 | 1 | Ho |
| 1895.73 | 315.2 | 1.12 | + 5.5 | +.07 | 2 | See |
| 1895.73 | 313.4 | ..... | + 3.7 | ..... | 2 | Mo |
| 1895.80 | 309.6 | 1.00 | − .3 | −.06 | 3 | Com |
| 1896.65 | 311.9 | 1.07 | − .5 | −.04 | 3 | Com |
| 1896.70 | 310.3 | 0.84 | − 2.3 | −.27 | 3 | A |
| 1897.54 | 311.1 | 1.00 | − 3.7 | −.16 | 4 | A |
| 1898.00 | 308.1 | 0.93 | − 7.8 | −.26 | 5 | Gr O |
| 1898.15 | 315.2 | 1.07 | − 1.1 | −.13 | 4 | Com |
| 1898.33 | 320.2 | 1.15 | + 3.4 | −.06 | 3 | β |
| 1899.29 | 319.7 | 1.23 | + 1.3 | −.03 | 2 | Hu |
| 1899.34 | 317.3 | 1.25 | − 1.8 | −.01 | 3 | A |
| 1899.41 | 317.7 | 1.37 | − 1.5 | +.10 | 3 | Doo |
| 1900.54 | 319.0 | 1.27 | − 2.6 | −.04 | 4 | A |
| 1901.53 | 322.1 | 1.31 | − 1.3 | −.03 | 4 | A |
| 1902.33 | 323.1 | 1.29 | − 1.8 | −.07 | 2 | A |
| 1902.49 | 317.2 | 1.20 | − 8.0 | −.17 | 2 | Gr O |
| 1902.59 | 325.2 | 1.49 | − .2 | +.12 | 9 | Com |
| 1903.37 | 332.2 | 1.54 | + 5.4 | +.15 | 7 | Doo |
| 1903.65 | 324.0 | 1.42 | − 3.3 | −.02 | 1 | A |
| 1903.71 | 324.5 | 1.54 | − 2.9 | +.14 | 3 | VBs |
| 1905.51 | 332.4 | 1.34 | + 2.9 | −.12 | 2 | VBs |
| 1905.62 | 327.4 | 1.31 | − 3.2 | −.15 | 2 | A |
| 1905.66 | 324.2 | 1.37 | − 6.5 | −.09 | 2 | β |
| 1906.11 | 329.5 | 1.43 | − 1.9 | −.04 | 4 | Com |
| 1906.66 | 331.1 | 1.49 | − 1.0 | +.01 | 3 | Lo |
| 1908.45 | 333.3 | 1.46 | − 1.8 | −.02 | 4 | Com |
| 1908.83 | 337.1 | 1.56 | + 1.4 | +.04 | 6 | Doo |
| 1909.00 | 337.6 | 1.36 | + 1.7 | −.17 | 6 | VBs |
| 1910.29 | 342.0 | 1.50 | + 4.2 | −.04 | 3 | A |
| 1910.48 | 337.9 | 1.50 | − .2 | −.05 | 2 | Jon |
| 1910.69 | 335.4 | 1.60 | − 3.0 | +.05 | 1 | Wirtz |
| 1910.94 | 336.9 | 1.64 | − 1.8 | +.08 | 4 | Com |
| 1911.30 | 340.5 | 1.46 | + 1.2 | −.10 | 4 | VBs |
| 1912.57 | 340.3 | 1.42 | − .8 | −.14 | 2 | A |
| 1912.62 | 340°.8 | 1″.56 | − °.4 | ″.00 | 3 | Com |
| 1912.64 | 340.6 | 1.62 | − .6 | +.06 | 4 | Fox |
| 1912.76 | 345.2 | 1.49 | + 3.8 | −.07 | 4 | VBs |
| 1913.41 | 342.9 | 1.58 | + .6 | +.02 | 3 | Voûte |
| 1913.56 | 345.7 | 1.53 | + 3.2 | −.03 | 2 | Gr O |
| 1914.49 | 346.1 | 1.50 | + 2.2 | −.06 | 3 | VBs |
| 1914.61 | 343.2 | 1.45 | − .8 | −.10 | 2 | A |
| 1914.86 | 345.5 | 1.75 | + 1.1 | −.20 | 7 | Com |
| 1915.86 | 346.4 | 1.44 | + .6 | −.11 | 4 | VBs |
| 1917.06 | 350.3 | 1.70 | + 2.7 | +.17 | 6 | Com |
| 1917.56 | 350.6 | 1.63 | + 2.3 | +.09 | 4 | Baize |
| 1917.58 | 349.1 | 1.47 | + .8 | −.07 | 3 | VBs |
| 1918.64 | 348.9 | 1.50 | − .9 | −.04 | 2 | A |
| 1919.21 | 350.4 | 1.71 | − .4 | +.18 | 5 | Com |
| 1919.61 | 350.8 | 1.54 | − .4 | +.02 | 4 | Lv |
| 1921.47 | 355.9 | 1.45 | + 1.7 | −.05 | 4 | Bos |
| 1921.49 | 7.4 | 1.18 | +13.1 | −.32 | 4 | Prz |
| 1921.55 | 353.6 | 1.55 | − .8 | +.07 | 2 | A |
| 1921.59 | 358.2 | 1.51 | + 3.8 | +.02 | 5 | Ber O |
| 1922.59 | 358.0 | 1.64 | + 1.9 | +.17 | 9 | Ber O |
| 1922.64 | 356.2 | 1.75 | + .1 | +.28 | 2 | Lv |
| 1923.24 | 358.5 | 1.56 | + 1.4 | +.11 | 4 | Bos |
| 1923.62 | 359.8 | 1.51 | + 2.1 | +.07 | 4 | VBs |
| 1923.60 | 359.4 | 1.66 | + 1.7 | +.22 | 4 | Ber O |
| 1923.80 | 358.8 | 1.48 | + .8 | +.05 | 4 | Jackson |
| 1924.46 | 359.7 | 1.61 | + 1.2 | +.19 | 4 | Bos |
| 1924.73 | 0.2 | 1.61 | + .7 | +.19 | 4 | Ber O |
| 1925.34 | 1.8 | 1.42 | + 1.1 | +.02 | 4 | Bos |
| 1925.46 | 3.8 | 1.47 | + 2.9 | +.08 | 2 | VBs |
| 1925.48 | 3.1 | 1.46 | + 2.1 | +.07 | 5 | Ber O |
| 1925.60 | 7.0 | 1.19 | + 5.8 | −.19 | 1 | Pal |
| 1926.56 | 6.5 | 1.53 | + 3.5 | +.17 | 1 | Lv |

As it seemed impossible to improve the orbit any further by a modification of the period, the question of interpreting the large and consistent deviations before and after periastron passage was considered. The positive trend in the residuals after 1920, corresponding to that at the beginning, lent support to the theory that the differences might be due to perturbations arising from the presence of a third star or to the oblateness of the principal component. On the other hand there were only observations by Burnham on which to base conclusions, and these may have been affected by systematic error. Accordingly it seemed wise before pursuing the investigation, to represent the individual measures which were made available through the kindness of Dr. Aitken. The comparison yielded the following results:

Comparison with Burnham's Measurements in Position Angle

| | Computed | Observed | (o−c) | |
|---|---|---|---|---|
| 1878.458 | 22°.23 | 26°.6 | + 4°.4 | |
| .460 | 22.24 | 20.6 | − 1.6 | |
| .473 | 22.28 | 26.1 | + 3.8 | 18½″ |
| 1879.468 | 25.96 | 26.5 | + .5 | Dearborn |
| 1880.455 | 29.76 | 30.1 | + .3 | refractor |
| .613 | 30.41 | 33.1 | + 2.7 | |
| 1881.43 | 34.40 | 29.4 | − 5.0 | "excessively difficult" |

| | Computed | Observed | (o−c) | |
|---|---|---|---|---|
| 1888.733 | 242.77 | Apparently single | | |
| 1889.502 | 266.60 | 281.2 | +14.6 | |
| 1890.406 | 275.23 | 285.2 | +10.0 | |
| .463 | 280.00 | 285.0 | + 5.0 | |
| .466 | 280.04 | 285.0 | + 5.0 | 36-inch refractor |
| 1891.540 | 290.42 | 299.2 | + 8.8 | |
| .559 | 290.59 | 293.1 | + 2.5 | |
| .578 | 290.75 | 290.9 | + .1 | |
| 1892.400 | 296.05 | 300.0 | + 4.0 | |
| .403 | 296.07 | 298.8 | + 2.7 | |
| .422 | 296.16 | 298.9 | + 2.7 | |

Since the companion is again approaching periastron (1943) the development of the perturbations has been deferred in the hope that a large number of observations, including radial velocities, may be secured during the critical period. The system of 99 *Herculis* is of particular interest since the eccentricity is fifty per cent higher than the average and the period correspondingly smaller. The spectral type of the principal star is F8.

The radial velocities of the brighter component, measured at Lick Observatory, are as follows:

| | | |
|---|---|---|
| 1900 | June 25 | +1.2 km/sec |
| 1900 | July 10 | −0.7 |
| 1901 | July 3 | −0.6 |
| 1905 | Aug. 23 | +0.2 |
| 1907 | Sept. 1 | −0.6 |
| 1908 | June 14 | +2.0 |
| 1926 | July 26 | +2.6 |

An attempt was made to represent these radial velocities, extending over practically equal intervals on either side of apastron (1915), by assuming values for the parallax and deriving the mass of the system in terms of the Sun's mass by Kepler's harmonic law. The mass of the principal component was arbitrarily taken as 0.6 the total mass, and its radial velocity was computed for different values of the parallax, according to the formula found in Plummer's Dynamical Astronomy, p. 127:

$$V-\gamma=1730\mu \sin i \sec \varphi \,(\cos u+e\cos\omega)\,(a''/\pi''\,\frac{m_1}{m_1+m_2})$$

| | | Computed radial velocity for $m_1$ | | |
|---|---|---|---|---|
| $\pi''$ | $m_1+m_2$ | 1900.52 | 1908.45 | 1926.57 |
| 0".04 | 5.46 | −16.39 km/sec | −8.85 km/sec | +7.50 km/sec |
| 0.05 | 2.80 | − 6.71 | −3.62 | +3.07 |
| 0.06 | 1.62 | − 3.23 | −1.75 | +1.48 |
| 0.07 | 1.02 | − 1.75 | −0.94 | +0.80 |
| Observed velocities | | − 0.7 | −0.6 | +2.6 |
| (o−c) | | + 1.0 | +0.3 | +1.8 |

The value of $\gamma$, the radial component of the velocity of the center of mass, thus appears to be +1 km/sec. Other ratios for the masses would give somewhat different results. This particular value points to a measurable parallax.

From the proper motion (Boss) of −0$^s$.0073 in $\alpha$ and +0".063 in $\delta$ very little can be learned regarding the parallax, since the system is situated close to the apex of the Sun's Way. The small relative radial velocity indicates a motion of 99 *Herculis* in space nearly equal to that of the Sun.

The dynamical parallax and mass of the system were also determined by the method illustrated in Russell's Astronomy, Vol. II, p. 692, making use of the mass-luminosity curve. The values thus obtained were as follows:

$\pi''=0''.064$ $(m_1+m_2)=1.35$
$a$ = 16.1 astr. units
$q$ = 3.4 astr. units
$K$ = 4.67 km/sec. half range radial velocity for principal component

The final elements, together with the three orbits previously derived as well as an ephemeris for the approaching periastron passage, are as follows:

| | Aitken (1900) | Doberck (1903) | Lohse (1908) | Makemson (1929) |
|---|---|---|---|---|
| $P$ | 63.0 years | 64.52 years | 53.51 years | 56.0 years |
| $T$ | 1887.0 | 1884.00 | 1887.84 | 1887.69 |
| $e$ | 0.76 | 0.811 | 0.763 | 0.787 |
| $a$ | 1".00 | 1".28 | 1".11 | 1".03 |
| $\omega$ | 11°.3 | 120°.1 | 93°.7 | 71.61 |
| $i$ | 24.7 | 53.0 | 38.3 | 34.20 |
| $\Omega$ | 154.7 | 28.5 | 75.0 | 97.46 |

| Date | $\theta c$ | $\rho c$ | Date | $\theta c$ | $\rho c$ |
|---|---|---|---|---|---|
| 1928.50 | 6°.6 | 1".29 | 1937.50 | 34°.8 | 0".82 |
| 1929.50 | 8.8 | 1.25 | 1938.50 | 40.6 | .74 |
| 1930.50 | 11.1 | 1.21 | 1939.50 | 47.8 | .66 |
| 1931.50 | 13.6 | 1.17 | 1940.50 | 57.2 | .56 |
| 1932.50 | 16.3 | 1.12 | 1941.50 | 70.7 | .46 |
| 1933.50 | 19.2 | 1.07 | 1942.50 | 93.5 | .33 |
| 1934.50 | 22.3 | 1.01 | 1943.50 | 148.6 | .20 |
| 1935.50 | 25.7 | 0.95 | 1944.50 | 234.9 | .26 |
| 1936.50 | 30.0 | 0.89 | 1945.50 | 266.8 | 0.42 |

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ASTRONOMY

# LICK OBSERVATORY BULLETIN

NUMBER 410

# A PHOTOMETRIC STUDY OF THE CONTINUOUS SPECTRA OF GIANT AND DWARF STARS

BY

NORMAN WYMAN STORER

## INTRODUCTION

Recent work by Adams and Russell at the Mt. Wilson Observatory[1] on the intensities of absorption lines in high dispersion stellar spectrograms gives promise of a method of determining stellar temperatures by the application of quantum principles. It is still of great interest, however, to derive these from the form of the absolute energy curve of the continuous spectrum, and determinations of the intensity distribution in stellar spectra are of importance for other astrophysical problems as well. Much has been done in classifying stars for different purposes by means of color-indices and effective wave-lengths, both of which depend, empirically, at least, upon the spectral energy distribution. It is now possible to determine spectral types in this way, but as yet no method has been proposed for distinguishing between giants and dwarfs except by an analysis of the spectra themselves. This was originally done by relating absolute magnitude to the relative intensity of certain pairs of absorption lines. More recently Shapley,[2] Lindblad,[3] and Öhman,[4] have discussed cyanogen absorption as a criterion of absolute magnitude, a criterion which may be applied to low dispersion spectrograms. It was in the hope of extending these methods to a means of distinguishing between giants and dwarfs which might appear as a color-index, that the present investigation was undertaken. Incidentally a new and, it is believed, an improved method has been developed for determining the spectral energy curves of stars.

[1] *Ap. J.*, **68**, 1, 1928.

[2] *Harv. Bull.*, 805, 1924; 856, 857, 1928.

[3] *Mt. Wil. Cont.*, no. 228, 1921.

[4] *Arkiv för Matematik, Astronomi och Fysik*, 19B, no. 6, 1926; 20A, no. 23, 1927.

Various methods of doing this have been described. Wilsing, Scheiner, and Münch[5] made their measurements visually, using a Nicol prism to equalize the star's light of a given wave-length with the radiation of the same wave-length from a comparison lamp of known intensity distribution, a correcting factor depending on the setting of the Nicol being then applied to determine the intensity in the star spectrum relative to that in the comparison spectrum.

Rosenberg,[6] the pioneer worker in this field using photography, used an objective prism spectrograph, limiting his field, however, to one spectrum in the center. The spectra were photographed out of focus to secure breadth. Each plate contained, in addition to the spectra of a number of stars, several exposures of different duration on the spectrum of a comparison star, from which the relation between light intensity and plate blackening was derived for each wave-length. The blackening of the plate was measured with a Hartmann wedge microphotometer, and the Sun was used to reduce to relative absolute units.

Sampson[7] used slit spectrograms, studying them with a self-registering photo-electric cell microphotometer. His process of reduction was rather complicated. The galvanometer deflections (measurements of the microphotometer tracings) were changed to densities. Then assuming an approximate relation between light-intensity, density, and other factors, he derived constants for each plate, and finally arrived at the ratio of intensities of the radiation from each star and the comparison star, *Polaris*. *Capella* with an assumed temperature served as standard.

[5] *Publ. Potsdam Obs.*, **19**, no. 56, 1909; **24**, no. 74, 1919.

[6] *Kaiserl. Leop.-Carol Deutschen Akademie der Naturforscher*, **101**, no. 2; also, *A. N.*, **193**, 357, 1912.

[7] *Mon. Not.*, **83**, 174, 1923; **85**, 212, 1925.

H. H. Plaskett[8] applied the wedge method to several bright stars. A neutral tint wedge, uniformly illuminated by the star, was placed before the slit, producing a spectrum which was the wider in the more intense portions of the spectrum. The measurements determined how much the star radiation of a given wave-length had to be diminished by the calibrated wedge in order to produce a standard blackening. The comparison spectrum and ultimate standard was that of a tungsten lamp photographed on each plate and measured to the same standard blackening.

Yü[9] has obtained a fine series of stellar energy curves using the method of sensitometer circles. Six standard exposures were impressed on each plate through an ultra-violet filter, the intensities increasing in the ratio 1 : 2. The densities of these circles, measured with a Hartmann microphotometer, furnished values from which could be obtained graphically the intensity in different portions of the spectrum relative to the faintest standard circle. The disturbing factors, such as instrumental absorption, varying dispersion, and plate sensitivity, were eliminated and the transition to relative absolute units was accomplished by assuming a black body curve and definite temperature for one star.

Radiometric measurements of stellar energy curves have been made by Coblentz,[10] and by Nicholson and Pettit.[11] These have the advantage of being extended far into the infra-red.

The method used by Wilsing[12] on the Sun is perhaps the one most nearly like that of the present investigation. The solar spectrum was photographed on a plate along with several exposures of a comparison spectrum of known intensity distribution, the latter being varied by known amounts by means of a Nicol prism. These furnished the scale for determining the intensity.

Most of the precise spectrophotometry at the Harvard College Observatory has been applied to a study of absorption lines in stellar spectra. The intensity scale has been calibrated in various ways. Shapley[13] used several apertured objective prism spectra of the same star, and Dunham[14] used standard exposure strips similar to those employed by Yü. Miss Payne and Hogg[15] in summarizing all the methods of spectrophotometry in use at Harvard, mention the possibility, when working with objective prism spectrograms, of using several stars of the same type and of known magnitude difference for calibrating the intensity scale of the plate. Hogg[16] applies this method to a determination of the effective temperatures of 36 stars in the *Pleiades*, and discusses its application to the large number of fine unstandardized objective prism plates at the Harvard Observatory. Unfortunately only a small spectral interval will be available for this work, λ4050 to λ4860.

A method quite different from any of those so far mentioned is described by Greaves, Davidson, and Martin.[17] A coarse wire grating is placed in the optical train with its dispersion perpendicular to that of the prism. The central spectrum of the star is then flanked by fainter spectra of calculable magnitude differences. These serve to establish the photometric scale.

Some of the advantages and weak points of these various methods may well be pointed out here. The visual method, of course, involves difficult observing and rather large personal errors. Rosenberg's method is theoretically sound and has produced good results, but an extension to fainter stars and to a greater range of the spectrum is desirable. A possible objection is that it involves the assumption, as does Shapley's method of apertured exposures, that the transparency of the sky has remained absolutely uniform during the standardizing exposures.

The microphotometer tracings of Sampson appear much like those of the writer and his reductions are handled in such a way that the results should be good. However, our experience, about to be described, makes it appear that such an involved procedure is unnecessary. The assumption of a temperature for the standard star might also be criticized when applied to fundamental work.

The wedge method leaves little to be desired theoretically, but it is difficult of application. Uniform illumination of the entire slit length is hard to obtain, and the exposure times are excessively long. For example, Plaskett, using four times the light gathering power of the writer (though with five and one-half times the dispersion) required exposures about 40 times as long.

The chief drawback in the work of Yü is the assumption that the curve connecting intensity and density obtained from the sensitometer circles for λ3600± will hold out as far as λ4800. The difference may be a second order quantity when, as he has pointed out,[18] the photographic densities at a given wave-length are practically the same for all his stars, which are of much the same type. However for an investigation in which stars of radically different types are to be compared, this assumption is not valid.

[8] *Publ. Dom. Astroph. Obs.*, **2**, no. 12, 1923.
[9] *Lick Obs. Bull.*, **12**, no. 375, 1926.
[10] *Bur. of Stand. Sc. Papers*, **17**, no. 438, 1922.
[11] *Ap. J.*, **56**, 293, 1923.
[12] *Publ. Potsdam Obs.*, **22**, No. 66, 1913.
[13] *Harv. Bull.*, 805, 1924.
[14] *Harv. Bull.*, 853, 1927.
[15] *Harv. Cir.*, 301, 1927.
[16] *Harv. Cir.*, 309, 1927.
[17] *Mon. Not.*, **87**, 352, 1927.
[18] *Observatory*, **49**, 370, 1926.

The method used by Wilsing on the Sun differs from the one to be described chiefly in that his comparison spectra were varied in intensity by a Nicol prism, while we have accomplished this result by changing the distance of the comparison lamp from the slit. It is believed that the latter procedure gives as accurate results, and it is unencumbered by the effect of varying absorption in the Nicol prism.

The present investigation, it is believed, combines the strong features of the methods of other observers. Red-sensitive plates have been used to secure as wide a range of spectrum as possible. The spectra have been studied with a Moll self-registering microphotometer, thus eliminating all personal error in the measurement of densities. A minimum of assumptions has been made as to the photographic process and those which were necessary have been justified. And finally, the Sun, that celestial body which has been the most thoroughly studied, has served as standard of intensities.

## Method

The procedure, in brief, has been to impress on each plate two exposures of a star spectrum in the ratio 4 : 1, using a slit spectrograph attached to the Crossley Reflector; and also, while the plate was in the spectrograph for the star, to give four exposures to the spectrum of a comparison lamp, the four being of the same duration, but of different intensities, in the ratio 1 : 4 : 16 : 64, approximately. Tracings of all six spectra were then secured with the Moll microphotometer and these were measured at fourteen different wave-lengths between λ3780 and λ7425. The measures of the four comparison spectra permitted an empirical determination at each wave-length of the curve connecting intensity with blackening, and that, with the measures of the star spectra, gave the intensity of the star's radiation at that wave-length in terms of the faintest lamp intensity taken as unit. The intensity distribution in the lamp spectrum was then determined, and selective reflection and absorption in the apparatus were eliminated by taking spectrograms of the Sun similar to those of the stars, deriving its intensity along the spectrum in terms of the comparison spectrum, and then, by assuming Abbot's values for the solar intensity in absolute units, passing to the energy curve of the lamp. A correction for atmospheric absorption is of course applied to both Sun and stars individually.

We may begin a more thorough examination of the details of the work with the spectrograph itself. This consisted of a 60° light flint prism, a collimating lens of 8¼ inches focus, and a 6-inch camera. The combination gave a dispersion varying from 115A to the millimeter at λ4000 to 624A to the millimeter at λ7000. Since only the continuous spectrum or regions of heavy absorption were to be studied, a wide slit, 0.24 millimeter, was used in order to obscure the fainter absorption lines. The six spectra were secured on the plate by disclosing a different portion of the slit for each. This was accomplished by means of a sliding diaphragm operated by a lever. A stop-slot arrangement, the action of which could be felt, held the slide with the holes accurately over the slit, and it was easily possible to open any desired portion of the slit entirely by touch, as was necessary from the position of the spectrograph in the telescope.

The comparison spectrum was that of an automobile headlight operated by a storage battery at a current of 2.50 amperes. This was controlled by a rheostat. Tests showed that the character of the spectrum did not change at all with a change of current of ±0.10 amp., and though the intensity did change by 0.30 magnitude, the error introduced by this is believed to be negligible as the current seldom varied over 0.02 amp. and that for only very short intervals as the ammeter was watched constantly.

The lamp was enclosed in a box with a 1-inch ground glass window, and to secure better diffusion a second piece of ground glass was placed one inch nearer the lamp. The box was fixed to a board which slid in grooves on the spectrograph case, and the intensity of the comparison spectrum was varied by sliding this box to different distances from the slit. In order to insure having the collimator filled, a small piece of ground glass was placed in a frame before the slit during comparison exposures, and the distances were measured from this to the window in the lamp box. The sliding board was held at the proper places by a stop-slot. The distances used were 5, 10, 20, and 40 inches, in the expectation that the intensities would vary in the ratio 64 : 16 : 4 : 1. But the inverse square law failed, due to two possible causes: imperfect diffusion of the light by the ground glass window, and more reflection of the light by the sliding board at the greater distances. The true intensity ratios were calibrated photographically by comparison with a brighter light at distances where the inverse square law could be relied upon. A test was made which showed a very slight departure of the slit from uniform width, but any possible error from this was eliminated in the calibration of the intensity ratios. The fact that all portions of the slit used were within 3/16 inch of the axis of the collimator made the optical paths through the instrument practically identical. The exposure was accurately 4 minutes on each of the four comparison spectra, and the openings over the slit were so used that the two star exposures came among the other four. A reproduction of a sample spectrogram, enlarged about seven times, is shown in Plate I.

The procedure when photographing the solar spectrum was identical with that for a star, except that most of the telescope mirror had to be covered to get suitable exposure times. A large piece of beaver board was adjusted over the end of the telescope and a hole about 1 millimeter in diameter was left open. Even this required an exposure of only 1 second. This opening was used over six parts of the mirror (equally spaced and about half the radius out from the center) in an effort to allow for possible irregularity in the silver surface. Each part of the mirror used was examined and appeared to be free from tarnish. The image of the Sun was about two inches in diameter, and the center of the disc was directed to the slit. The values of the solar intensities in terms of the comparison spectrum were derived for each of the six plates, and the mean taken for the calibration of the comparison spectrum.

Since it was desired to get as much as possible of the visual spectrum of each star for discussion, the plates finally adopted were Eastman 33's stained with Wallace's three-dye solution, consisting of Pinacyanol, Pinaverdol, and Homocol.[19] These gave best results when developed for five minutes in Rodinal diluted 80 to 1, the usual precautions being taken to secure uniformity. As will be seen from the mathematical discussion to follow, changes in the conditions of development should introduce no errors into the final result since the intensity-blackening curve is to be derived independently for each wave-length on each plate; but in addition to the general desirability of uniformity, it has been convenient to have the plates as much alike as possible, as will appear later from the method of reduction.

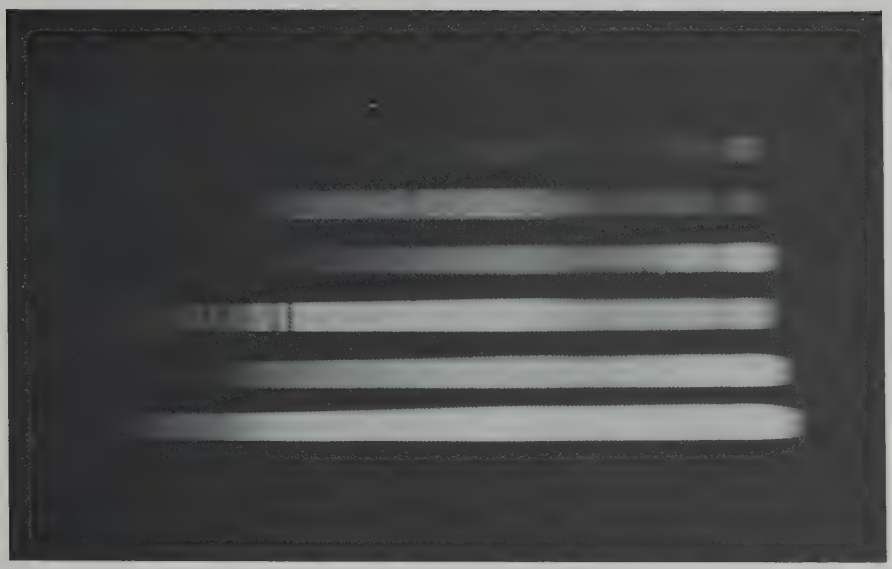
With the method of obtaining the stellar and comparison spectra fully outlined, some description of the Moll microphotometer may not be out of place. The instrument really records the transparency of the spectrum. This is done by illuminating the spectrum on the plate with a bright electric lamp run by a storage battery, and then focusing the spectrum on the slit-plate before a thermo-couple. The plate is moved slowly by a motor so that different portions of the spectrum are brought successively before the slit. The light which does get through the spectrum causes the thermo-couple to actuate a moving coil galvanometer, which reflects a beam of light onto a scale at a distance of 7 feet. Under the scale is a slit in the wall of a box inside of which a piece of bromide paper is turned on a drum by the same motor which moves the plate. When this slit is open the beam falls on the paper, and thus the deflections caused by different parts of the spectrum are recorded, the combination of gears used giving a magnification of the wave-length scale of 6.9. The brightness of the lamp and the accuracy with which it is focused on the spectrogram determine the amount of the deflection, but 10 centimeters between "clear glass" and "complete opacity" has been found most convenient. Due to the position of the drum in the box, the deflection is perpendicular to the edge of the paper and hence all measurements of the height of the curve must be made in that direction.

The characteristic curve of a thermo-couple is a parabola, but over the very small range of temperatures dealt with it may be considered linear. The small swing of the galvanometer coil may also be taken as linear with the impressed E. M. F., so that the photographic record on the tracing is a linear function of the transparency. Due to this fact any change in the lamp intensity will produce a linear change in the deflection, and hence, since the scale is variable, all measurements of the height of the curve are made as fractions of the distance from "clear glass" to "complete opacity." This is necessary partly because it is not always convenient to set the total deflection at exactly 10 centimeters, and partly because this may change in the course of one spectrum. Therefore each curve is made to consist of the spectrum itself, a short portion of "clear glass" at each end, and beyond these, short marks showing "complete opacity" in which the light was covered up entirely by a cap. All six spectra were treated in the same way. A sample set of microphotometer curves is shown in Plate II.

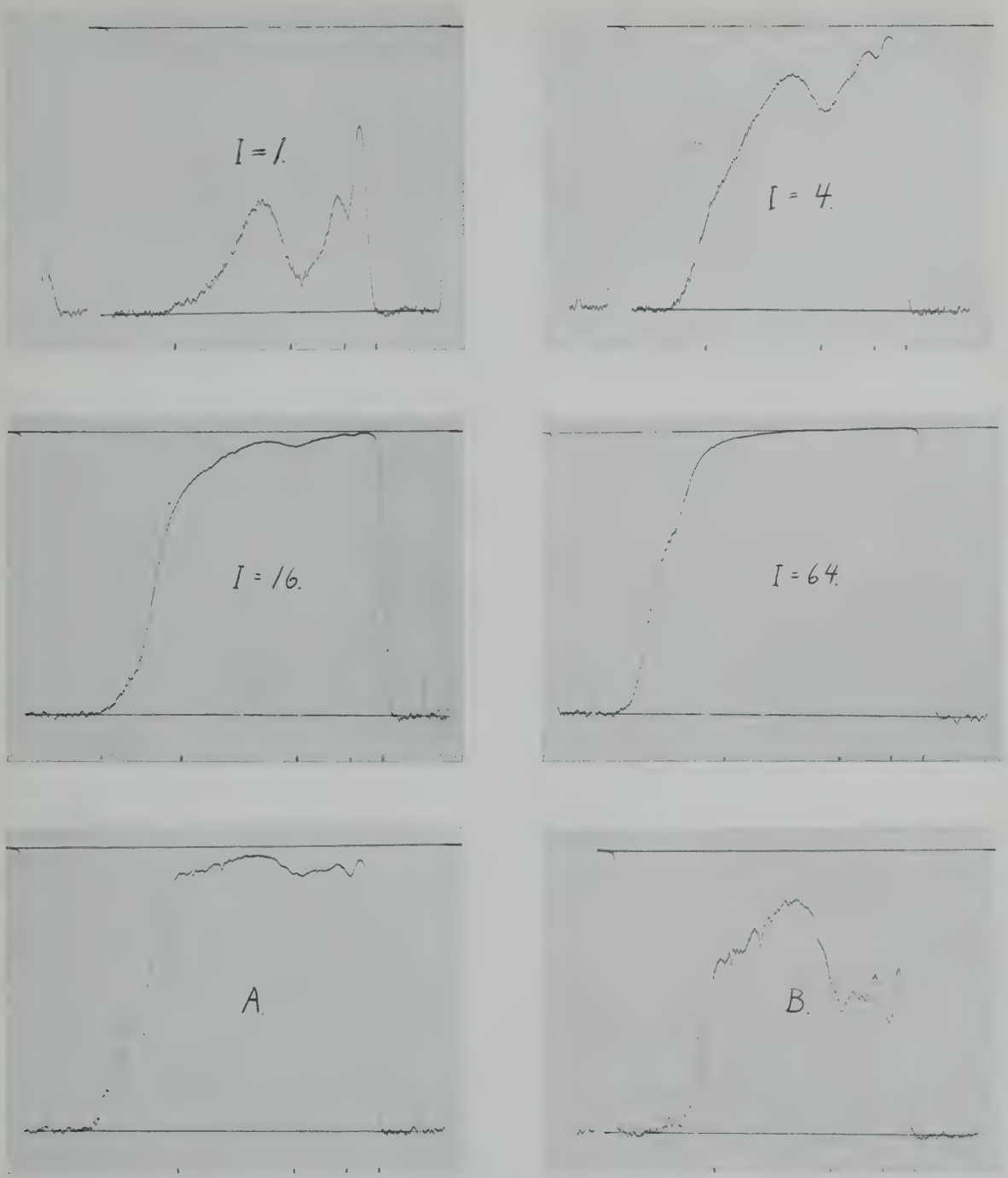
The question of fog on the plates has been given careful consideration. The glass itself of course has a certain opacity, and a uniform fog merely has the effect of increasing that slightly. It will to a certain extent alter the form of the microphotometer curve, but all six spectra will be affected in a way which will be eliminated in our process of reducing the measurements. A uniformly progressive fog from one end of the plate to the other, which occasionally occurs, is taken care of by the method of measuring the height of the curve as a fraction of the complete deflection at that point. To obtain that, lines are drawn on the tracing connecting the "complete opacity" marks and "clear glass" marks at the ends of the spectrum. If the fog is uniformly progressive these "clear glass" marks may be considerably tilted but still in a straight line. Irregular fog is of course hopeless for photometric work and all plates so fogged have been rejected.

To facilitate the measurement of the curves a glass reseau was made on which a number of parallel lines were crossed by 21 lines converging so that the lines of the first series were all divided accurately into 20 equal parts. To measure the height of the curve, then, that one of the parallel lines was selected on which the outside distance was equal to the distance

[19] *Ap. J.*, **26**, 316, 1907.




Spectrogram of $\eta$ *Pegasi*, gG0, Enlarged 7 times (Plate St 242X).




Sample Set of Microphotometer Curves.

between the "clear glass" and "complete opacity" lines at the desired wave-length on the curve. The height of the curve was then read to the 200th part.

In general the height of the curve itself was measured. Occasionally, however, if an absorption line occurred at the point to be measured, the curve as smoothed over the top of the line was measured when this could be done safely. In later type spectra, where lines and bands were too numerous to be sure what actually represented continuous spectrum, the curve itself was always measured.

The wave-lengths to be measured were marked at the bottom of the sheet, the calcium K-line or one of the hydrogen lines usually serving as an identification point for the star spectra. For the comparison spectra, the method adopted was to scratch lines on the film with a razor blade beyond the ends of the spectra and perpendicular to them. These would leave sharply defined marks on the tracings which would have the same relation to all six spectra as to wave-length.

It was originally planned to measure the spectra at intervals of 200A from λ3600 to λ7400, shifting these slightly to avoid absorption lines in the solar spectrum. It was found however, that very few of the star spectra had measurable intensities at λ3600, due to absorption in the spectrograph, and also, that measurements every 400A were ample for the portion to the red of λ5400. Thus the final results are based on fourteen readings from λ3780 to λ7425, with the first and last frequently missing or of very low weight.

With the measurements of the microphotometer curves in hand, it remains but to explain the process of reducing these to the relative intensity values for the star spectra outside the Earth's atmosphere. Let $R_\lambda$ represent the measurement of one curve at the wave-length, λ. This is a function of the photographic density, which in turn is a function of other quantities,[20] so that we may write:

$$R_\lambda = f(I'_\lambda \cdot t^{p_\lambda}, i_\lambda, \gamma_\lambda) \qquad (1)$$

where $I'_\lambda$ is the intensity of light reaching the photographic plate, $t$ is the exposure time, $p_\lambda$ is Schwarzschild's exponent, $i_\lambda$ is the plate inertia, and $\gamma_\lambda$ the photographic development constant. Assuming for the moment that $p_\lambda$ is constant with wave-length, (an assumption which will be justified later), the quantity $t^{p_\lambda}$ will necessarily be constant for all spectra of the same exposure time. (For $t$ is obviously the same for all parts of one spectrum.) Further, if $p_\lambda$ is constant with wave-length, the quantity $t^{p_\lambda}$ will merely have the effect, when $t$ does vary, of increasing $I'_\lambda$ in the same proportion for all wave-lengths. Hence, for one spectrum $t^{p_\lambda}$ is a constant. Then, assuming $i_\lambda$ and $\gamma_\lambda$ uniform all over one plate, (an assumption which is inherent in all photographic photometry), we may plot $R_\lambda$, equation (1), with $I'_\lambda$ as the only variable. The measurements of the comparison spectra of known intensity ratios furnish four points for the determination of this curve for each wave-length, and from it is then obtained the intensity corresponding to the measurement of the curve of the star's spectrum. This value is in terms of the faintest comparison spectrum taken as unit, and is called $k_\lambda$. It may be defined:

$$k_\lambda = \frac{J'_\lambda}{I'_\lambda} \qquad (2)$$

where $I'_\lambda$ = intensity at wave-length, λ, of light from comparison apparatus at greatest distance as it reaches the photographic plate.

$J'_\lambda$ = intensity of star light as it reaches the plate.

In plotting the curve of equation (1), it is convenient to have the intensities as abscissae expressed logarithmically, and then, following the practice of other investigators, to change these to the magnitude scale.

To avoid drawing fourteen free hand curves for each of about seventy-five plates, a device was constructed which saved a great deal of time and probably increased the accuracy of the results. A piece of coordinate paper such as would be used for drawing the curves was pasted to a board. Four vertical rows of holes were punched in this at abscissa values corresponding to comparison spectrum intensities. (0.00, 1.36, 2.70, and 3.85 magnitudes were the values resulting from the calibration.) Pins were stuck in these holes at ordinates corresponding to measurements of the comparison spectra, and a flexible hard rubber ruler was then laid along the four pins to represent the curve; from this the intensities corresponding to star spectra measurements were read off. This gave the values of $k_\lambda$ expressed in magnitudes for each of the two star spectra. It is believed that the curve so obtained is better than one drawn free hand, and with an assistant to read off measurements and record intensities, a great deal of time was saved.

Occasionally the form of the curve was uncertain at the point to be used, and in such cases, and also when an extrapolation was necessary, another plan was followed. Advantage was taken of the fact that the curves for adjacent wavelengths should be quite similar in shape. A mean curve for the different wavelengths was drawn and tables constructed which gave, for each comparison ordinate other than the mean, a correction to the magnitude read from the mean curve. These corrections were plotted against comparison measurements and then, using the measurement of the star spectrum, the correction was obtained and applied to the mean curve value. This plan was originally intended for all the reductions, but it was found that the individual curves departed so much from the mean, due to the fact that the sensitivities of different

[20] Hurter and Driffield, *Photographic Researches Memorial Volume*, p. 76, 1920.

stained plates are not always the same, that the corrections were too large to give accurate values easily. Accordingly this method was used only when the other failed.

It has been stated that two exposures of a star were secured on each plate. If all intensities on the spectra could be determined with perfect precision, there should be a constant difference in magnitude between the two determinations at each wave-length. This of course is never realized, but a method of searching for errors is suggested. The differences in magnitude were taken for all pairs of determinations on each plate, and any which differed from the mean by as much as 0.10 magnitude were examined more carefully. Frequently the discrepancy could be decreased. Some mention of the average amount of these deviations may be of assistance in forming a judgment as to the accuracy of an intensity determination. From a number of plates chosen at random, the average deviation from the mean difference was $\pm 0.05$ magnitude, giving a probable error for one magnitude difference of approximately $\pm 0^m.04$. This would give a probable error of $\pm 0^m.02$ for the mean of the two determinations. The best plate reduced gave an average deviation of $\pm 0^m.03$ from the mean, while the worst gave $\pm 0^m.10$. The latter was on a *K*5 star in which the measurements are necessarily uncertain due to the steepness of the microphotometer curve between absorption bands.

In combining the two sets of intensities for the star's spectrum, the weighted mean was taken of the two determinations for each wave-length, each being weighted according to the slope of both the microphotometer tracing and the intensity-blackening curve. The result is the final value of $k_\lambda$ for the plate, expressed in magnitudes.

The atmospheric correction was next applied (or in cases where the zenith distance changed greatly between exposures, this was applied before means of the two were taken). The transmission coefficients used were those determined at Mt. Wilson, corrected to the altitude of the Crossley Reflector, 1200 meters. The temperature and humidity for each night when observations were made, were taken from the observatory's meteorological records, with the expectation of making a correction for variation in these, using the formula given by Fowle.[21] Investigation showed, however, that the extremes of temperature and precipitable water vapor in the atmosphere could produce errors of 0.01 magnitude at the most, so mean values of both were adopted in computing the factors to be applied, the only variables being wave-length and zenith distance. The approximate relation $a_\lambda = a_{o\lambda}^{\sec z}$ has been used, and a table giving values of $a_\lambda$ expressed in magnitudes was constructed for the wave-lengths treated and for values of sec $z$ at intervals of 0.05. The appropriate column was then applied to the values of $k_\lambda$ giving $k_\lambda/a_\lambda$ in magnitudes. These values are homogeneous for all stars, being the ratios of intensities of the star radiation, $J'_\lambda$, and comparison lamp radiation, $I'_\lambda$, as they reach the photographic plate, with the effect of atmospheric absorption, which varies from star to star, eliminated. Instrumental reflection, absorption, and dispersion are still unaccounted for, but these are the same for all stars.

To treat these we must consider in order the various influences which may change the character of the radiation coming from the star. The combined effect of these at any wave-length, $\lambda$, may be expressed in the following equation where, as before, $J'_\lambda$ is the intensity of the star's radiation as it reaches the photographic plate:

$$J'_\lambda = J_\lambda a_\lambda r_\lambda b_\lambda d_\lambda. \tag{3}$$

In this equation, for the wave-length $\lambda$,

$J_\lambda$ = true intensity of star's radiation.
$a_\lambda$ = atmospheric transmission coefficient.
$r_\lambda$ = reflectivity coefficient of silver mirror.
$b_\lambda$ = transmission coefficient of spectrograph.
$d_\lambda$ = dispersion coefficient.

If $I_\lambda$ is the initial intensity of the radiation from the comparison apparatus (including the three pieces of ground glass), we shall have an equation similar to (3) for the comparison spectrum, except that $a_\lambda$ and $r_\lambda$ are omitted:

$$I'_\lambda = I_\lambda b_\lambda d_\lambda. \tag{4}$$

Dividing (3) by (4) we obtain:

$$\frac{J'_\lambda}{I'_\lambda} = \frac{J_\lambda a_\lambda r_\lambda}{I_\lambda} = k_\lambda \qquad \text{(See equation 2.)}$$

or

$$J_\lambda = \frac{k_\lambda}{a_\lambda} \cdot \frac{I_\lambda}{r_\lambda} = \frac{k_\lambda}{a_\lambda} \cdot C_\lambda \tag{5}$$

If we let the subscript zero apply to values referring to the Sun, equation (5) may be rewritten:

$$C_\lambda = J_{o\lambda} \div \frac{k_{o\lambda}}{a_{o\lambda}} \tag{6}$$

In this equation $\frac{k_{o\lambda}}{a_{o\lambda}}$ is determined as explained above from the six plates of the solar spectrum using different portions of the Crossley mirror. If for $J_{o\lambda}$ we employ the measurements of Abbot[22] on the spectral energy distribution of the Sun, we have the material in hand for deriving from equation (6) the values of $C_\lambda$. These values are computed once for all. Then by equation (5), these quantities, applied to the values of $\frac{k_\lambda}{a_\lambda}$

[21] *Smithsonian Physical Tables*, Reprint of 7th Revised Edition, p. 419, 1921.

[22] *Smithsonian Misc. Coll.*, **74**, no. 7, 1923.

obtained as explained before, give the final relative intensity values in the star spectrum as viewed from outside the Earth's atmosphere.

For convenience in combining intensity measures for different stars, or in comparing different stars by inspection, the quantities $J_\lambda$ have been changed into values expressing the difference in magnitude between the radiation at each wave-length and that at λ5400. These last quantities have been designated $M_\lambda$. A positive value means that the radiation is stronger than that at λ5400.

in columns 8 and 9 indicate values taken from a poorly determined part of the intensity-blackening curve.

The derivation of the quantities $C_\lambda$ requires further mention. As explained previously the six spectrograms from which the mean values of $k_{o\lambda}/a_{o\lambda}$ were derived were obtained from observations of the center of the Sun's disc. Hence we should expect to use values of $J_{o\lambda}$ derived from that portion. After careful consideration, however, Abbot's latest values (and he believes his best) for integrated sunlight have been used.

TABLE I

MEASUREMENTS AND REDUCTIONS OF SAMPLE PLATE

Plate ST210R Star: HR7479 gG0 m=4.4 M=−0.9. Sec $z$=1.05

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| λ | $R_{84}$ | $R_{16}$ | $R_4$ | $R_1$ | $R_A$ | $R_B$ | $k_A$ | $k_B$ | $d$ | $v$ | $k_B+D$ | $k$ | $a$ | $k/a$ | $C$ | $J$ | $M$ | $w$ |
| 3780 | 440 | 050 | 0 | 0 | 100 | 030 | 2.97 | | | | | 2.97 | −0.47 | 3.44 | 0.92 | 4.36 | −0.88 | . |
| 4004 | 965 | 750 | 160 | 015 | 750 | *600* | 2.70 | 2.35 | 0.35 | 7 | 2.77 | 2.71 | −0.35 | 3.08 | 1.77 | 4.85 | −0.39 | . |
| 4184 | 985 | 885 | 410 | 030 | 855 | 750 | 2.53 | 2.13 | 0.40 | 2 | 2.55 | 2.55 | −0.30 | 2.86 | 2.22 | 5.08 | −0.16 | + |
| 4380 | 995 | 955 | 680 | 130 | 915 | 845 | 2.32 | 1.92 | 0.40 | 2 | 2.34 | 2.34 | −0.28 | 2.63 | 2.65 | 5.28 | +0.04 | . |
| 4595 | 1000 | 975 | 825 | 290 | 945 | 895 | 2.25 | 1.79 | 0.46 | 4 | 2.21 | 2.23 | −0.22 | 2.47 | 2.98 | 5.45 | +0.21 | . |
| 4807 | 1000 | 975 | 820 | 260 | 915 | 845 | 1.96 | 1.49 | 0.47 | 5 | 1.91 | 1.93 | −0.20 | 2.14 | 3.28 | 5.42 | +0.18 | . |
| 4990 | 1000 | 960 | 700 | 100 | 770 | 615 | 1.57 | *1.17* | 0.40 | 2 | 1.59 | 1.58 | −0.18 | 1.76 | 3.57 | 5.33 | +0.09 | . |
| 5190 | 1000 | 970 | 670 | 065 | 670 | 430 | 1.36 | *0.90* | 0.46 | 4 | 1.32 | 1.34 | −0.18 | 1.52 | 3.72 | 5.24 | 0.00 | . |
| 5400 | 1000 | 980 | 755 | 105 | 680 | 450 | 1.18 | *0.75* | 0.43 | 1 | 1.17 | 1.18 | −0.15 | 1.34 | 3.90 | 5.24 | 0.00 | + |
| 5780 | 1000 | 990 | 900 | 295 | 755 | 550 | *0.85* | *0.45* | 0.40 | 2 | 0.87 | 0.86 | −0.10 | 0.96 | 4.29 | 5.25 | +0.01 | + |
| 6120 | 1000 | 995 | 915 | *350* | 650 | 430 | *0.50* | 0.15 | 0.35 | 7 | 0.57 | 0.54 | −0.08 | 0.62 | 4.50 | 5.12 | −0.12 | + |
| 6540 | 1000 | 995 | 965 | *500* | *720* | *500* | *0.45* | 00.00 | 0.45 | 3 | 0.42 | 0.44 | −0.08 | 0.52 | 4.68 | 5.20 | −0.04 | . |
| 6925 | 985 | *870* | *300* | 020 | 050 | 000 | *0.30* | | | | | 0.30 | −0.05 | 0.35 | 4.59 | 4.94 | −0.30 | − |
| 7425 | 150 | 000 | 000 | 000 | 000 | 000 | | Av.:−0.42 | | 3½ | | | −0.02 | | 4.10 | | | |

$M$ gives the radiation intensities measured at the various wave-lengths, expressed in magnitude differences, brighter than (+) or fainter than (−) the intensity at λ5400.

The arrangement of the measurements and reductions for one plate is shown in Table I. Column 1 contains the wave-length, and columns 2 to 7 inclusive contain the measurements, $R$, of the four comparison spectrum curves and the two star curves, $A$ and $B$, expressed in thousandths of the total deflection. Beginning with column 8, all values are expressed in magnitudes, so that all processes indicated as multiplication or division are carried out as addition or subtraction. Columns 8 and 9 give the values of $k$ for each of the two star spectra, 10 gives the difference in magnitude between $k_A$ and $k_B$, and 11 gives the deviation of each difference from the mean. Column 12 gives the mean difference added to $k_B$, and 13 gives the weighted mean of columns 8 and 12. The headings of the remaining columns are self explanatory. The weights (Col. 19) are merely a rough indication of the probable accuracy, considering the steepness of the curves from which the values were derived. A dot means average, and + and − mean above and below average respectively. Italicized values of $R$ in the first few columns indicate measurements which were made where the curve was very steep, and which were therefore unreliable. Similarly, italicized values of $k$

The *Smithsonian Physical Tables*[23] give the energy distribution in the spectrum of integrated sunlight, and also in that of light from the center of the disc. The relative differences are small, (usually less than 0.05 magnitude), and they are much smaller than the differences between those integrated sunlight values and the later ones which we have used. It therefore appears that the departure of the energy curve of integrated sunlight from that of light from the center of the Sun's disc is less than the uncertainty of the measurements, and hence, since no values for the center of the disc are given with the later publication, the integrated values have been used. Abbot mentions the fact[24] that the contrast between center and limb may be increased at the time of sun spot maximum, at which time our observations were made, but the effect is very slight, probably less than 1%, and hence it will not increase the previous error sensibly.

Table II contains the magnitude intensities for the wave-lengths measured derived from Abbot's published data. These values constitute our $J_{o\lambda}$. The mean values of $k_{o\lambda}/a_{o\lambda}$ and the computation of $C_\lambda$ are also given.

[23] Second Printing, 7th Edition, p. 418, 1921.

[24] *An. Astroph. Obs. Smithsonian Inst.*, **4**, 222, 1922.

TABLE II

COMPUTATION OF $C_\lambda$

| $\lambda$ | $J_{o\lambda}$ | $k_{o\lambda}/a_{o\lambda}$ | $C_\lambda$ |
|---|---|---|---|
| 3780 | 3.45 | +2.53 | 0.92 |
| 4004 | 3.65 | +1.88 | 1.77 |
| 4184 | 3.79 | +1.57 | 2.22 |
| 4380 | 3.90 | +1.25 | 2.65 |
| 4595 | 3.98 | +1.00 | 2.98 |
| 4807 | 4.02 | +0.74 | 3.28 |
| 4990 | 3.99 | +0.42 | 3.57 |
| 5190 | 3.94 | +0.22 | 3.72 |
| 5400 | 3.90 | 0.00 | 3.90 |
| 5780 | 3.86 | −0.43 | 4.29 |
| 6120 | 3.82 | −0.68 | 4.50 |
| 6540 | 3.72 | −0.96 | 4.68 |
| 6925 | 3.65 | −0.94(?) | 4.59 |
| 7425 | 3.49 | −0.61(?) | 4.10 |

It was originally intended to get observations of the Sun at frequent intervals during the observing period. This was made difficult, however, by a rather strenuous observing program which required all-night observing for several nights in succession while the telescope was available. Plates were secured, nevertheless, on six days, but the only ones entirely satisfactory for photometric purposes were those taken on the last two days, which were unfortunately two weeks after observations of stars had been completed. Spectrograms of stars repeated immediately after that yielded accordant results, however, and it is believed that no error has crept in.

The method of reducing the observations has now been explained in complete detail, and it is believed to be theoretically rigorous except for the one assumption that Schwarzschild's exponent, $p_\lambda$, is constant with wave-length. This assumption will now be justified.

It was pointed out that if $p_\lambda$ is constant with wave-length, a variation of $t$ will merely have the effect of increasing $I'_\lambda$ in a definite proportion for all wave-lengths. If, however, $p_\lambda$ does vary with wave-length, then a change in $t$ will cause different parts of the spectrum to change in different proportions. The result will be that the same spectrum photographed with two different exposure times will show different intensity distributions.

This effect, often called the photographic Purkinje effect, would certainly show itself in our observations if it existed, for we have exposure times ranging from one-half second on the Sun to one hour on a few stars. To test for its presence the following experiment was performed: Two of the usual stained plates were given the four regular comparison exposures of four minutes each. On one of the plates was made another exposure of the comparison lamp with the distance so small that a one-second exposure was sufficient. On the other plate the distance was increased and a one-hour exposure made. Both of these were reduced as unknowns in the same manner as star spectra. If the effect were absent the values of $k$ for all wave-lengths should be the same except for errors of observation; hence, deviations of all measured intensities from the mean were taken for each plate, and these were examined. For the points which were well determined the residuals were of the same order as those found on other plates, but there were slight indications of variation with wave-length and these were in opposite directions for the two plates.

The residuals from these plates are plotted in Figure 1, in the sense: Observed minus Mean. Unfortunately, a defect occurred in the center of the long exposure spectrum, but the determinations at the ends are sufficient to show its trend. The curves suggest that the short exposure is slightly weaker at the red end, while the longer exposure is stronger in the red. Now it is to be noticed that if this is a real effect it is in the reverse sense from that to be expected. Parkhurst,[25] summarizing the investigations of others on this subject, says, "As the intensity of the light increases, the position of maximum sensitiveness shifts toward the longer wave-lengths." That is, we should expect the short exposure to be stronger in the red, whereas it is, if anything, slightly weaker. It seems justifiable, therefore, to consider the apparent effect as merely the outcome of accidental errors, and to assume that the long and the short exposures result in the same energy distribution in the spectrum. This justifies the assumption that for the plates used $p_\lambda$ is constant for all the wave-lengths investigated.

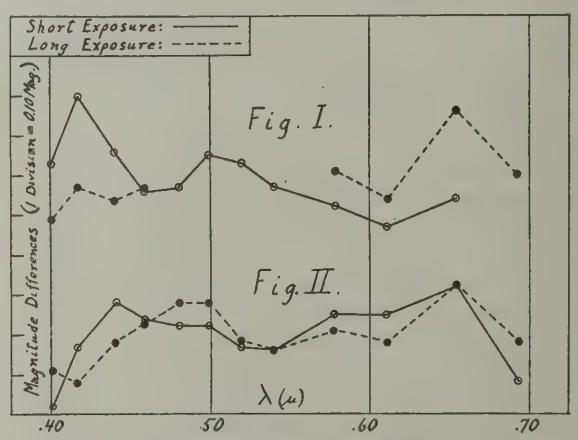



Fig. 1. Test of Constancy of $p$ with $\lambda$.
Fig. 2. Standard Comparison Exposures reduced as Unknowns.

Figure 2 has been placed beside Figure 1 for comparison. In this are plotted the residuals from two plates on each of which an additional four-minute exposure was given to the comparison spectrum at an

[25] *Ap. J.*, **49**, 203, 1919.

arbitrary distance, these two exposures being reduced as unknowns. Here obviously the intensity should be constant along the spectrum, for $t$ is the same as for the comparison spectra. It will be seen that the residuals are of the same order as those in Figure 1.

With the method of observation and reduction described in detail, we may now turn to the observing program and the results obtained from it.

## Observations and Temperatures

Since the purpose of this investigation has been to study the differences between the continuous spectra of giant and dwarf stars, the observing list has consisted for the most part of "late" type stars selected from Adams and Joy's catalogue of spectroscopic parallaxes,[26] the measured absolute magnitudes distinguishing between giants and dwarfs. Four stars of each half type were chosen on both giant and dwarf branches, and one star for each half type among the hotter stars to make the sequence complete. The distinction between the giants and dwarfs is probably artificial among the early $F$ stars, the intrinsically brightest being called giants and the faintest, dwarfs. Only two dwarf $K5$ stars could be found which were bright enough for observation, and no dwarf $M$'s. The final program consisted of 55 stars for which good spectrograms have been secured. Six of these were observed twice to test various matters which will be discussed.

The stars and data concerning them are contained in Table 3. The spectral intensity values for all the stars in this table have been derived as explained before, and the values of $M_\lambda$ are given in Table IV. This table also contains for reference the values of $k_{o\lambda}/a_{o\lambda}$ obtained from the spectrograms of the Sun.

Since temperature accounts for most of the variation in the continuous spectrum from star to star, this quantity has been determined first in order that it may be eliminated in the search for differences between giants and dwarfs. The values of $M_\lambda$ contained in Table IV were plotted for each star individually, and the curves so obtained were compared with Planck curves plotted to the same scale. The latter were drawn for temperatures from 2,500° up to 10,000° at intervals of 500°, and for a few higher or intermediate values when necessary. Samples of the derived energy curves and the corresponding temperature curves are given in Figure 3. There were but four cases where it seemed desirable to attempt to get the temperature closer than the nearest 500° by direct comparison with the Planck curve, and these were with cooler stars where the form of the curve changes more rapidly with temperature in the visual part of the spectrum. It would of course have been possible to compute the temperature rigorously, using all the intensity values which had been obtained, but it is doubtful whether the apparently more accurate temperatures so secured would have had meaning. For the graphical method makes it possible more easily to eliminate those points which obviously depart from the Planck curve for some such reason as heavy general absorption or error of observation. This has almost always been true for the first three points measured at the violet end of the spectrum, but the remaining portion from λ4400 to λ6900, which is usually well determined, is long enough to give good temperatures.

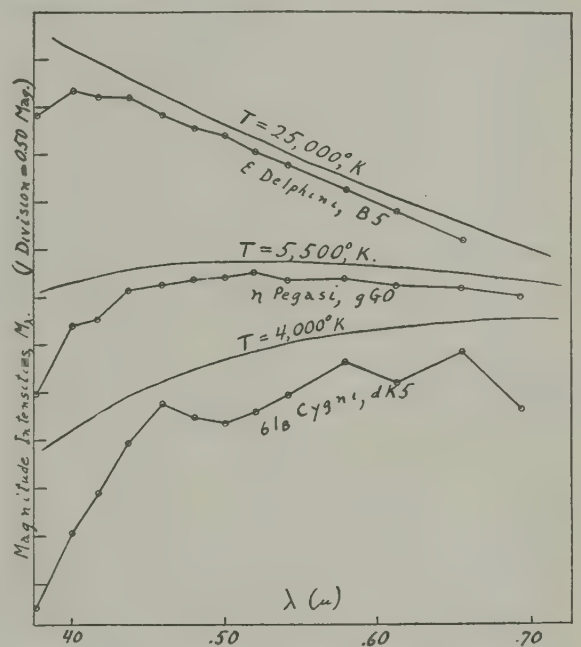



Fig. 3. Examples of Stellar Energy Curves.

One advantage of having the intensities expressed in magnitudes is that a given difference between stellar curve and Planck curve represents the same relative accuracy in all parts of the spectrum. This is helpful in fitting the temperature curves. Further, when the Planck curves are plotted in magnitudes it is readily seen that very great accuracy must be claimed for intensity measurements before temperatures closer than the nearest 500° will be significant. For example, for temperatures in the neighborhood of 6,000°, a change of 250° will produce a difference of but 0.05 magnitude at the extremes of the spectral interval treated in the present discussion. Naturally this difference will be less with a smaller range of spectrum, yet some investigators have given temperatures to the nearest 100°, or even less, with spectral intervals of only 1,000A.

Temperatures for the later types, $K$ and $M$, are uncertain when obtained from energy curves because

[26] *Contr. Mt. Wilson Obs.*, no. 199, 1921.

of the difficulty in deciding just what portion of the spectrum is unaffected by absorption bands. The accuracy of the determination is increased, however, by plotting the intensities in magnitudes, for then it is merely the slope of the curve which determines the temperature. Thus practically the same value will be obtained whether it is assumed that the waves in the observed curve are errors of observation and a mean curve is taken through them all, or whether it is assumed that the waves represent continuous spectrum and the dips absorption. The slopes over all are the same.

TABLE III

OBSERVATIONS

| Star | HR | α(1900) | δ(1900) | Sp. | m | Plate | Exp. | Z. D. | Date (1927) |
|---|---|---|---|---|---|---|---|---|---|
| Cas | 5 | $0^h\ 1^m0$ | 57° 53′ | dG5 | 6.1 | ST281V | $46,^m\ 12^m$ | 22° | Aug. 18 |
| χ Peg | 45 | 0 9.4 | 19 39 | gMa | 4.9 | ST289U | 20, 5 | 20° | Aug. 20 |
| 28 And | 114 | 0 24.8 | 29 12 | dF0 | 5.3 | ST276V | 20, 6 | 12° | Aug. 17 |
| ρ Psc | 413 | 1 20.9 | 18 39 | dF0 | 5.3 | ST290V | 20, 5 | 18° | Aug. 20 |
| α Tri | 544 | 1 47.4 | 29 06 | gF5 | 3.6 | ST291X | 3, 1 | 9° | Aug. 20 |
| ξ Boo | 5544 | 14 46.8 | 19 31 | dG5 | 4.6 | ST227U | 16, 8 | 27° | July 11 |
| λ Ser | 5868 | 15 41.6 | 7 40 | dG0 | 4.4 | ST216W | 8, 4 | 30° | July 9 |
| δ CrB | 5889 | 15 45.4 | 26 22 | gG5 | 4.7 | ST228V | 10, 5 | 25° | July 11 (1) |
| δ CrB | 5889 | 15 45.4 | 26 22 | gG5 | 4.7 | ST229W | 10, 5 | 29° | July 11 (2) |
| γ Ser | 5933 | 15 51.8 | 15 59 | dF5 | 3.9 | ST256R | 5, 1 | 34° | Aug. 9 |
| υ CrB | 6108 | 16 18.7 | 33 56 | gK5 | 5.3 | ST265O | 60, 15 | 23° | Aug. 11 |
| Her | 6183 | 16 33.3 | 46 49 | gG5 | 6.0 | ST251R | 40, 10 | 22° | Aug. 8 |
| 43 Her | 6228 | 16 41.0 | 8 46 | gK2 | 5.4 | ST224W | 22, 11 | 30° | July 10 |
| 32 Her | 6337 | 16 58.5 | 14 14 | gMa | 5.1 | ST268O | 32, 9 | 34° | Aug. 15 |
| π Her | 6418 | 17 11.6 | 36 55 | gK5 | 3.4 | ST208V | 2½, 1¼ | 8° | July 7 |
| α Oph | 6556 | 17 30.3 | 12 38 | A5 | 2.1 | ST292O | 45, 15 | 26° | Aug. 21 |
| θ Her | 6695 | 17 52.8 | 37 16 | gK0 | 4.0 | ST217U | 5, 2½ | 8° | July 9 |
| $70_A$ Oph | 6752 | 18 0.4 | 2 31 | dK0 | 4.3 | ST213U | 7, 3½ | 36° | July 8 |
| 104 Her | 6815 | 18 8.1 | 31 23 | gMa | 5.0 | ST263R | 20, 7 | 34° | Aug. 10 |
| 105 Her | 6860 | 18 15.1 | 24 24 | gK5 | 5.5 | ST219X | 24, 12 | 16° | July 9 |
| 106 Her | 6868 | 18 16.1 | 21 55 | gK5 | 5.0 | ST269R | 20, 7 | 26° | Aug. 15 |
| Oph | 6902 | 18 20.8 | 7 58 | gG0 | 5.7 | ST266R | 30, 8 | 32° | Aug. 11 |
| Oph | 6987 | 18 31.8 | 6 36 | gF2 | 5.4 | ST262O | 22, 6 | 36° | Aug. 10 |
| 110 Her | 7061 | 18 41.4 | 20 27 | dF5 | 4.3 | ST270S | 7, 2 | 30° | Aug. 15 |
| 113 Her | 7133 | 18 50.5 | 22 31 | gG0 | 4.6 | ST240U | 10, 2½ | 18° | July 28 |
| Aql | 7165 | 18 53.8 | 17 14 | gF5 | 5.4 | ST209X | 22, 11 | 20° | July 7 |
| γ Lyr | 7178 | 18 55.2 | 32 33 | A0p | 3.3 | ST293R | 2, ½ | 4° | Aug. 21 |
| γ Lyr | 7178 | 18 55.2 | 32 33 | A0p | 3.3 | ST296V | 2, ½ | 26° | Aug. 21 |
| Aql | 7373 | 19 20.2 | 11 44 | dG5 | 5.2 | ST215V | 18, 9 | 26° | July 8 |
| 3 Vul | 7386 | 19 21.3 | 24 44 | dF8 | 6.2 | ST220U | 52, 26 | 19° | July 9 |
| θ Cyg | 7469 | 19 33.8 | 49 59 | dF5 | 4.6 | ST252S | 9, 2 | 12° | Aug. 8 |
| α Sae | 7479 | 19 35.6 | 17 47 | gG0 | 4.4 | ST210R | 8, 4 | 20° | July 7 |
| β Sae | 7488 | 19 36.6 | 17 15 | gK0 | 4.4 | ST258V | 8, 2 | 22° | Aug. 9 |
| Cyg | 7495 | 19 37.7 | 45 17 | gF2 | 5.0 | ST253U | 14, 4 | 14° | Aug. 8 |
| 19 Cyg | 7566 | 19 47.0 | 38 27 | Ma | 5.4 | ST277O | 52, 14 | 4° | Aug. 18 |
| β Aql | 7602 | 19 50.4 | 6 9 | dK0 | 3.9 | ST238R | 4, 1 | 32° | July 28 (3) |
| β Aql | 7602 | 19 50.4 | 6 9 | dK0 | 3.9 | ST239U | 40, 10 | 32° | July 28 (4) |
| Cyg | 7606 | 19 51.1 | 36 44 | gG5 | 5.8 | ST278R | 34, 10 | 12° | Aug. 18 |
| Cyg | 7670 | 19 59.5 | 29 38 | dK0 | 5.7 | ST241V | 30, 9 | 10° | July 29 |
| 15 Sae | 7672 | 19 59.7 | 16 48 | dG0 | 5.9 | ST282O | 38, 10 | 24° | Aug. 19 |
| 27 Cyg | 7689 | 20 2.7 | 35 42 | dK0 | 5.5 | ST286O | 26, 7 | 7° | Aug. 20 |
| ε Del | 7852 | 20 28.4 | 10 58 | B5 | 4.0 | ST294S | 4, 1 | 32° | Aug. 21 |
| 49 Cyg | 7921 | 20 37.0 | 31 57 | gK0 | 5.8 | ST264S | 37, 10 | 21° | Aug. 10 |
| Cyg | 8084 | 21 2.3 | 30 47 | gF5 | 5.9 | ST287R | 38, 10 | 17° | Aug. 20 |
| $61_A$ Cyg | 8085 | 21 2.4 | 38 15 | dK5 | 5.6 | ST284S | 28, 7 | 32° | Aug. 19 |
| $61_B$ Cyg | 8086 | 21 2.4 | 38 15 | dK5 | 6.3 | ST283R | 58, 14 | 20° | Aug. 19 |
| τ Cyg | 8130 | 21 10.8 | 37 37 | gF0 | 3.8 | ST273R | 5, 1 | 11° | Aug. 17 |
| α Equ | 8131 | 21 10.8 | 4 50 | gF8 | 4.1 | ST272O | 5, 1½ | 33° | Aug. 17 |
| Cyg | 8220 | 21 23.8 | 31 47 | dF0 | 5.7 | ST211U | 30, 15 | 4° | July 7 |

TABLE III (*Continued*)

| Star | HR | α(1900) | δ(1900) | Sp. | m | Plate | Exp. | Z. D. | Date (1927) |
|---|---|---|---|---|---|---|---|---|---|
| π Aqr | 8539 | 22 20.2 | 0 52 | B1p | 4.6 | ST297X | 9, 2 | 36° | Aug. 21 |
| π Aqr | 8539 | 22 20.2 | 0 52 | B1p | 4.6 | ST298O | 9, 2 | 37° | Aug. 21 (5) |
| 31 Cep | 8615 | 22 33.3 | 73 7 | gF0 | 5.2 | ST279S | 20, 5 | 36° | Aug. 18 |
| 10 Lac | 8622 | 22 34.8 | 38 32 | Oe5 | 4.9 | ST295U | 12, 3 | 25° | Aug. 21 |
| Peg | 8631 | 22 35.9 | 14 1 | dG5 | 5.8 | ST260X | 34, 8 | 24° | Aug. 9 |
| η Peg | 8650 | 22 38.3 | 29 42 | gG0 | 3.1 | ST242X | 2, ½ | 11° | July 29 |
| η Peg | 8650 | 22 38.3 | 29 42 | gG0 | 3.1 | ST306X | 2, ½ | 21° | Sept. 4 (6) |
| ξ Peg | 8665 | 22 41.6 | 11 40 | dF5 | 4.3 | ST274S | 7, 2 | 26° | Aug. 17 |
| ξ Peg | 8665 | 22 41.6 | 11 40 | dF5 | 4.3 | ST305X | 7, 2 | 28° | Sept. 4 (6) |
| 51 Peg | 8729 | 22 52.5 | 20 14 | dG0 | 5.6 | ST261O | 28, 8 | 22° | Aug. 9 |
| 7 And | 8830 | 23 8.0 | 48 52 | dF0 | 4.6 | ST280U | 9, 2 | 11° | Aug. 18 |
| 85 Peg | 9088 | 23 56 9 | 26 33 | dG0 | 5.8 | ST299R | 30, 6 | 10° | Aug. 21 |
| Sun—Opening No. 1 | | | | | | ST233Q | 2ˢ, ½ˢ | 45° | July 27 (7) |
| Sun—Opening No. 1 | | | | | | ST300M | 1½, ½ | 30° | Sept. 4 |
| Sun—Opening No. 2 | | | | | | ST301O | 1½, ½ | 30° | Sept. 4 |
| Sun—Opening No. 3 | | | | | | ST302Q | 1½, ½ | 46° | Sept. 4 |
| Sun—Opening No. 4 | | | | | | ST303T | 1½, ½ | 48° | Sept. 4 |
| Sun—Opening No. 5 | | | | | | ST307T | 1, ¼ | 48° | Sept. 5 |
| Sun—Opening No. 6 | | | | | | ST308X | 1, ¼ | 51° | Sept. 5 |

(1) Star drifting along slit.
(2) Slow motion trailing.
(3) Without Diaphragm.
(4) With Diaphragm.
(5) Slit vertical.
(6) Red focus corrected.
(7) Not included in mean, but results agree well with those of Plate ST300M.

TABLE IV

VALUES OF Mλ FOR 55 STARS

(Values in italics are of low weight)

| Star HR | Type \ λ | 3780 | 4004 | 4184 | 4380 | 4595 | 4807 | 4990 | 5190 | 5400 | 5780 | 6120 | 6540 | 6925 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8622 | Oe5 | +0.85 | +0.92 | +0.96 | +0.89 | +0.63 | +0.55 | +0.41 | +0.20 | 0.00 | −0.15 | −0.45 | *−0.32* | *−0.04* |
| 7852 | B5 | +0.52 | +0.78 | +0.72 | +0.70 | +0.51 | +0.39 | +0.32 | +0.14 | 0.00 | −0.25 | −0.48 | −0.77 | |
| 8539 | B1p | +0.65 | +0.81 | +0.79 | +0.73 | +0.55 | +0.42 | +0.38 | +0.17 | 0.00 | −0.10 | −0.27 | *−0.12* | *−0.37* |
| 8539 | B1p | +0.40 | +0.46 | +0.57 | +0.61 | +0.45 | +0.36 | +0.29 | +0.14 | 0.00 | −0.20 | −0.38 | −0.23 | *−0.74* |
| 7178 | A0p | +0.69 | +0.70 | +0.69 | +0.66 | +0.49 | +0.40 | +0.34 | +0.18 | 0.00 | −0.14 | −0.25 | −0.48 | |
| 7178 | A0p | +0.65 | +0.78 | +0.79 | +0.71 | +0.49 | +0.41 | +0.38 | +0.16 | 0.00 | −0.20 | −0.38 | −0.59 | *−0.15* |
| 6556 | A5 | +0.03 | +0.42 | +0.52 | +0.43 | +0.44 | +0.30 | +0.24 | +0.11 | 0.00 | −0.11 | −0.31 | −0.46 | *−0.47* |
| 8130 | gF0 | −0.27 | +0.18 | +0.31 | +0.44 | +0.38 | +0.30 | +0.24 | +0.12 | 0.00 | −0.08 | −0.28 | −0.33 | *−0.26* |
| 6987 | gF2 | −0.15 | +0.19 | +0.32 | +0.36 | +0.35 | +0.21 | +0.21 | +0.06 | 0.00 | −0.13 | −0.28 | *−0.11* | |
| 7495 | gF2 | −0.08 | +0.27 | +0.38 | +0.51 | +0.38 | +0.36 | +0.30 | +0.09 | 0.00 | −0.01 | −0.13 | *−0.28* | |
| 8084 | gF5 | −0.57 | +0.03 | −0.05 | +0.36 | +0.32 | +0.22 | +0.16 | +0.08 | 0.00 | −0.03 | +0.04 | −0.35 | *−0.62* |
| 8615 | gF0 | −0.38 | +0.14 | +0.27 | +0.34 | +0.34 | +0.27 | +0.22 | +0.03 | 0.00 | −0.07 | −0.29 | −0.21 | *−0.33* |
| 8131 | gF8 | −0.52 | +0.16 | +0.23 | +0.34 | +0.33 | +0.25 | +0.17 | +0.12 | 0.00 | −0.05 | −0.15 | −0.24 | *−0.31* |
| 544 | gF5 | −0.39 | +0.04 | +0.16 | +0.25 | +0.28 | +0.19 | −0.13 | +0.07 | 0.00 | −0.05 | −0.12 | −0.36 | *−0.58* |
| 7165 | gF5 | −0.61 | −0.06 | +0.13 | +0.22 | +0.11 | +0.22 | +0.10 | +0.04 | 0.00 | −0.03 | −0.16 | −0.05 | *−0.50* |
| 7479 | gG0 | −0.88 | −0.39 | −0.16 | +0.04 | +0.21 | +0.18 | +0.09 | 0.00 | 0.00 | +0.01 | −0.12 | −0.04 | *−0.30* |
| 5889 | gG5 | −0.48 | −0.14 | −0.06 | 0.00 | +0.22 | +0.19 | +0.12 | 0.00 | 0.00 | +0.01 | −0.09 | −0.05 | *−0.37* |
| 5889 | gG5 | −0.83 | −0.36 | −0.19 | −0.02 | +0.12 | +0.19 | +0.15 | +0.02 | 0.00 | +0.04 | −0.01 | *+0.30* | *−0.30* |
| 7133 | gG0 | | −0.27 | −0.22 | +0.04 | +0.14 | +0.16 | +0.05 | 0.00 | 0.00 | +0.09 | −0.09 | −0.08 | *−0.34* |
| 6183 | gG5 | *−1.44* | −0.83 | −0.78* | −0.09 | +0.04 | +0.07 | −0.04 | −0.03 | 0.00 | +0.07 | 0.00 | *+0.10* | |
| 7606 | gG5 | −0.86 | −0.22 | −0.25 | +0.12 | +0.11 | +0.18 | +0.13 | +0.07 | 0.00 | +0.01 | 0.00 | −0.01 | *−0.27* |
| 8650 | gG0 | −1.10 | −0.42 | −0.39 | −0.08 | −0.07 | +0.02 | +0.02 | +0.08 | 0.00 | +0.02 | −0.02 | −0.09 | *−0.12* |
| 8650 | gG0 | −1.25 | −0.55 | −0.43 | −0.14 | −0.03 | −0.01 | +0.04 | +0.08 | 0.00 | +0.03 | −0.10 | −0.08 | *−0.25* |
| 6902 | gG0 | *−0.95* | −0.42 | −0.44 | −0.14 | −0.07 | +0.09 | +0.05 | −0.06 | 0.00 | +0.02 | +0.22 | *+0.19* | *−0.18* |
| 7488 | gK0 | | −0.73 | −0.76* | −0.20* | +0.02 | +0.11 | +0.07 | −0.07 | 0.00 | +0.10 | +0.04 | *+0.20* | |
| 7921 | gK0 | −1.29 | −0.53 | −0.45 | −0.09 | −0.12 | +0.18 | +0.01 | −0.07 | 0.00 | +0.08 | +0.04 | +0.18 | *+0.24* |

TABLE IV (*Continued*)

| Star HR | Type \ λ | 3780 | 4004 | 4184 | 4380 | 4595 | 4807 | 4990 | 5190 | 5400 | 5780 | 6120 | 6540 | 6925 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6228 | gK2 | | −1.89 | −1.55 | −0.66 | −0.23 | −0.23 | −0.21 | −0.22 | 0.00 | +0.23 | +0.15 | +0.28 | *−0.08* |
| 6108 | gK5 | | −1.87 | −1.55 | −0.88 | −0.22 | −0.18 | −0.23 | −0.19 | 0.00 | +0.30 | +0.19 | +0.42 | *0.00* |
| 6418 | gK5 | | *−1.75* | −1.45 | −0.72 | −0.29 | −0.14 | −0.09 | −0.19 | 0.00 | +0.22 | +0.12 | +0.25 | *−0.35* |
| 6695 | gK0 | | −1.34 | −1.36* | −0.48 | −0.18 | +0.12 | +0.03 | −0.13 | 0.00 | +0.15 | +0.16 | +0.44 | *+0.10* |
| 6868 | gK5 | | *−2.11* | −1.66 | −0.96 | −0.32 | −0.38 | −0.29 | −0.20 | 0.00 | +0.25 | +0.05 | +0.28 | *−0.04* |
| 6860 | gK5 | | *−1.61* | −1.54 | −0.80 | −0.26 | −0.08 | −0.17 | −0.27 | 0.00 | +0.17 | +0.19 | +0.42 | *+0.35* |
| 45 | gMa | | −2.20 | −1.51 | −1.00 | −0.39 | −0.45 | −0.39 | −0.30 | 0.00 | +0.16 | −0.07 | +0.36 | *+0.43* |
| 6815 | gMa | | −2.25 | −1.67 | −1.03 | −0.49 | −0.53 | −0.41 | −0.23 | 0.00 | +0.20 | −0.11 | +0.34 | *+0.20* |
| 6337 | gMa | | −2.13 | −1.66 | −0.95 | −0.45 | −0.53 | −0.47 | −0.40 | 0.00 | +0.20 | +0.16 | +0.45 | *−0.03* |
| 7566 | gMa | | −2.43 | −1.75 | −0.99 | −0.52 | −0.54 | −0.39 | −0.38* | 0.00 | +0.22 | +0.15 | +0.45 | *+0.31* |
| 8830 | dF0 | +0.01 | +0.60 | +0.69 | +0.67 | +0.49 | +0.42 | +0.32 | +0.15 | 0.00 | −0.09 | −0.24 | *−0.23* | |
| 8220 | dF0 | −0.03 | +0.48 | +0.65 | +0.62 | +0.47 | +0.47 | +0.29 | +0.16 | 0.00 | −0.06 | −0.16 | −0.41 | *−0.72* |
| 114 | dF0 | −0.13 | +0.42 | +0.48* | +0.53 | +0.45 | +0.33 | +0.31 | +0.14 | 0.00 | −0.08 | −0.23 | −0.52 | *−0.95* |
| 413 | dF0 | −0.31 | +0.22 | +0.35 | +0.39 | +0.33 | +0.31 | +0.23 | +0.04 | 0.00 | −0.07 | −0.20 | −0.47 | *−0.79* |
| 7386 | dF8 | −0.13 | +0.25 | +0.36 | +0.36 | +0.39 | +0.36 | +0.23 | 0.00 | 0.00 | −0.06 | −0.17 | −0.30 | *−0.81* |
| 7061 | dF5 | −0.18 | +0.18 | +0.30 | +0.34 | +0.28 | +0.19 | +0.15 | +0.06 | 0.00 | −0.07 | −0.21 | −0.32 | *−0.66* |
| 7469 | dF5 | −0.16 | +0.19 | +0.30 | +0.35 | +0.28 | +0.26 | +0.24 | +0.10 | 0.00 | −0.09 | −0.12 | *−0.29* | |
| 8665 | dF5 | −0.39 | +0.23 | +0.24 | +0.31 | +0.29 | +0.26 | +0.23 | +0.14 | 0.00 | −0.04 | −0.04 | *−0.08* | *−0.27* |
| 8665 | dF5 | −0.40 | +0.10 | +0.14 | +0.22 | +0.19 | +0.20 | +0.19 | +0.08 | 0.00 | −0.04 | −0.08 | *−0.09* | *−0.68* |
| 5933 | dF5 | *−0.11* | +0.19 | +0.22 | +0.27 | +0.25 | +0.19 | +0.17 | +0.08 | 0.00 | −0.04 | −0.06 | *−0.26* | |
| 5868 | dG0 | −0.46 | −0.02 | +0.06 | +0.17 | +0.25 | +0.25 | +0.16 | +0.02 | 0.00 | −0.02 | −0.08 | −0.19 | |
| 7672 | dG0 | −0.53 | −0.07 | +0.04 | +0.13 | +0.20 | +0.22 | +0.14 | +0.04 | 0.00 | +0.01 | −0.10 | −0.09 | *−0.46* |
| 9088 | dG0 | −0.72 | −0.15 | −0.03 | +0.07 | +0.14 | +0.19 | +0.14 | 0.00 | 0.00 | −0.09 | −0.10 | −0.09 | *−0.60* |
| 8729 | dG0 | −0.52 | −0.18 | −0.05 | +0.15 | +0.24 | +0.20 | +0.07 | 0.00 | 0.00 | −0.03 | −0.08 | −0.16 | *−0.33* |
| 5 | dG5 | −0.67 | −0.14 | +0.02 | +0.17 | +0.23 | +0.21 | +0.15 | +0.04 | 0.00 | +0.01 | −0.01 | −0.11 | *−0.37* |
| 8631 | dG5 | −0.55 | −0.25 | −0.08 | +0.14 | +0.19 | +0.26 | +0.16 | +0.26 | 0.00 | +0.06 | −0.08 | *−0.06* | |
| 5544 | dG5 | −0.60 | −0.25 | −0.12 | +0.11 | +0.24 | +0.18 | +0.05 | −0.07 | 0.00 | +0.03 | −0.05 | −0.05 | *−0.31* |
| 7670 | dK0 | *−0.78* | −0.25 | −0.15 | +0.04 | +0.18 | +0.22 | +0.11 | +0.07 | 0.00 | +0.03 | −0.01 | −0.37 | *−0.33* |
| 7602 | dK0 | *−0.93* | −0.48 | −0.23 | *−0.02* | +0.13 | +0.15 | +0.19 | −0.04 | 0.00 | +0.09 | −0.03 | *+0.02* | *−0.42* |
| 7602 | dK0 | −0.93 | −0.44 | −0.24 | −0.04 | +0.15 | +0.15 | +0.13 | −0.01 | 0.00 | +0.01 | −0.11 | *−0.18* | *−0.60* |
| 7373 | dG5 | −0.77 | −0.25 | −0.18 | +0.09 | +0.16 | +0.18 | +0.09 | −0.01 | 0.00 | +0.02 | −0.08 | −0.09 | *−0.31* |
| 6752 | dK0 | *−0.91* | −0.52 | −0.38 | −0.02 | +0.09 | +0.16 | +0.10 | −0.08 | 0.00 | +0.01 | −0.13 | +0.22 | |
| 7689 | dK0 | *−1.16* | −0.45 | −0.32 | +0.03 | +0.09 | +0.19 | +0.09 | +0.02 | 0.00 | 0.00 | +0.18 | −0.01 | *−0.34* |
| 8085 | dK5 | −1.88 | −1.17 | −0.84 | −0.38* | −0.03 | −0.07 | −0.21 | −0.18 | 0.00 | +0.22 | +0.02 | +0.13 | *−0.10* |
| 8086 | dK5 | *−2.23* | −1.45 | −1.03 | −0.51 | −0.10 | −0.23 | −0.30 | −0.18 | 0.00 | +0.34 | +0.13 | +0.45 | *−0.14* |
| Sun | | | | | | | | | | | | | | |
| Opening 1†..... | | +2.31 | +1.63 | +1.58 | +1.27 | +1.00 | +0.75 | +0.51 | +0.21 | 0.00 | −0.44 | −0.70 | −0.89 | −0.87 |
| Opening 1..... | | +2.15 | +1.76 | +1.48 | +1.30 | +1.04 | +0.76 | +0.44 | +0.18 | 0.00 | −0.41 | −0.65 | −0.96 | −1.13 |
| Opening 2..... | | +2.37 | +1.80 | +1.59 | +1.17 | +0.94 | +0.73 | +0.41 | +0.21 | 0.00 | −0.39 | −0.68 | −0.84 | −0.49 |
| Opening 3..... | | +2.43 | +1.82 | +1.46 | +1.13 | +0.91 | +0.67 | +0.34 | +0.19 | 0.00 | −0.46 | −0.71 | −0.95 | −0.71 |
| Opening 4..... | | +2.60 | +1.87 | +1.45 | +1.19 | +1.02 | +0.68 | +0.40 | +0.26 | 0.00 | −0.44 | −0.62 | −1.13 | −1.34 |
| Opening 5..... | | +3.04 | +2.04 | +1.79 | +1.46 | +1.12 | +0.78 | +0.39 | +0.24 | 0.00 | −0.39 | −0.64 | −0.89 | −1.02 |
| Opening 6..... | | +2.58 | +2.02 | +1.64 | +1.27 | +0.99 | +0.81 | +0.57 | +0.23 | 0.00 | −0.47 | −0.76 | −1.00 | −0.97 |

* Measured in an absorption line.

† Not used in reductions.

Table V shows the temperatures derived for each of the stars observed, arranged in order of temperature, with the giant and dwarf sequences grouped separately. The absolute magnitudes, Harvard spectral types, and Mt. Wilson types, both estimated and measured, are also given for comparison. The spectral types vary of course with temperature, but as might be expected, there are numerous deviations from absolute correlation. When average spectral types are taken, grouping the stars by three's or even by two's, the correlation is closer, but still not enough closer in any one column to afford a means of determining which system of classification is best. There is little or no correlation between the temperatures and absolute magnitudes of the giants as a whole, but within each class it will be seen that those stars with the lower temperatures tend to the greater absolute luminosity. This will be noticed best among the F0's, G0's, and K0's. The effect is merely a less pronounced result of the same difference that exists between giants and dwarfs, the less luminous star of a given type being at the higher temperature. For the dwarfs in Table V the parallelism between temperature and absolute magnitude is good.

TABLE V

TEMPERATURES

| | HR | T | M | Sp. Har | Sp. M. W. −E. | Sp. M. W. −M. | Residuals |
|---|---|---|---|---|---|---|---|
| | 8622 | 35,000 | | Oe5 | | | |
| | 7852 | 25,000 | | B5 | | | |
| Early | 8539 | 15,000 | | B1p | | | |
| | 7178 | 15,000 | −0.2 | A0p | B9 | | |
| | 6556 | 11,000 | +0.7 | A5 | A0 | | |
| | 8130 | 9,000 | +1.9 | gF0 | F1 | F3 | − 700 |
| | 6987 | 9,000 | +1.6 | gF2 | F0 | F1 | + 200 |
| | 7495 | 9,000 | +1.5 | gF2 | F4 | F4 | + 200 |
| | 8084 | 8,500 | +0.7 | gF5 | F7 | F6 | + 700 |
| | 8615 | 8,000 | +0.2 | gF0 | F2 | F3 | −1700 |
| | 8131 | 8,000 | +1.9 | gF8 | F6 | F7 | +1000 |
| | 544 | 7,500 | +1.2 | gF5 | F2 | F4 | − 300 |
| | 7165 | 7,000 | +0.3 | gF5 | F8 | F7 | − 800 |
| | 7479 | 6,500 | −0.9 | gG0 | F9 | F8 | 0 |
| | 5889 | 6,250 | +1.7 | gG5 | G4 | G3 | + 750 |
| | 7133 | 6,000 | +1.7 | gG0 | F9 | G0 | − 500 |
| | 6183 | 5,750 | −0.3 | gG5 | G6 | G6 | + 250 |
| | 7606 | 5,500 | +0.9 | gG5 | F6 | F6 | 0 |
| Giants | 8650 | 5,500 | −0.2 | gG0 | G1 | G3 | −1000 |
| | 6902 | 5,000 | −0.8 | gG0 | G0 | G0 | −1500 |
| | 7488 | 5,000 | +0.0 | gK0 | G8 | G7 | + 250 |
| | 7921 | 4,750 | +0.0 | gK0 | G8 | G4 | 0 |
| | 6228 | 4,250 | +0.8 | gK2 | K5 | K3 | − 200 |
| | 6108 | 4,250 | +0.6 | gK5 | K6 | K2 | + 250 |
| | 6418 | 4,000 | +0.4 | gK5 | K4 | K1 | 0 |
| | 6695 | 4,000 | −0.3 | gK0 | G9 | G7 | − 750 |
| | 6868 | 4,000 | +0.2 | gK5 | K6 | K3 | 0 |
| | 6860 | 4,000 | +0.2 | gK5 | K2 | K0 | 0 |
| | 45 | 3,800 | +0.0 | gMa | Ma | K2 | + 50 |
| | 6815 | 3,800 | −0.1 | gMa | Ma | K0 | + 50 |
| | 6337 | 3,700 | −0.1 | gMa | Mb | K0 | 0 |
| | 7566 | 3,500 | | gMa | | | − 200 |
| | 8830 | 11,000 | +2.3 | dF0 | F0 | F0 | + 500 |
| | 8220 | 11,000 | +2.9 | dF0 | F0 | F1 | + 500 |
| | 114 | 11,000 | +2.5 | dF0 | F2 | F3 | + 500 |
| | 413 | 10,000 | +2.7 | dF0 | F0 | F1 | − 500 |
| | 7386 | 8,500 | +3.9 | dF8 | F6 | F4 | +1000 |
| | 7061 | 8,000 | +3.3 | dF5 | F4 | F6 | − 300 |
| | 7469 | 8,000 | +3.4 | dF5 | F2 | F5 | − 300 |
| | 8665 | 7,500 | +3.8 | dF5 | F5 | F6 | − 800 |
| | 5933 | 7,500 | +4.3 | dF5 | F5 | F6 | − 800 |
| | 5868 | 7,000 | +4.4 | dG0 | G0 | G0 | 0 |
| | 7672 | 7,000 | +5.0 | dG0 | G1 | G0 | 0 |
| | 9088 | 7,000 | +5.6 | dG0 | G3 | G2 | 0 |
| Dwarfs | 8729 | 7,000 | +4.6 | dG0 | G4 | G3 | 0 |
| | 5 | 6,500 | +5.3 | dG5 | G4 | G2 | + 200 |
| | 8631 | 6,500 | +4.5 | dG5 | G4 | G5 | + 200 |
| | 5544 | 6,500 | +5.8 | dG5 | G6 | G6 | + 200 |
| | 7670 | 6,500 | +4.8 | dK0 | G7 | G7 | + 900 |
| | 7602 | 6,500 | +3.9 | dK0 | G8 | G7 | + 900 |
| | 7373 | 6,000 | +5.1 | dG5 | G8 | G6 | − 300 |
| | 6752 | 6,000 | +5.8 | dK0 | K0 | G9 | + 400 |
| | 7689 | 5,500 | +4.1 | dK0 | G8 | G6 | − 100 |
| | 8085 | 5,000 | +8.3 | dK5 | K7 | K7 | + 200 |
| | 8086 | 4,000 | +8.7 | dK5 | K8 | K7 | − 800 |

In a few cases the temperatures given in Table V are not those originally derived from the comparison of stellar and Planck curves. A correction has been applied depending on residuals from the temperature curves. For each star on the program the energy curve has been laid over the Planck curve which seemed to fit it best, and differences in magnitude were read off for the points determined on the star curve. The two curves were arbitrarily made to coincide at λ5780, for this point seemed to be the least affected by systematic deviations from the Planck curve. The magnitude differences have been averaged according to spectral type, and curves similar to those shown in Figure 4 have resulted from plotting these group values in the sense: Observed Curve minus Temperature Curve. The curves so obtained would be straight horizontal lines if the agreement between stellar and temperature curves were perfect, and systematic error in the original temperatures will show up as a slight slope of this line.

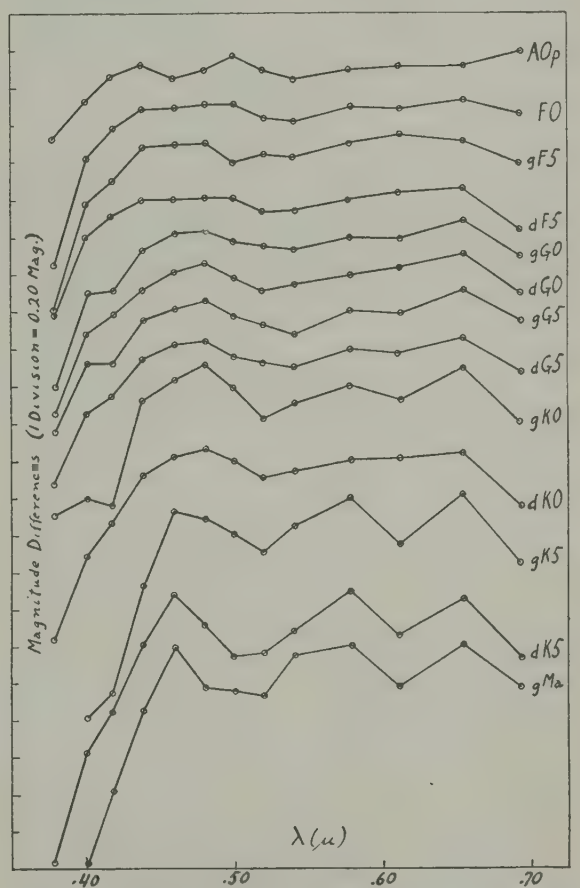



Fig. 4. Mean Deviations from Planck Curves, by Types.

This effect was observed in several cases. In all but one the curves were improved merely by examining the stellar energy curves again and giving greater weight to a few observations which had previously been disregarded because of apparent error. In one case, however, the giant G5's, a rise of the curve at the red end persisted. Hence a correction of −250° was applied to all the temperatures involved, this

being derived from a comparison of the slope of the residual curve and the difference between the Planck curves for 6,000° and 6,500°. The curves of Figure 4 are based on the finally adopted temperatures, those given in Table V.

The curves of Figure 4 are interesting in themselves. A fall of all types at the violet end of the spectrum is clearly indicated. This is partly due to the stars themselves, as other investigators have found the same effect, but in the present instance it is probably also due to the wide slit which has been used in the spectrograph. This makes it improbable that the true level of the continuous spectrum is reached at all, due to the large number of deep absorption lines and bands. All the curves show a slight dip between λ5,000 and λ5,500, as is also found in the Sun. Finally it is seen that the earlier type stars fit the Planck curves much better than the later types, an effect which is to be expected from the increased absorption in the latter.

The deviations from the Planck curves as shown in Figure 4 strengthen the remarks already made about the desirability of a wide range of spectrum when temperatures are to be derived from the energy distribution. For obviously when small ranges are employed, these deviations will suggest erroneous slopes for the energy curves. This may possibly have some bearing on the fact, soon to be discussed, that our temperatures are higher than those of other observers which are usually based on a smaller portion of the spectrum.

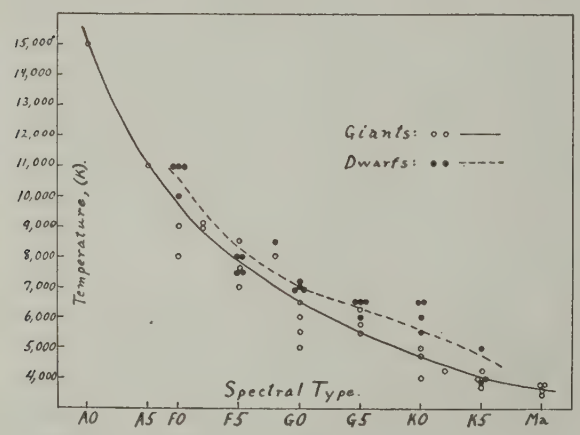



Fig. 5. Variation of Temperature with Spectral Type. Sample Set of Microphotometer Curves.

The temperatures given in Table V have been plotted against spectral type in Figure 5, the giants and dwarfs being indicated differently. Mean curves for both branches are drawn, and the residuals between each star and the appropriate mean curve are given in the last column of Table V. As is to be expected, the giants show a greater range of temperature within a given class than the dwarfs. The coefficient of dispersion has been computed for the two branches, the resulting values being ±197° for the giants and ±162° for the dwarfs.

The mean curves in Figure 5 show clearly the difference of temperature between the two sequences. The two converge for the early types and would of course come completely together if more stars were included. A convergence is also suggested for the late *K* and the *M* stars, though we have no *M*-type dwarfs.

Unfortunately most of the stars observed were faint, only eight of the fifty-five being brighter than magnitude 4.0; and as most other observers have confined themselves to the brighter stars, we can compare individual values for only eleven. Table VI gives

TABLE VI

Comparison of Individual Temperatures

| Star | Spec. | Storer | Wilsing | Yü | Rosenberg |
|---|---|---|---|---|---|
| γ Lyrae | A0p | 15,000 | 10,100 | 10,600 | |
| α Ophiuchi | A5 | 11,000 | 8,100 | 12,000 | 14,000 |
| τ Cygni | gF0 | 9,000 | 8,300 | | |
| α Equulei | gF8 | 8,000 | 6,300 | | |
| α Trianguli | gF5 | 7,500 | 6,100 | | |
| η Pegasi | gG0 | 5,500 | 4,700 | | |
| π Herculis | gK5 | 4,000 | 3,900 | | |
| θ Herculis | gK0 | 4,000 | 4,300 | | |
| γ Serpentis | dF5 | 7,500 | 6,400 | | |
| β Aquilae | dK0 | 6,500 | 4,800 | | |
| 70 Ophiuchi Br. | dK0 | 6,000 | 5,000 | | |

this comparison. Mean values for different spectral types may be compared, however. Table VII gives

TABLE VII

Comparison of Temperatures by Types

| Type | Storer | | | Russell |
|---|---|---|---|---|
| | No. of Stars | Mean Abs. Mag. | Mean Temp. | Adopted Temp. |
| Oe5 | 1 | | 35,000° | |
| B1p | 1 | | 15,000 | 23,000° (B0) |
| B5 | 1 | | 25,000 | 15,000 |
| A0p | 1 | −0.2 | 15,000 | 11,000 (A0) |
| A5 | 1 | +0.7 | 11,000 | 8,600 |
| F0 | 8 | +2.0 | 10,100 | 7,400 |
| F5 | 9 | +2.4 | 7,800 | 6,500 |
| gG0 | 4 | 0.0 | 5,750 | 5,600 |
| gG5 | 3 | +1.0 | 5,850 | 4,700 |
| gK0 | 4 | +0.1 | 4,500 | 4,200 |
| gK5 | 4 | +0.4 | 4,100 | 3,400 |
| gMa | 4 | 0.0 | 3,700 | 3,100 (gM2) |
| dG0 | 4 | +4.9 | 7,000 | 6,000 |
| dG5 | 4 | +5.2 | 6,400 | 5,600 |
| dK0 | 4 | +4.6 | 6,100 | 5,100 |
| dK5 | 2 | +8.5 | 4,500 | 4,400 |

the mean values as obtained by the writer, and the values adopted by Russell[27] as the mean of all previous determinations.

[27] *Astronomy*, Russell, Dugan, and Stewart, Vol. II, p. 753.

Examination of the above tables will reveal the fact that our temperatures are higher in nearly all cases than those obtained by other observers. Among themselves the temperatures are consistent: there is a decrease in temperature from early type to late, and the dwarfs are hotter than giants of the same type. Yet on the whole the values are higher than those of others. In comparing group values this discrepancy may be due in part to the small number of stars we have used, but the consistency of the difference suggests a systematic error, and an examination of the data has been made in search of this.

### Accuracy of Observations

The first possible source of systematic error is the atmospheric absorption, but it must be mentioned at the outset that this absorption will enter only in differential amounts since our standard of intensities is also affected by it. It is true that the average zenith distance for observations of the Sun was 42° while the greatest for any star was but 37°. It is also true that a haze in the sky, which is more likely to occur in the day time than at night, will cause increased absorption in the violet with the Sun and a correspondingly lower temperature for our standard. However, since all observations of the Sun were made on days which were apparently free from haze, and since it is generally considered that the absorption coefficients are fairly well known in the range of the spectrum most effective in deriving our temperatures, (λ4400 to λ6900), we may conclude that the possibility of serious error from this cause is small.

The possibility has been considered that atmospheric refraction has caused sufficient dispersion to prevent all the star's light from entering the slit. To test this $\pi$ *Aquarii* was observed at a zenith distance of 37° with the slit both horizontal and vertical. The resulting energy curves agree very closely from λ4400 to λ6600, and this fact, together with the thought of the wide slit used, lead us to discard refraction as a source of error.

It might be questioned whether the six openings used over the telescope in photographing the solar spectrum constitute a fair sample of the reflectivity of the mirror. The six sets of intensity measurements were in fair agreement, however, and since two plates of $\beta$ *Aquilae* taken with full and with one-tenth apertures, respectively, give almost identical energy curves, indicating a uniform reflecting surface, it is believed that our adopted values cannot be seriously in error.

Varying sensitivity from plate to plate is eliminated in the reductions, but a change in sensitivity of one plate between the photographing of the stellar and comparison spectra might produce a systematic difference. This possibility is excluded by the fact that on half of the plates the star spectra were exposed first, while on the others the comparison exposures were first.

After the regular observing program had been completed it was discovered that a slight broadening of the spectra at the red end, an effect which had been attributed to photographic spreading due to the intensity of the light, was in reality due to poor focusing in the camera. Accordingly the focus was corrected and observations of two stars were repeated. The resulting energy curves were in excellent agreement with those previously derived, thus showing that the faulty focus had not impaired the results. This was not to be expected in any event since the spectra were wide enough to prevent diminution of intensity along the center.

The two stars, observations of which were repeated, may be called upon in support of two other conclusions which have been drawn, namely, that the reflectivity of the silver mirror and the character of the comparison spectrum did not change during the course of the observing period. These conclusions are based mainly on the fact that the residuals in Table V show no progressive change when plotted against the dates of observation, but they are borne out by the repeated observations.

One further remark should be added about the variation of Schwarzschild's exponent, $p_\lambda$, with wavelength, previously discussed. If the slight indication of variation then noted were real, and a correction had been made for it, the resulting temperatures would have been even higher. For the long exposure spectrum was slightly low at the violet end, and correcting for that would have raised the temperature.

The foregoing paragraphs have considered the chief possible sources of systematic error in the process of securing the spectrograms. The microphotometer of course introduces new errors as does also the process of measuring the curves, but it is difficult to see how these could be systematic. The accidental error of measurement is greater at the red end of the spectrum due to the lower dispersion there, but it is still as likely to be in one direction as in the other.

It has been said that the probable error of one intensity determination is $\pm 0^m.02$. This of course results merely from the process of reduction. Since, however, the two spectra of one star have been photographed and measured independently, though they were on the same plate, and since it has been shown that the errors in the comparison spectra must be small, it is perhaps justifiable to take $\pm 0^m.02$ as the final probable error of a value of $k_\lambda$. The resulting probable error of $J_\lambda$ must be somewhat larger than that, due to possible errors both in $a_\lambda$ and in $C_\lambda$.

In summarizing this discussion of the accuracy of our results we may say that little chance has been discovered for a systematic error which would cause our temperatures to be too high. The atmospheric correction is generally considered to be fairly well known in the part of the spectrum with which we are most concerned, (λ4400 to λ6900), and unless Abbot's solar measures are seriously in error, we are not prepared to admit that there is any great inaccuracy in our temperatures.

The greatest cause for concern in the discrepancy between our temperatures and those of others has been the fact that four stars of supposedly the same type as the Sun have each given, independently, a temperature of 7000°, whereas 6000° is usually accepted for dwarf G0 stars. In support of the reality of this discrepancy, we may quote the work of Bottlinger.[28] His photo-electric color-indices were changed to the scale of King, and then, using an empirical relation based on the temperatures of Wilsing, Rosenberg, and Saha, the value of $C_2/T$ was derived for each of his stars. His table includes eleven dwarf stars between F8 and G2 with a mean temperature of 6500°, and with a range from 5800° to 7000°. Bottlinger's absolute scale may be in error, but it is not to be expected that, with photo-electric accuracy, his wide range of temperatures should be without foundation. Hence it may well be that the difference between our dwarf G0 stars and the Sun is real. Only one of the above mentioned eleven stars is on our program, 85 *Pegasi*, and Bottlinger derives for it a temperature of 6100° against our 7000°, but individual differences are perhaps not surprising.

## Comparison of Giants and Dwarfs

As was stated at the beginning of this paper, the primary purpose of the present investigation has been to search for possible differences between the continuous spectra of giant and dwarf stars. Before discussing the method used in this search it should be pointed out that even though our temperatures may be systematically in error, this will not in any way impair the validity of our conclusions on the other topic, for giants and dwarfs will be affected in the same manner. The effect we are searching for is differential.

Since both giant and dwarf sequences show a continuous gradation of temperature from the hottest to the coolest, any difference between their spectra which will be of service in enabling us to distinguish between them must show itself over and above any difference that is due to the temperature. Therefore the effect of temperature has first to be eliminated. This has in fact already been done, and the curves comprising Figure 4 show the results. The differences among the different types of spectra, after the effect of temperature has been removed, stand out clearly. These have been discussed before.

These curves also contain accidental errors due to the fact that so few stars are involved in each. In searching for real differences larger groups are desirable to obviate this difficulty. Accordingly the residuals shown in the curves of Figure 4 have been combined into three groups, and in so doing the accidental errors have been very well smoothed out. All F0 stars have been combined with the four stars of types A and B; and all stars from F5 to K2 inclusive have been grouped as giants or dwarfs. The K5 and M stars have not been included in these groups because the accidental errors in the measurements of such stars are great, and they would only have caused apparent fluctuations in the mean curves.

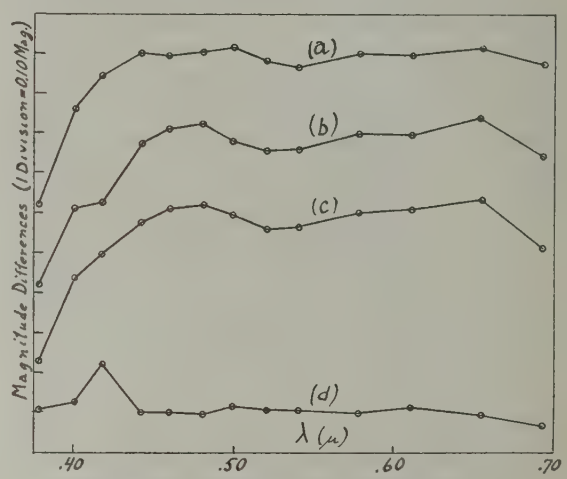



Fig. 6. Mean Deviations from Planck Curves of
(*a*) 12 Early Type Stars,
(*b*) 15 Giants,
(*c*) 17 Dwarfs.
(*d*) Dwarf Curve (*c*) minus Giant Curve (*b*).

The three groups referred to,—the early type stars, the giants, and the dwarfs, are shown graphically in Figure 6. These curves, then, show the deviations of the spectra of the three groups of stars from the Planck curves. All three curves are low at the violet end, an effect already discussed, and all show clearly the dip between λ5000 and λ5500. The dip at λ4200 on the giant curve is due to a cyanogen band which occurs in the G and K giants near that wave-length.

A glance at the giant and dwarf curves in Figure 6 will reveal the fact that there is no appreciable difference between the two in the visible portion of the spectrum. To bring this out more clearly, the differences between the two, in the sense: Dwarf minus Giant, are plotted in the fourth curve of Figure 6. Aside from the rise at λ4200 due to the cyanogen

[28] *Publ. Ber. Babel. Obs.*, **3**, no. 4, p. 21.

absorption in the giants, the curves show differences which average only $\pm 0.02$ magnitude. It would be expected that this curve would continue to rise at the violet end from the well known fact that the spectra of giants break off much more rapidly in the violet than do those of dwarfs. It has been pointed out, however, that our values, for various reasons, are unreliable in the violet.

The lack of any indication of considerable difference between the continuous spectra of giant and dwarf stars obviously makes it quite impossible to distinguish between the two using a color-index involving only that part of the spectrum between λ4400 and λ6900. Differences in the ultra-violet are known to exist however, and it would appear to be in this part of the spectrum that a means of distinguishing between the two classes is to be sought.

The plan which is suggested by the present result, therefore, is to use a color-index involving only that portion of the spectrum which we have discussed to determine the temperature of a star. This would seem feasible using color screens with transmission bands centering approximately at λ4700 and λ6500 respectively. These regions appear free from systematic deviations from the Planck curve. (See Figure 6.) Then, combining these observations with one through an ultra-violet filter, it might be possible to distinguish a giant from a dwarf. But the exact nature of this latter differentiation, involving the ultra-violet, is beyond the scope of the present investigation.

## Summary

The purpose of this investigation has been a search for differences between the continuous spectra of giant and dwarf stars which might make it possible to distinguish between the two by means of a color-index involving only the visual spectrum. Since this involves a knowledge of the spectral energy curves of stars, a study has been made of the methods used by a number of investigators in determining these, and the relative advantages and disadvantages of those methods have been indicated.

The method used in the present investigation has been described in detail. Spectrograms of 55 stars and the Sun were secured with a slit spectrograph attached to the Crossley Reflector. Plates were used which gave a measurable range of spectrum from λ3800 to λ7400. On each plate were impressed four exposures of different intensities of a standard spectrum, these providing the intensity scale for each wave-length independently. The Sun served as ultimate standard. The spectra were studied by means of a Moll self-registering microphotometer. A simplified method of making the reductions has been described.

The energy curves for all the stars as viewed from outside the Earth's atmosphere have been plotted, and from these the effective temperatures have been derived by comparison with Planck curves.

The temperatures, both individually and by groups, are shown to be somewhat higher than those derived by other investigators, and therefore a search for sources of systematic error has been made. Atmospheric absorption, loss of light at the slit, uneven reflectivity of the mirror, and errors in the process of reduction have been considered, but the first of these alone has been found capable of producing temperatures which are too high, and the possibility of this has been shown to be slight. On the other hand the wide range of spectrum involved lends confidence to our temperatures.

Deviations of each stellar curve from its temperature curve have been obtained, and these have been combined into three groups: early type stars, giants, and dwarfs. The two latter curves are nearly identical, showing that *no color-index involving only the visual spectum can distinguish a giant from a dwarf*. This negative result is probably the chief contribution resulting from this investigation. The suggestion is made, however, that the temperatures might be derived from the visual spectrum, using spectral regions centering approximately at λ4700 and λ6500; and that such observations together with one in the ultra-violet might distinguish a giant from a dwarf.

## Acknowledgments

In conclusion I wish to express my deep sense of obligation to Associate Director Robert G. Aitken for placing at my disposal the excellent equipment of the Lick Observatory during the course of this investigation. For extending my Fellowship beyond its normal length and for many other acts of friendly interest I am also indebted to him. Many other members of the Observatory staff have rendered assistance which is much appreciated. But I wish to express my deepest sense of gratitude to Dr. Robert Trumpler, Chairman of the committee of the faculty which has supervised my work, and to Dr. Joseph H. Moore, a member of that committee. They not only suggested and outlined the investigation which has been described, but have been most generous in their continued helpful suggestions and friendly encouragement.

April, 1929
Issued May 18, 1929.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

# LICK OBSERVATORY BULLETIN

NUMBER 411

## ELEMENTS AND EPHEMERIS OF MINOR PLANET Y.0.6. (1922 MY).

By

C. J. KRIEGER and D. C. DUNCAN

Approximate positions of 1922 MY were published by G. Van Biesbroeck, O. Struve and I. Yamamoto in the *Astronomical Journal* No. 822, June 18, 1923. A second paper by G. Van Biesbroeck and O. Struve, (*A. J.* No. 833, January 9, 1924) contains accurate measures of the positions previously announced:

| GMT 1922 | α(1922.0) | δ(1922.0) | Mag. | Obs. | Place |
|---|---|---|---|---|---|
| Nov. 24.78348 | $4^h32^m26^s.18$ | +37°06′31″.6 | | VB | Yerkes |
| Nov. 28.86667 | 4 27 21.68 | +37 00 06.3 | 16.2 | VB | Yerkes |
| Dec. 1.96286 | 4 23 29.04 | +36 52 08.4 | 16.2 | VB | Yerkes |
| Dec. 9.62764 | 4 14 11.75 | +36 22 14.9 | 16.0 | VB | Yerkes |
| Dec. 15.80139 | 4 07 29.65 | +35 49 09.6 | 16.1 | VB | Yerkes |
| Dec. 22.70735 | 4 01 22.71 | +35 05 46.7 | 16.2 | VB | Yerkes |

The asteroid was also found by M. Wolf, who published the observations in the *Beobachtungszirkular der Astronomischen Nachrichten.* 1922, No. 23, p. 60. Upon request the following remeasured positions were sent to Berkeley:

| Plate | Date, 1922 | Kgst. M.Z. | α(1922.0) | δ(1922.0) | Place |
|---|---|---|---|---|---|
| D2506 | Dec. 14 | $8^h45^m3$ | $4^h08^m59^s.29$ | +35°57′34″.8 | Königstuhl |
| D2514 | Dec. 21 | 10 14.4 | 4 02 24.05 | +35 14 14.6 | Königstuhl |

Wolf adds that the positions will not be very exact.

Elements by Kahrstedt for 1922 MY are published in *Astronomische Nachrichten* 5290, p. 163, 1924:

$T$ 1922 Nov. 24.5 GMT (Old Style)
$M$ 0° 36′ 47″.7
$\omega$ 90° 43′ 39″.5
$\Omega$ 335° 50′ 55″.5 } 1922.0
$i$ 8° 05′ 46″.2
$\varphi$ 5° 15′ 45″.0
$\mu$ 1005″.340
log $a$ 0.365129

It is not stated what observations were used for the computation, nor what residuals these elements give. G. Stracke mentions that not sufficient observations are available to assign a number.

By means of Leuschner's Short Method the following elements were derived from the three observations:

1922 Nov. 24.78348
1922 Dec. 9.62764
1922 Dec. 22.70735

### ELEMENTS

$T$ 1922 Nov. 18.7412 GMT (Old Style)
$\omega$ 89° 23′ 11″.7
$\Omega$ 335° 58′ 39″.0 } 1922.0
$i$ 8° 05′ 03″.7
$\varphi$ 5° 18′ 11″.9 ($e$=0.092428)
$\mu$ 1006″.649
log $a$ .364753

### CONSTANTS FOR THE EQUATOR 1922.0

$x = r[9.999288] \sin (155° 34′ 34″.3 + v)$
$y = r[9.934038] \sin (\ 63° 37′\ \ 5″.9 + v)$
$z = r[9.711823] \sin (\ 71°\ \ 3′ 10″.5 + v)$

The representation of the observations is as follows:

| No. | Date | O-C Δα cos δ | O-C Δδ |
|---|---|---|---|
| 1 | 1922 Nov. 24.78348* | −0″.25 | +0″.55 |
| 2 | Nov. 28.86667 | −4.2 | +1.6 |
| 3 | Dec. 1.96286 | −2.3 | +0.7 |
| 4 | Dec. 9.62764* | 0.0 | +0.35 |
| 5 | Dec. 14.34055 | −2.9 | +1.1 |
| 6 | Dec. 15.80139 | +4.2 | +0.6 |
| 7 | Dec. 21.40243 | −3.2 | −3.4 |
| 8 | Dec. 22.70735* | −0.57 | −0.23 |

* The orbit is based on these observations.

The elements and constants for the equator were reduced to the equinox of 1929.0 and an opposition ephemeris was computed:

## Elements

$T$ 1929, Dec. 7.115 U. T.

$\omega$ 89° 23′ 04″.38
$\Omega$ 336° 04′ 38″.06 } 1929.0
$i$ 8° 05′ 06″.83

$\mu$ 1006″.649
$e$ 0.092428
$\log a$ 0.364753

## Constants for the Equator 1929.0

$x = r[9.999293] \sin (155° 40′ 23″.48 + v)$
$y = r[9.934013] \sin (\ 63° 43′ 20″.68 + v)$
$z = r[9.711877] \sin (\ 71° \ 7′ 41″.14 + v)$

## Opposition Ephemeris

| 1929 U. T. | α(1929.0) | δ(1929.0) | log Δ | log r | Mag. |
|---|---|---|---|---|---|
| Nov. 1.0 | 4ʰ04ᵐ46ˢ.4 | 35°05′53″ | .0764 | .3235 | 16.1 |
| Nov. 9.0 | 3 57 02.3 | 35 20 13 | .0637 | .3231 | 16.0 |
| Nov. 17.0 | 3 47 50.4 | 35 17 22 | .0562 | .3229 | 16.0 |
| Nov. 25.0 | 3 38 07.7 | 34 56 56 | .0546 | .3227 | 16.0 |
| Dec. 3.0 | 3 28 57.3 | 34 21 02 | .0590 | .3226 | 16.0 |
| Dec. 11.0 | 3 21 18.9 | 33 34 20 | .0691 | .3226 | 16.1 |
| Dec. 19.0 | 3 15 54.2 | 32 42 43 | .0841 | .3227 | 16.1 |

The magnitudes are based on Van Biesbroeck's estimates during the 1922 opposition. (*A. J.* 833, 1924.)

Lick Observatory,
Mt. Hamilton, California,
April 12, 1929.

Issued May 18, 1929

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

# LICK OBSERVATORY BULLETIN

NUMBER 412

## THE RADIAL VELOCITY CURVE OF THE CEPHEID VARIABLE $\kappa$ PAVONIS

By T. S. Jacobsen

The light curve of $\kappa$ *Pavonis* ($\alpha = 18^h\ 46^m.6$; $\delta = -67°\ 21'$) has been discussed by Roberts,[1] Chandler,[2] and Cousins.[3] A comparison of their work shows that the period is irregular. Robert's maxima from 1891 to 1903 may be satisfactorily represented by a formula having a simple periodic term. If later light observations are to be included, the constants must be changed; —and the formula itself fails to harmonize light and velocity observations, however much the constants are altered. The variation is thus more complicated, or irregular. But the changes are slow during any one of the short series of velocity observations dealt with. Each velocity curve will therefore be discussed on the basis of the period which fits it the best.

The radial velocity material consists of 12 three-prism and 21 two-prism spectrograms, secured with the 37¼-inch reflector of the Chile Station of the Lick Observatory. All plates have been measured by one or both of the original observers,[4] and many also by the writer. Systematic corrections have been applied to bring the three-prism velocities into accord with the system used by Campbell and Moore.[5]

The data are entered in Table I. Phases in column two are reckoned from the separate epochs of periastron passages by the periods for each series. The O-C column lists residuals from the elliptic curve shown at the bottom in Figure 1. At the top is a reproduction of Robert's light curve.[6] Short crosslines mark the points of maximum and minimum light and velocity, and of passage through the apsides. The curves are arbitrarily oriented with light maximum and velocity minimum coinciding in time.

[1] *A. J.* **21**, 87, 1901; **24**, 91, 1904; *Ap. J.* **34**, 164, 1911.
[2] *A. J.* **24**, 7, 1904.
[3] *Mon. Not. R. A. S.* **88**, 108, 1927.
[4] Huffer and Wilson.
[5] *Publ. Lick Obs.* **16**.
[6] *Ap. J.* **34**, 167, 1911.

TABLE I. RADIAL VELOCITIES OF $\kappa$ PAVONIS

Three-prism series, 1904–07.

| J. D. (helioc.) | Helio-centric phase | Radial velocity, km/sec | O-C | High minus low |
|---|---|---|---|---|
| 2416654.640 | 0.493 | +25.5 | +1.8 | +0.3 |
| 6719.606 | 1.847 | +24.9 | +0.8 | −2.8 |
| 7692.828 | 2.717 | +27.6 | −0.5 | −3.8 |
| 6638.843 | 2.870 | +27.5 | −1.3 | −1.2 |
| 7702.794 | 3.596 | +31.6 | −0.3 | −3.6 |
| 6613.792 | 5.082 | +38.4 | +0.2 | −0.7 |
| 7713.732 | 5.446 | +40.1 | +0.3 | −1.6 |
| 7686.813 | 5.789 | +41.2 | −0.1 | −3.0 |
| 7714.743 | 6.457 | +44.6 | +0.2 | −0.9 |
| 7715.745 | 7.459 | +49.1 | −0.4 | −3.3 |
| 7697.784 | 7.673 | +51.4 | +0.8 | −1.0 |
| 7707.758 | 8.560 | +49.2 | −1.6 | −0.7 |

Two-prism series, 1916–18.

| | | | | |
|---|---|---|---|---|
| 2421162.518 | 0.080 | +33.5 | −1.3 | |
| 1163.543 | 1.105 | +20.0 | −1.5 | |
| 1527.516 | 1.150 | +26.1 | +4.6 | |
| 1145.543 | 1.302 | +21.0 | −0.9 | |
| 1138.531 | 3.388 | +29.5 | −1.5 | |
| 1120.561 | 3.614 | +36.9 | +4.8 | |
| 1693.863 | 3.730 | +32.0 | −0.5 | |
| 1384.862 | 4.068 | +29.4 | −4.5 | |
| 1530.509 | 4.143 | +33.6 | −0.6 | |
| 1139.514 | 4.371 | +33.9 | −1.3 | |
| 1130.523 | 4.478 | +39.8 | +4.1 | |
| 1113.577 | 5.729 | +39.6 | −1.4 | |
| 1532.510 | 6.144 | +44.0 | +1.0 | |
| 1141.508 | 6.365 | +45.9 | +1.8 | |
| 1469.667 | 6.989 | +49.6 | +2.5 | |
| 1142.533 | 7.390 | +51.1 | +3.0 | |
| 1470.678 | 8.000 | +48.7 | −3.2 | |
| 1143.585 | 8.442 | +50.4 | −1.5 | |
| 1134.536 | 8.491 | +49.3 | −2.2 | |
| 1680.822 | 8.885 | +44.3 | −0.5 | |
| 1471.666 | 8.988 | +41.7 | +0.2 | |

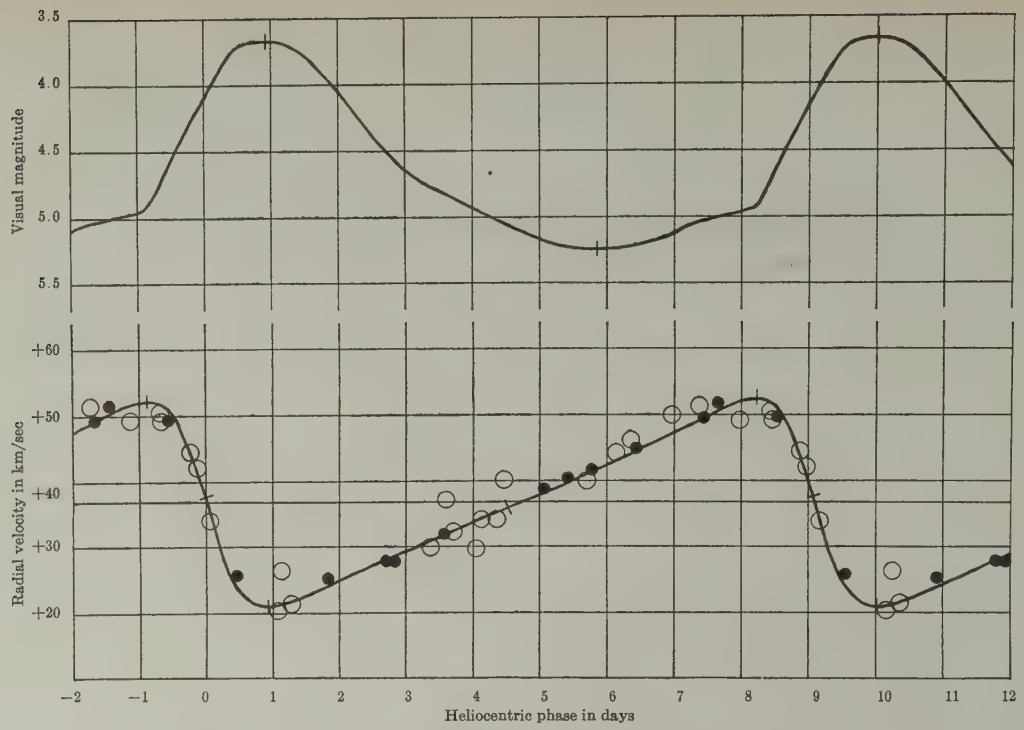



Fig. 1. Light and radial velocity curves of $\kappa$ *Pavonis*. Above, Robert's light curve. Below, radial velocity curve. Black dots, three-prism observations; open circles, two-prism observations.

The spectroscopic elements of the plotted curve, derived from the three-prism observations, are as follows:

$K = 15.5$ km/sec
$e = 0.49$
$\omega = 86°.84$
$\gamma = 36.5$ km/sec (Mills system)
$P = 9^d.0874$ (1904–07)
$T =$ J. D. 2416654.147

The values of the last two, derived from the two-prism observations, are as follows:

$P = 9^d.0982$ (1916–18)
$T =$ J. D. 2421162.438

The two series are plotted with their $\gamma$-axes coinciding. This is effected by subtracting the observed difference, 3.4 km/sec, from the two-prism series. Of this, 2.58 is a systematic error due to a certain standard velocity plate used in the Hartmann comparator measurements, and the remainder, 0.82, is too small to be seriously taken as indicating a real change in $\gamma$. Differences of the same sign and order of magnitude exist between two- and three-prism series of $\beta$ *Doradus* and of 1 *Carinae*.

The last column in Table I lists the velocity differences for groups of lines whose mean levels in the Sun are 843 and 472 km. For comparison, the same groups have been studied in several spectrograms of each of the stars $\alpha$ *Carinae*, $\alpha$ *Aquarii*, and $\delta$ *Sagittarii*, with the differences $-0.28$, $-0.79$, and $-2.79$ km/sec, corresponding to their spectral types F, G, and K. The viewpoint has been advanced in connection with other Cepheids, that much, if not all, of such velocity differences is due to uncompensated wavelength changes, and that they are thus not truly level effects. A smooth curve through the points plotted from columns two and five shows differences which are to be expected with the given spectral type variation, F5 to G5. Thus the changes with phase most probably result directly from this variation.

## Summary

1. The variations in the length of period are discussed on the basis of both light and velocity observations.

2. Radial velocity curves are constructed from two- and three-prism observations.

3. The velocity differences between groups of high and low level lines are such as are to be expected from the changes in blend effect accompanying spectral type variation.

University of Washington,
Seattle, Washington,
April 10, 1929.
Issued May 18, 1929.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

# LICK OBSERVATORY BULLETIN

NUMBER 413

## MEASURES OF 296 A DOUBLE STARS

BY

ROBERT G. AITKEN

It was a definite part of my observing program to remeasure every double star I had discovered after a suitable interval following the completion of my survey of the northern sky. Some of the close, bright pairs, which might, *a priori*, be expected to be in rapid orbital motion were, in fact, remeasured repeatedly while the survey was still in progress, but there was time then for but little work of this kind. From the second set of measures of the entire list, completed some years ago, it was found that in the average interval of ten years between the two sets of measures a considerable percentage of the new pairs showed change in angle or distance, or in both coordinates, that might reasonably be ascribed to actual relative motion. Efforts were therefore made to secure additional measures of such pairs. At the same time the systematic reobservation of the entire list at a third epoch was undertaken as a prerequisite to the publication of a general catalogue.

It will require four or five more years to complete the main program and it has therefore seemed appropriate to publish at this time the measures of those stars for which the evidence of orbital or relative proper motion seems fairly definite. The chief object is to direct the attention of other observers with powerful telescopes to these pairs, the great majority of which have been measured only by myself.

Nearly all the pairs listed have been measured in at least three different years, but 26, in which the motion is well defined, are included though measured at but two epochs. Pairs found single at the second and later examinations are excluded, though there is no doubt of the fact that the stars are double. Unless there is also definite change in the angular distance no pair is included with a measured change in angle of less than 10°, though there are many pairs measured at three or more epochs in which the observed motion, somewhat smaller in amount, is certainly real. While it is believed that the observed motion in the pairs which are listed is real, it is, of course, possible that, in a few instances, it is only the effect of error of observation.

The maximum measured angular separation in 223 of the pairs listed is less than 0″.40, in 100 of these it is 0″.25 or less and in but 17 pairs is it in excess of 1″.00. It is probable that a few of these wider pairs are not binary systems. Forty-three of the stars, including several brighter than 8.5 B. D. magnitude, are not listed in the *Henry Draper Catalogue*. The following table shows the magnitude and spectral class of the remaining 253:

| Spectral Class | No.[1] | Mag. | No. |
|---|---|---|---|
| B8 to A3 | 47½ | To 6.00 | 11 |
| A5 to F2 | 43½ | 6.01– 7.00 | 28 |
| F5 to G2 | 107 | 7.01– 8.00 | 68 |
| G5 to K2 | 55 | 9.01–10.00 | 23 |

It would, of course, be possible to compute dynamic parallaxes of all of these systems, but since this paper is purely a list of measures this has not been done. Notes are, in general, omitted, and orbits computed at earlier dates have not been revised. The tabulation gives for each pair the number in Burnham's General Catalogue, or in Jonckheere's Catalogue[2], the designation of the pair, the BD number, the photometric magnitude and spectral class for those in the *Henry Draper Catalogue*,[3] the star place for 1950.0 and my revised

[1] Three of the stars have composite spectra: this explains the half units.

[2] One or two pairs are not found in either catalogue.

[3] The BD magnitude is given for the 43 stars not in the *HDC*.

estimates of the magnitudes of the components. The measures are mean results, and the average deviation of the individual measures from the mean is added as a guide to the weight of each mean.

12705 **A 428**

$0^h\ 00^m\ 05^s$; $-8°\ 46'$ $-9°.6310$

9.1−9.1 8.7 G0

| | | | |
|---|---|---|---|
| 1902.74 | 111°.1±1°.0 | 0″.22±0″.02 | 3n |
| 1903.72 | 112.7±2.7 | 0.20±0.01 | 2 |
| 1915.67 | 91.6±1.5 | 0.25±0.02 | 2 |
| 1921.61 | 84.6±2.2 | 0.24±0.01 | 2 |
| 1925.79 | 74.7±1.4 | 0.22±0.02 | 2 |

13665 **A 1250**

$0^h\ 02^m\ 17^s$; $+29°\ 49'$ $+29°.5057$

8.2−10.5 8.2 F5

| | | | |
|---|---|---|---|
| 1905.86 | 90°.2±1°.1 | 0″.44±0″.01 | 3n |
| 1916.29 | 84.7±0.5 | 0.46±0.02 | 2 |
| 1928.72 | 78.3 | 0.50 | 1 |

*34* **A 1256**

$0^h\ 12^m\ 40^s$; $+43°\ 56'$ $+43°.33$

A and B, 7.5−7.5 6.64 B9

| | | | |
|---|---|---|---|
| 1906.78 | 4°.0±3°.2 | 0″.13±0.02 | 3n |
| 1908.95 | 11.2 | 0.12 | 1 |
| 1914.52 | 16.9 | 0.15 | 1 |
| 1918.63 | 19.8±1.0 | 0.15±0.01 | 2 |
| 1921.66 | 20.0±4.4 | 0.16±0.02 | 6 |
| 1928.32 | 23.4±3.9 | 0.15±0.01 | 3 |
| | AB and C, −13.0 | | |
| 1906.77 | 344°.8 | 18″.70 | 1 |
| | AB and D, −14.5 | | |
| 1906.77 | 133.6 | 27.60 | 1 |

*42* **A 1803**

$0^h\ 14^m\ 40^s$; $+8°\ 36'$ $+8°.24$

A and B, 7.5−7.6 6.62 F5

| | | | |
|---|---|---|---|
| 1908.75 | 300°.1±4°.3 | 0″.14±0″.01 | 4n |
| 1909.76 | 319.5±10.4 | 0.13±0.01 | 2 |
| 1917.36 | 149.1±2.5 | 0.15±0.00 | 2 |
| 1921.64 | 122.4±3.8 | 0.16±0.01 | 2 |
| 1922.70 | 123.7 | 0.20 | 1 |
| 1926.74 | 122.7±1.5 | 0.15±0.00 | 2 |
| | AB and C, =Σ22, −7.8 | | |
| 1908.70 | 237°.2 | 4″.13 | 1 |
| 1922.70 | 236.5 | 4.25 | 1 |
| 1926.74 | 236.3 | 4.14 | 1 |

The position angle measures in 1909 are quite uncertain. In 1917 and later years I thought the south following star was slightly the fainter, but it is possible that this is not correct. Additional measures will be necessary to settle the question. The star is 38 *Piscium* and the struve pair is number 116 in Burnhams' Catalogue.

12771 **A 647**

$0^h\ 17^m\ 53^s$; $+45°\ 14'$ $+44°.62$

7.5−9.5 7.02 F5

| | | | |
|---|---|---|---|
| 1904.54 | 215°.8±0°.2 | 0″.66±0″.02 | 3n |
| 1918.66 | 204.6±2.3 | 0.67±0.05 | 3 |
| 1921.68 | 200.5±3.5 | 0.61±0.01 | 2 |
| 1928.64 | 192.7±0.5 | 0.64±0.05 | 2 |

211 **A 431**

$0^h\ 25^m\ 35^s$; $-8°\ 09'$ $-8°.65$

8.5−8.6 8.3 G5

| | | | |
|---|---|---|---|
| 1903.75 | 353°.1±3°.4 | 0″.19±0″.02 | 3n |
| 1915.70 | 268.4±1.0 | 0.24±0.02 | 2 |
| 1917.85 | 260.6±0.6 | 0.25±0.03 | 2 |
| 1920.71 | 249.6±1.3 | 0.28±0.02 | 3 |
| 1921.66 | 247.0±0.9 | 0.34±0.02 | 2 |
| 1922.62 | 243.2±0.4 | 0.29±0.02 | 2 |
| 1925.80 | 239.6±0.4 | 0.34±0.00 | 2 |
| 1928.97 | 236.4±1.2 | 0.37±0.01 | 2 |

270 **A 111**

$0^h\ 29^m\ 55^s$; $-5°\ 27'$ $-5°.77$

A and B, 9.1−9.1 8.9 G5

| | | | |
|---|---|---|---|
| 1915.70 | 262°.5±1°.4 | 0″.22±0″.02 | 2n |
| 1916.71 | 241.2±0.9 | 0.18±0.01 | 2 |
| 1917.90 | 208.1±0.9 | 0.15±0.01 | 3 |
| 1918.56 | 172.2 | 0.11 | 1 |
| 1921.64 | 332.7±2.2 | 0.14±0.01 | 2 |
| 1922.68 | 313.8±2.3 | 0.16±0.01 | 3 |
| 1923.73 | 296.5±1.9 | 0.20±0.02 | 3 |
| 1925.80 | 280.0±0.8 | 0.18±0.01 | 2 |
| | AB and C, −12.5 | | |
| 1900.71 | 212°.6±1°.3 | 2″.13±0″.08 | 3n |
| 1902.67 | 212.1±0.4 | 2.26±0.05 | 2 |
| 1904.96 | 210.6 | 2.30 | 1 |
| 1906.81 | 209.4±1.8 | 2.22±0.03 | 2 |
| 1908.97 | 209.6 | 2.05 | 1 |
| 1911.84 | 209.2±1.0 | 2.19±0.08 | 2 |
| 1915.70 | 207.3±0.7 | 2.30±0.01 | 2 |
| 1916.71 | 206.0±2.2 | 2.28±0.03 | 2 |
| 1917.72 | 207.6 | 2.29 | 1 |
| 1921.64 | 205.4±0.9 | 2.19±0.07 | 2 |
| 1923.64 | 206.6±0.4 | 2.24±0.06 | 2 |
| 1925.80 | 204.8±1.1 | 2.20±0.05 | 2 |

My orbit of the close pair, computed in 1923, has the elements: $P=10.5$ years, $T=1919.75$, $e=0.405$, $a=0″.18$, $\omega=30°.45$, $i=37.85$, $\Omega=125°.5$. This gives for the later measure (1925.80) the residuals (O−C), $+11°.2$, $-0″.05$. The proper motion of the system is $0″.38$ in 99°. It is evident that this applies also to the third star, and that the system is a triple one.

12790 **A 914**

$0^h\ 33^m\ 45^s$; $+55°\ 52'$ $+55°.117$

8.6−8.6 8.0 F5

| | | | |
|---|---|---|---|
| 1905.80 | 257°.1±1°.0 | 0″.26±0″.01 | 3n |
| 1917.69 | 243.4±1.4 | 0.26±0.03 | 2 |
| 1921.73 | 242.4±0.7 | 0.30±0.01 | 2 |
| 1928.77 | 237.2 | 0.29 | 1 |

*79* **A 1505**

$0^h\ 33^m\ 50^s$; +37° 08′ +36°.93

9.6−9.6 9.0−

| | | | |
|---|---|---|---|
| 1907.60 | 50°.4±0°.3 | 0″.22±0″.01 | 2n |
| 1919.15 | 39.7±0.3 | 0.20±0.03 | 2 |
| 1921.78 | 37.8±1.3 | 0.22±0.01 | 2 |

There is also a star 14.5 magnitude 12″.32 from AB in 76°.9 (1919.64).

**12794** **A 650**

$0^h\ 38^m\ 21^s$; +46° 42′ +46°.136

8.3−15.2 8.4 A5

| | | | |
|---|---|---|---|
| 1904.55 | 24°.1±0°.1 | 3″.78±0″.01 | 2n |
| 1919.19 | 19.7±0.2 | 3.94±0.03 | 2 |
| 1928.71 | 14.4 | 4.00 | 1 |

**377** **A 435**

$0^h\ 39^m\ 54^s$; −6° 00′ −6°.119

9.7−10.2 10.0 G0

| | | | |
|---|---|---|---|
| 1903.78 | 221°.4±2°.5 | 0″.48±0″.03 | 3n |
| 1915.78 | 227.7±2.4 | 0.52±0.05 | 2 |
| 1928.97 | 236.2±0.7 | 0.56±0.01 | 2 |

**12799½** **A 651**

$0^h\ 40^m\ 35^s$ +47° 10′ +46°.145

9.0−10.0 8.9−

| | | | |
|---|---|---|---|
| 1904.57 | 200°.2±2°.1 | 0″.58±0″.01 | 3n |
| 1916.67 | 193.1 | 0.71 | 1 |
| 1919.19 | 189.6±0.0 | 0.63±0.01 | 2 |
| 1928.68 | 187.0±1.4 | 0.66±0.03 | 2 |

**12810** **A 921**

$0^h\ 46^m\ 14^s$; +57° 03′ +56°.130

A and B, 9.2−9.7 8.7−

| | | | |
|---|---|---|---|
| 1905.62 | 38°.9±1°.4 | 0″.17±0″.00 | 3n |
| 1915.70 | 47.2±1.7 | 0.21±0.02 | 2 |
| 1925.82 | 60.4 | 0.19 | 1 |
| | AB and C, −14.2 | | |
| 1905.60 | 265°.4±0°.3 | 4″.32±0″.05 | 3n |
| 1915.66 | 265.2 | 4.28 | 1 |
| 1925.82 | 265.2 | 4.25 | 1 |

*120* **A 2307**

$0^h\ 50^m\ 45^s$; +3° 49′ +3°.120

7.5−8.5 7.02 F0

| | | | |
|---|---|---|---|
| 1910.76 | 30°.5±1°.2 | 0″.24±0″.01 | 3n |
| 1911.68 | 33.7±1.8 | 0.25±0.01 | 2 |
| 1919.74 | 37.5±0.3 | 0.32±0.01 | 2 |
| 1927.79 | 45.6 | 0.30 | 1 |

*141* **A 1902**

$0^h\ 56^m\ 47^s$; −0° 57′ −1°.124

8.0−8.8 7.7 F5

| | | | |
|---|---|---|---|
| 1908.87 | 198°.6±2°.5 | 0″.31±0″.01 | 3n |
| 1916.71 | 207.6±0.8 | 0.32±0.02 | 2 |
| 1921.64 | 215.0±1.0 | 0.33±0.03 | 3 |
| 1923.65 | 220.6±2.4 | 0.38±0.02 | 2 |

*142* **A 1903**

$0^h\ 57^m\ 01^s$; −0° 28′ −1°.125

A and B, 9.2−9.5 8.67 G5

| | | | |
|---|---|---|---|
| 1908.87 | 86°.9±0°.9 | 0″.37±0″.03 | 3n |
| 1916.71 | 104.0±0.4 | 0.37±0.01 | 2 |
| 1920.75 | 116.1±3.3 | 0.34±0.02 | 2 |
| 1921.64 | 114.0±1.4 | 0.40±0.02 | 3 |
| 1923.65 | 122.7±0.7 | 0.40±0.01 | 2 |
| | AB and C, −13.7 | | |
| 1908.87 | 30°.0±0°.6 | 5″.72±0″.02 | 2n |
| 1916.71 | 29.4 | 5.82 | 1 |
| 1921.66 | 28.7 | 5.80 | 1 |
| 1923.68 | 27.8 | 5.66 | 1 |

—— **A 2210**

$0^h\ 57^m\ 36^s$; +17° 56′ +17°.135

7.4−13.4 7.32 G5

| | | | |
|---|---|---|---|
| 1910.66 | 210°.8±0°.4 | 5″.06±0″.00 | 2n |
| 1919.81 | 218.9±0.5 | 4.86±0.05 | 2 |
| 1921.65 | 219.4±0.3 | 4.55±0.00 | 2 |

**12820** **A 926**

$0^h\ 58^m\ 02^s$; +60° 05′ +59°.166

8.5−9.0 8.0 F5

| | | | |
|---|---|---|---|
| 1905.62 | 248°.4±2°.8 | 0″.25±0″.01 | 3n |
| 1915.69 | 258.2±0.4 | 0.28±0.02 | 2 |
| 1923.83 | 264.8±0.8 | 0.33±0.01 | 2 |
| 1925.81 | 268.5±1.9 | 0.32±0.02 | 2 |

*147* **A 2003**

$0^h\ 59^m\ 41^s$; +5° 36′ +5°.138

9.3−9.3 8.76 G0

| | | | |
|---|---|---|---|
| 1909.75 | 302°.8±3°.7 | 0″.20 | 4n |
| 1917.74 | 336.7±2.8 | 0.20 | 2 |
| 1921.63 | 357: | 0.15 | 1 |

*161* **A 1516**

$1^h\ 04^m\ 20^s$; +38° 23′ +37°.210

8.0−8.0 7.30 F8

| | | | |
|---|---|---|---|
| 1907.71 | 334°.7±1°.9 | 0″.16±0″.01 | 3n |
| 1918.72 | 61.0±0.1 | 0.15±0.00 | 2 |
| 1920.63 | 68.9±2.2 | 0.17±0.02 | 2 |
| 1921.61 | 83.4±1.5 | 0.16±0.01 | 2 |

*171* **A 1518**

$1^h\ 07^m\ 11^s$; +40° 23′ +39°.273

8.3−13.5 8.92 G

| | | | |
|---|---|---|---|
| 1907.69 | 239°.0±1°.0 | 3″.40±0″.04 | 2n |
| 1919.62 | 247.9 | 4.10 | 1 |
| 1920.67 | 251.2 | 4.32 | 1 |
| 1921.55 | 252.8±0.1 | 4.62±0.02 | 2 |

**12831** **A 655**

$1^h\ 08^m\ 23^s$; +40° 57′ +40°.235

8.2−8.7 7.87 G5

| | | | |
|---|---|---|---|
| 1904.05 | 144°.6±2°.3 | 0″.28±0″.02 | 3n |
| 1912.75 | 162.7 | 0.35 | 1 |
| 1918.63 | 168.0±0.4 | 0.34±0.03 | 2 |
| 1921.24 | 176.5±1.3 | 0.30±0.01 | 3 |
| 1928.68 | 189.8±4.0 | 0.28±0.03 | 2 |

704 **A 313**

$1^h\ 17^m\ 05^s$; $-5°\ 35'$ $-6°.251$

8.5−9.0 8.3 F8

| | | | |
|---|---|---|---|
| 1902.77 | 207°.2±0°.6 | 0″.19±0″.01 | 3n |
| 1903.72 | 201.6±1.4 | 0.19±0.01 | 2 |
| 1915.70 | 178.0±1.8 | 0.28±0.01 | 2 |
| 1920.14 | 169.3±1.4 | 0.25±0.01 | 3 |
| 1926.03 | 163.4±1.2 | 0.25±0.01 | 2 |

12846 **A 939**

$1^h\ 22^m\ 09^s$; $+45°\ 21'$ $+44°.296$

8.8−8.8 8.22 F0

| | | | |
|---|---|---|---|
| 1905.69 | 263°.8±3°.4 | 0″.20±0″.01 | 4n |
| 1918.70 | 287.4±0.4 | 0.22±0.01 | 2 |
| 1921.98 | 297.4±1.6 | 0.18±0.01 | 3 |
| 1927.78 | 310.0±0.8 | 0.20±0.02 | 2 |

*222* **A 1910**

$1^h\ 26^m\ 57^s$; $+22°\ 34'$ $+22°.236$

7.2−7.2 6.75 A0

| | | | |
|---|---|---|---|
| 1908.91 | 126°.7±0°.8 | 0″.14±0″.01 | 3n |
| 1917.62 | (73.0) | 0.13 | 1 |
| 1921.62 | 86.3±2.8 | 0.15±0.01 | 3 |

The seeing on the single night in 1917 was only fair, and the pair was a very difficult one to measure.

*229* **A 1913**

$1^h\ 31^m\ 38^s$; $+34°\ 24'$ $+33°.257$

9.7−9.7 9.1−

| | | | |
|---|---|---|---|
| 1908.87 | 267°.9±2°.7 | 0″.22±0″.02 | 3n |
| 1919.66 | 247.1±2.2 | 0.22±0.01 | 2 |
| 1921.70 | 246.8±3.3 | 0.23±0.02 | 3 |

812 **A 314**

$1^h\ 31^m\ 50^s$; $-8°\ 43'$ $-9°.301$

8.5−8.7 8.20 F5

| | | | |
|---|---|---|---|
| 1902.77 | 360°.0±0°.4 | 0″.33±0″.01 | 3n |
| 1915.70 | 357.0±0.3 | 0.39±0.01 | 2 |
| 1926.03 | 354.7±0.4 | 0.30±0.01 | 2 |
| 1929.02 | 348.8 | 0.38±0.01 | 1 |

*248* **A 2320**

$1^h\ 38^m\ 16^s$; $+11°\ 02'$ $+10.221$

9.5−9.5 8.9 G5

| | | | |
|---|---|---|---|
| 1911.20 | 317°.0±0°.1 | 0″.16±0″.02 | 2n |
| 1919.84 | 31.0±5.5 | 0.14±0.01 | 3 |
| 1921.60 | 108.4 | 0.12: | 1 |

*249* **A 2321**

$1^h\ 38^m\ 22^s$; $+18°\ 01'$ $+17°.244$

9.0−12.0 8.9−

| | | | |
|---|---|---|---|
| 1911.60 | 81°.2±0°.2 | 2″.17±0″.05 | 3n |
| 1919.69 | 88.2±0.6 | 1.40±0.01 | 2 |
| 1921.61 | 87.8±0.6 | 1.26±0.06 | 2 |

*250* **A 2404**

$1^h\ 38^m\ 27^s$; $+1°\ 22'$ $+0°.270$

9.2−9.2 8.49 F8

| | | | |
|---|---|---|---|
| 1911.86 | 360°.9±3°.1 | 0″.26±0″.02 | 3n |
| 1919.89 | 346.0 | 0.23 | 1 |
| 1921.63 | 341.4±5.1 | 0.20±0.02 | 3 |

888 **A 1**

$1^h\ 39^m\ 54^s$; $-7°\ 00'$ $-7°.282$

8.6−8.7 8.01 F2

| | | | |
|---|---|---|---|
| 1899.78 | 165°.2±1°.1 | 0″.31±0″.01 | 3n |
| 1902.75 | 172.5±1.8 | 0.38±0.02 | 4 |
| 1915.70 | 189.6±0.6 | 0.46±0.00 | 2 |
| 1920.14 | 193.7±1.1 | 0.41±0.03 | 3 |
| 1926.03 | 199.2±1.1 | 0.44±0.02 | 2 |
| 1928.97 | 204.2±1.7 | 0.44±0.03 | 2 |

12862 **A 948**

$1^h\ 43^m\ 17^s$; $+44°\ 24'$ $+43°.364$

A and B, 8.5−10.5 8.6 A5

| | | | |
|---|---|---|---|
| 1905.82 | 27°.5±2°.1 | 0″.26±0″.01 | 3n |
| 1917.97 | 34.4±3.9 | 0.26±0.01 | 2 |
| 1921.61 | 42.6±1.1 | 0.27±0.05 | 2 |

C and D, 13.5−13.8

| | | | |
|---|---|---|---|
| 1905.80 | 307°.5±0°.6 | 1″.99±0″.04 | 2n |
| 1917.94 | 307.7 | 2.47 | 1 |
| 1921.59 | 307.3 | 2.50 | 1 |

AB and C

| | | | |
|---|---|---|---|
| 1905.80 | 202°.6 | 52″.60 | 1n |
| 1917.94 | 204.3 | 52.27 | 1 |
| 1921.59 | 204.0 | 53.41 | 1 |

12865 **A 950**

$1^h\ 46^m\ 11^s$; $+56°\ 30'$ $+56°.357$

A and B, 9.2−9.7 8.8−

| | | | |
|---|---|---|---|
| 1905.82 | 118°.2±0°.2 | 0″.38±0″.02 | 3n |
| 1915.67 | 126.3±0.3 | 0.42±0.01 | 2 |
| 1925.82 | 131.4 | 0.43 | 1 |

AB and C = h2082, −10.0

| | | | |
|---|---|---|---|
| 1905.80 | 125°.5 | 16″.09 | 1n |
| 1915.70 | 125.6 | 16.30 | 1 |

*291* **A 2407**

$1^h\ 52^m\ 47^s$; $+2°\ 43'$ $+2°.296$

9.0−11.0 9.0−

| | | | |
|---|---|---|---|
| 1912.74 | 103°.7±2°.9 | 0″.53±0″.04 | 3n |
| 1919.88 | 117.6±1.8 | 0.52±0.02 | 2 |

*317* **A 1813**

$1^h\ 59^m\ 13^s$; $+36°\ 29'$ $+36°.391$

A and B, 8.5−8.6 8.0 G5

| | | | |
|---|---|---|---|
| 1908.82 | 8°.5±0°.2 | 0″.20±0″.01 | 3n |
| 1918.66 | 163.0 | 0.21 | 1 |
| 1919.68 | 173.2±3.2 | 0.18±0.01 | 2 |
| 1921.67 | 188.9±2.2 | 0.19±0.01 | 3 |

AB and C, −11.5

| | | | |
|---|---|---|---|
| 1908.82 | 338°.9±1°.2 | 0″.70±0.02 | 3n |
| 1918.66 | 324.8 | 0.85 | 1 |
| 1919.68 | 325.6±1.3 | 0.75±0.00 | 2 |
| 1921.66 | 322.2 | 0.83 | 1 |

**12876** **A 957**

$2^h\ 01^m\ 15^s$; +60° 16′ +59°.400

7.8−10.0 8.06 G0

| | | | |
|---|---|---|---|
| 1905.82 | 107°.2±3°.1 | 0″.45±0″.05 | 3n |
| 1915.64 | 100.4 | 0.57 | 1 |
| 1917.82 | 94.2±0.8 | 0.58±0.02 | 2 |
| 1921.63 | 93.9±2.5 | 0.54±0.03 | 2 |
| 1925.82 | 89.8 | 0.48 | 1 |

*339* **A 2013**

$2^h\ 13^m\ 16^s$; +6° 24′ +5°.307

9.5−9.5 9.0 K2

| | | | |
|---|---|---|---|
| 1909.84 | 145°.7±1°.0 | 0″.23±0″.00 | 2n |
| 1918.06 | 118.8 | 0.32 | 1 |
| 1919.39 | 110.2±0.1 | 0.43±0.02 | 2 |
| 1920.74 | 108.9 | 0.42 | 1 |
| 1921.67 | 105.2±0.2 | 0.44±0.02 | 2 |

**1190** **A 207**

$2^h\ 15^m\ 06^s$; +39° 06′ +38°.453

9.6−9.7 9.0−

| | | | |
|---|---|---|---|
| 1902.00 | 126°.4±0°.2 | 0″.25±0″.02 | 2n |
| 1917.67 | 150.4 | 0.25 | 1 |
| 1918.67 | 158.3±1.1 | 0.27±0.02 | 2 |
| 1921.62 | 163.0±1.6 | 0.28±0.01 | 2 |

**12885** **A 961**

$2^h\ 17^m\ 15^s$; +29° 21′ +29°.393

9.0−9.0 8.5 F5

| | | | |
|---|---|---|---|
| 1905.87 | 57°.4±1°.5 | 0″.18±0″.01 | 3n |
| 1917.13 | 358.0±2.2 | 0.29±0.03 | 3 |
| 1921.28 | 354.7±2.0 | 0.29±0.02 | 3 |
| 1922.64 | 353.0±2.3 | 0.33±0.04 | 2 |
| 1928.77 | 341.5 | 0.35 | 1 |

*355* **A 2605**

$2^h\ 22^m\ 02^s$; +14° 00′ +13°.386

8.7−11.7 8.7−

| | | | |
|---|---|---|---|
| 1912.88 | 127°.9 | 0″.26 | 1n |
| 1914.10 | 131.0 | 0.34 | 1 |
| 1924.07 | 142.0±2°.8 | 0.36±0″.05 | 2 |

**12889** **A 964**

$2^h\ 23^m\ 45^s$; +31° 54′ +31°.424

9.5−10.0 9.2−

| | | | |
|---|---|---|---|
| 1905.87 | 93°.5±2°.7 | 0″.24±0″.01 | 3n |
| 1917.26 | 81.0±0.7 | 0.24±0.01 | 2 |
| 1921.71 | 80.0±0.2 | 0.25±0.01 | 2 |
| 1928.77 | 61.5 | 0.23 | 1 |

*371* **A 2016**

$2^h\ 26^m\ 00^s$; +8′ 26′ +8°.380

9.5−9.5 9.1 G0

| | | | |
|---|---|---|---|
| 1909.82 | 104°.0±2°.5 | 0″.26±0″.01 | 3n |
| 1919.50 | 118.7±0.9 | 0.25±0.01 | 3 |

*388* **A 1818**

$2^h\ 32^m\ 07^s$; +40° 04′ +39°.570

9.2−9.2 8.7 F8

| | | | |
|---|---|---|---|
| 1908.78 | 331°.9±1°.8 | 0″.18±0″.00 | 3n |
| 1918.67 | 155.4±12.9 | 0.12±0.02 | 2 |
| 1919.68 | 95.2±2.5 | 0.12±0.01 | 2 |
| 1921.64 | 75.1±5.7 | 0.15±0.02 | 3 |

*401* **A 2219**

$2^h\ 35^m\ 06^s$; +19° 50′ +19°.391

9.0−9.0 8.6 F0

| | | | |
|---|---|---|---|
| 1910.65 | 300°.9±0°.9 | 0″.25±0″.00 | 3n |
| 1919.20 | 287.8±2.1 | 0.23±0.03 | 2 |
| 1922.71 | 282.8±1.0 | 0.20±0.01 | 2 |

*410* **A 1928**

$2^h\ 37^m\ 18^s$; −0° 04′ −0°.407

8.7−8.7 8.2 G0

| | | | |
|---|---|---|---|
| 1908.97 | 222°.5±1°.4 | 0″.18±0″.02 | 2n |
| 1916.71 | 292.0±9.2 | 0.14±0.02 | 2 |
| 1917.75 | 331.2±5.3 | 0.14±0.02 | 2 |
| 1919.84 | 43.6 | 0.13: | 1 |
| 1920.73 | 105.9±18.5 | 0.14±0.00 | 2 |
| 1921.70 | 161.0±15.0 | 0.13±0.02 | 3 |
| 1922.75 | 183.0±4.1 | 0.13±0.00 | 2 |
| 1926.69 | 213.0 | 0.21 | 1 |

The revolution period is about 19 years. It would be possible to compute an orbit, but this can be done to better advantage after the companion passes once more into the fourth quadrant.

*437* **A 1281**

$2^h\ 48^m\ 13^s$; +45° 47′ +45°.669

9.0−10.5 9.2 G5

| | | | |
|---|---|---|---|
| 1906.76 | 189°.0±1°.9 | 0″.45±0″.04 | 3n |
| 1917.18 | 217.2±4.4 | 0.37±0.01 | 4 |
| 1921.63 | 235.1±4.3 | 0.29±0.01 | 3 |
| 1922.69 | 246.1±2.2 | 0.33±0.05 | 2 |

This binary has the large proper motion 0″.63 in 124°.4 (Porter). The parallax is +0″.023 (*Yale Cat.*).

*456* **A 2413**

$2^h\ 54^m\ 39^s$; +1° 41′ +1°.515

8.3−8.4 7.8 G0

| | | | |
|---|---|---|---|
| 1912.79 | 181°.4±0°.6 | 0″.51±0″.01 | 3n |
| 1918.81 | 190.4±1.1 | 0.42±0.02 | 2 |
| 1922.05 | 196.6±0.4 | 0.34±0.01 | 2 |

*469* **A 2414**

$2^h\ 59^m\ 25^s$; +18° 07′ +17°.477

9.5−10.2 9.2−

| | | | |
|---|---|---|---|
| 1910.72 | 311°.2 | 0″.16 | 1n |
| 1912.82 | 319.2±0°.8 | 0.20±0″.03 | 2 |
| 1921.64 | 347.5±2.8 | 0.29±0.04 | 2 |
| 1922.70 | 347.6±0.5 | 0.31±0.01 | 2 |

*489* **A 2030**

$3^h\ 06^m\ 58^s$; $+4°\ 59'$ $+4°.501$

9.2–9.2 8.6 G0

| | | | |
|---|---|---|---|
| 1909.76 | 337°.2±0°.2 | 0″.31±0″.01 | 3n |
| 1919.39 | 304.5±0.8 | 0.36±0.03 | 2 |
| 1922.05 | 296.6±0.1 | 0.35±0.03 | 2 |

—— **A 2909**

$3^h\ 22^m\ 04^s$; $-15°\ 50'$ $-16°.630$

A and B, 7.4–7.4 7.55 G0

| | | | |
|---|---|---|---|
| 1918.02 | 162°.5±3°.3 | 0″.13±0″.00 | 2n |
| 1921.67 | 198.6±2.8 | 0.12±0.01 | 3 |

AB and C, −13.5

| | | | |
|---|---|---|---|
| 1918.00 | 252°.9 | 16″.69 | 1n |
| 1921.66 | 252.3 | 17.01 | 1 |

**12937** **A 829**

$3^h\ 23^m\ 24^s$; $+12°\ 19'$ $+11°.474$

8.5–10.0 8.1 G0

| | | | |
|---|---|---|---|
| 1904.85 | 40°.5±2°.5 | 0″.33±0″.01 | 3n |
| 1914.73 | 29.6±0.4 | 0.44±0.01 | 2 |
| 1922.05 | 18.2±0.2 | 0.42±0.02 | 2 |

**12936** **A 980**

$3^h\ 24^m\ 18^s$; $+60°\ 05'$ $+59°.657$

6.7–8.2 6.48 B8

| | | | |
|---|---|---|---|
| 1905.64 | 175°.6±0°.4 | 0″.34±0″.02 | 3n |
| 1908.84 | 170.4 | 0.35 | 1 |
| 1912.42 | 168.0±3.4 | 0.44±0.02 | 2 |
| 1916.20 | 162.6±0.5 | 0.40±0.01 | 2 |
| 1921.66 | 156.1±1.7 | 0.40±0.02 | 2 |
| 1925.81 | 154.9±1.2 | 0.35±0.01 | 2 |

*533* **A 1535**

$3^h\ 32^m\ 45^s$; $+42°\ 11'$ $+41°.714$

8.6–9.2 8.4 G5

| | | | |
|---|---|---|---|
| 1907.69 | 34°.1±1°.0 | 0″.48±0″.03 | 3n |
| 1916.87 | 61.4±2.1 | 0.36±0.01 | 2 |
| 1921.70 | 79.1±0.1 | 0.34±0.01 | 2 |
| 1922.67 | 84.0±1.3 | 0.28±0.01 | 2 |

**12944** **A 830**

$3^h\ 33^m\ 48^s$; $+10°\ 01'$ $+9°.459$

9.5–9.7 9.3 G0

| | | | |
|---|---|---|---|
| 1904.85 | 71°.1±1°.1 | 0″.24±0″.00 | 3n |
| 1914.73 | 57.0±3.2 | 0.34±0.02 | 2 |
| 1920.76 | 51.9 | 0.34 | 1 |
| 1921.68 | 48.9±1.8 | 0.34±0.02 | 2 |

*558* **A 1289**

$3^h\ 40^m\ 16^s$; $+52°\ 19'$ $+52°.709$

9.5–9.7 9.2 A

| | | | |
|---|---|---|---|
| 1906.68 | 10°.1±1°.5 | 0″.37±0″.01 | 3n |
| 1917.75 | 20.0±1.3 | 0.34±0.01 | 2 |
| 1922.70 | 23.8±0.8 | 0.34±0.02 | 2 |

**12955** **A 990**

$3^h\ 41^m\ 50^s$; $+57°\ 25'$ $+57°.742$

9.3–9.7 9.2 A0

| | | | |
|---|---|---|---|
| 1905.73 | 96°.8±1°.2 | 0″.23±0″.01 | 3n |
| 1908.84 | 94.4 | 0.26 | 1 |
| 1916.20 | 84.0±2.4 | 0.25±0.01 | 2 |
| 1922.70 | 82.5±1.2 | 0.22±0.01 | 2 |
| 1925.81 | 83.8±0.1 | 0.20±0.03 | 2 |

The measures in 1925 are marked "very difficult."

*581* **A 1293**

$3^h\ 49^m\ 55^s$; $+53°\ 08'$ $+52°.722$

8.2–8.6 7.9 G0

| | | | |
|---|---|---|---|
| 1906.68 | 216°.3±1°.0 | 0″.32±0″.03 | 3n |
| 1917.75 | 236.4±0.6 | 0.38±0.02 | 2 |
| 1921.66 | 244.6±1.7 | 0.40±0.01 | 2 |

*600* **A 1936**

$3^h\ 57^m\ 28^s$; $+8°\ 10'$ $+7°.582$

B and C, 9.6–10.0 9.0–

| | | | |
|---|---|---|---|
| 1908.84 | 136°.3±1°.9 | 0″.37±0″.05 | 3n |
| 1918.43 | 124.9±2.9 | 0.41±0.01 | 2 |
| 1922.69 | 119.7±1.0 | 0.38±0.04 | 2 |

A and BC=A. G. Leipzig II, 1472 9.5–

| | | | |
|---|---|---|---|
| 1908.84 | 27°.5±0°.2 | 4″.16±0°.5 | 2n |
| 1917.80 | 27.3 | 4.04 | 1 |
| 1922.69 | 27.3 | 4.04 | 1 |

*602* **A 1937**

$3^h\ 58^m\ 04^s$; $+4°\ 57'$ $+4°.619$

9.0–9.0 8.5 A3

| | | | |
|---|---|---|---|
| 1908.84 | 45°.2±7°.9 | 0″.18±0″.01 | 3n |
| 1919.06 | Too close | | 1 |
| 1919.73 | 117: | 0.14: | 2 |
| 1922.73 | 176.9±4.1 | 0.13±0.02 | 3 |

**2062** **A 469**

$4^h\ 06^m\ 55^s$; $-8°\ 04'$ $-8°.798$

8.0–8.0 7.28 A2

| | | | |
|---|---|---|---|
| 1903.88 | 343°.9±1°.6 | 0″.24±0″.00 | 3n |
| 1915.78 | 350.3±1.1 | 0.23±0.01 | 2 |
| 1924.07 | 355.5±0.8 | 0.25±0.01 | 2 |
| 1926.02 | 356.6 | 0.25 | 1 |
| 1929.02 | 1.6 | 0.22 | 1 |

*629* **A 2801**

$4^h\ 08^m\ 15^s$; $-5°\ 00'$ $-5°.841$

8.0–8.0 7.50 G0

| | | | |
|---|---|---|---|
| 1914.78 | 356°.0±0°.6 | 0″.19±0″.00 | 2n |
| 1921.70 | 15.1±0.7 | 0.20±0.01 | 2 |
| 1924.07 | 27.4±1.0 | 0.16±0.01 | 2 |

632 **A 1938**

$4^h\ 10^m\ 51^s$; +7° 35′ +7°.617

6.1−6.3 5.35 F0

| | | | |
|---|---|---|---|
| 1908.91 | 315°.0±0°.5 | 0″.14±0″.00 | 3n |
| 1914.71 | 293.4±1.4 | 0.12±0.01 | 3 |
| 1915.74 | Too close | | 1 |
| 1917.90 | 8.3 | 0.13 | 1 |
| 1919.73 | 156.3±2.3 | 0.13±0.02 | 2 |
| 1920.76 | Too close | | 1 |
| 1921.77 | 289.2±1.7 | 0.10±0.01 | 2 |
| 1922.70 | 310.2±5.6 | 0.14±0.02 | 2 |

The pair is 46 *Tauri*. It is always hard to measure but there has never been any question as to the fact that it is double. The means of the angle measures are probably less accurate than the residuals indicate.

13010 **A 1014**

$4^h\ 38^m\ 48^s$; +57° 06′ +56°.964

8.5−8.6 8.0 F8

| | | | |
|---|---|---|---|
| 1905.82 | 296°.6±0°.2 | 0″.25±0″.00 | 3n |
| 1908.84 | 312.4 | 0.24 | 1 |
| 1915.71 | 336.2±0.6 | 0.34±0.01 | 2 |
| 1921.76 | 347.6±0.0 | 0.34±0.03 | 2 |

690 **A 2424**

$4^h\ 39^m\ 37^s$; +2° 54′ +2°.751

8.6−8.6 7.8 A0

| | | | |
|---|---|---|---|
| 1911.75 | 63°.2 | 0″.18 | 1n |
| 1912.85 | 42.6±0°.8 | 0.16±0″.02 | 2 |
| 1920.11 | 216.6 | 0.17 | 1 |
| 1920.78 | 209.9 | 0.15±0.01 | 3 |

702 **A 1544**

$4^h\ 43^m\ 55^s$; +43° 19′ +43°.1057

A and B, 9.2−9.2 8.7 F0

| | | | |
|---|---|---|---|
| 1907.82 | 255°.8±0°.1 | 0″.16±0″.02 | 2n |
| 1919.35 | 219.7±2.4 | 0.20±0.01 | 3 |
| 1921.83 | 212.0±0.7 | 0.22±0.03 | 2 |

AB and C, −10.4

| | | | |
|---|---|---|---|
| 1907.79 | 42°.4±0°.0 | 1″.33 | 2n |
| 1917.94 | 40.2 | 1.27 | 1 |
| 1921.83 | 41.0±0.4 | 1.28 | 2 |

713 **A 2621**

$4^h\ 46^m\ 57^s$; +2° 07′ +1°.823

8.4−8.4 7.6 A0

| | | | |
|---|---|---|---|
| 1913.72 | 339°.2±2°.9 | 0″.19±0″.01 | 3n |
| 1920.75 | 354.4±2.8 | 0.18±0.01 | 3 |

742 **A 1303**

$4^h\ 55^m\ 54^s$; +53° 23′ +53°.836

8.5−8.8 8.0 F5

| | | | |
|---|---|---|---|
| 1906.76 | 37°.2±5°.4 | 0″.17±0″.00 | 4n |
| 1918.75 | 10.0±2.8 | 0.28±0.01 | 2 |
| 1921.83 | 6.9±1.7 | 0.28±0.02 | 2 |

755 **A 1844**

$4^h\ 58^m\ 38^s$; +26° 36′ +26°.775

A and B, 7.2−9.7 6.86 F5

| | | | |
|---|---|---|---|
| 1908.75 | 354°.5±1°.9 | 0″.26±0″.01 | 2n |
| 1917.74 | 73.3±3.1 | 0.26±0.02 | 2 |
| 1921.86 | 125.5 | 0.22 | 1 |

AB and C=S461, −8.5

| | | | |
|---|---|---|---|
| 1917.75 | 159°.0 | 78″.56 | 1n |
| 1921.86 | 159.2 | 78.74 | 1 |

771 **A 2803**

$5^h\ 01^m\ 25^s$; −3° 34′ −3°.992

9.5−9.6 9.3 G0

| | | | |
|---|---|---|---|
| 1914.83 | 65°.7±1°.0 | 0″.30±0″.01 | 2n |
| 1921.69 | 73.4 | 0.23 | 1 |
| 1922.75 | 77.4±3.2 | 0.21±0.01 | 2 |

2508 **A 481**

$5^h\ 01^m\ 51^s$; −6° 06′ −6°.1075

7.2−8.0 6.72 B9

| | | | |
|---|---|---|---|
| 1903.83 | 357°.0±3°.0 | 0″.24±0″.01 | 4n |
| 1915.78 | 345.0±1.1 | 0.28±0.02 | 2 |
| 1924.06 | 338.5±1.7 | 0.35±0.04 | 3 |

—— **A 3010**

$5^h\ 04^m\ 27^s$; +18° 35′ +18°.779

5.7−5.7 5.04 G0

| | | | |
|---|---|---|---|
| 1912.90 | 5°.9 | 0″.10 | 1n |
| 1914.85 | 90.3±8.5 | 0.10±0.00 | 2 |
| 1915.74 | 239.3 | 0.11 | 1 |
| 1921.94 | 267.6 | 0.09 | 1 |

The parallax of 104 *Tauri* is +0″.057, the proper motion 0″.542 in 88°.8 (*Yale Cat.*). All of the measures of position angle are necessarily subject to considerable uncertainty, but the image on all nights was unmistakably that of a double, not of a single star, and the elongation in the directions given seemed real.

2561 **A 484**

$5^h\ 08^m\ 04^s$; −7° 39′ −7°.993

8.8−8.8 8.2 F8

| | | | |
|---|---|---|---|
| 1903.92 | 298°.1±1°.0 | 0″.19±0″.02 | 3n |
| 1915.79 | 311.8±1.8 | 0.20±0.01 | 2 |
| 1924.06 | Too close | | 1 |

13044 **A 1031**

$5^h\ 11m\ 30^s$; +47° 07′ +47°.1117

7.0−10.0 6.97 G5

| | | | |
|---|---|---|---|
| 1905.82 | 349°.2±0°.9 | 0″.45±0″.02 | 3n |
| 1916.66 | 353.6 | 0.39 | 1 |
| 1919.71 | 361.2±1.5 | 0.45±0.00 | 2 |

13048 **A 844**

$5^h\ 13^m\ 53^s$; −1° 42′ −1°.841

9.4−9.4 8.7−

| | | | |
|---|---|---|---|
| 1904.86 | 350°.7±1°.0 | 0″.22±0″.02 | 3n |
| 1917.77 | 1.7 | 0.25 | 1 |
| 1918.94 | 3.7 | 0.23 | 1 |

*827* **A 2639**

$5^h\ 15^m\ 26^s$; +3° 38′ +3°.841

9.2−9.7 8.8 F5

| | | | |
|---|---|---|---|
| 1913.79 | 7°.6±1°.0 | 0″.34±0″.01 | 2n |
| 1920.76 | 348.6±0.3 | 0.36±0.01 | 2 |

2654 **A 53**

$5^h\ 16^m\ 40^s$; −3° 08′ −3°.1061

8.5−12.5 8.6 K0

| | | | |
|---|---|---|---|
| 1900.20 | 45°.6±0°.5 | 4″.94±0″.09 | 2n |
| 1904.08 | 44.8 | 4.69 | 1 |
| 1915.81 | 43.7±0.1 | 4.20±0.03 | 2 |
| 1920.80 | 42.1±1.5 | 3.94±0.07 | 2 |
| 1924.05 | 40.9±1.8 | 3.82±0.06 | 2 |

The proper motion of the brighter star is given as 0″.72 in 78°.1. Assuming the measures in 1900 to be correct, the position of B in 1924 would have been about 14″.4 in 267°, if the two stars were independent and B had no sensible proper motion. The two stars are undoubtedly moving together, and the change shown by the measures is probably the result of orbital motion.

*851* **A 2641**

$5^h\ 20^m\ 00^s$; +2° 34′ +2°.934

8.5−11.3 8.3 G5

| | | | |
|---|---|---|---|
| 1913.74 | 160°.7±0°.1 | 1″.14±0″.02 | 2n |
| 1920.75 | 154.4±1.1 | 1.05±0.05 | 2 |
| 1924.04 | 144.6 | 0.90 | 1 |

*861* **A 2703**

$5^h\ 21^m\ 49^s$; +9° 08′ +9°.821

8.5−9.2 7.9 F5

| | | | |
|---|---|---|---|
| 1914.74 | 302°.6±0°.7 | 0″.20±0.01 | 3n |
| 1921.43 | 284.5±1.8 | 0.20±0.00 | 2 |

2749 **A 319**

$5^h\ 23^m\ 15^s$; −4° 02′ −4°.1135

9.7−10.0 9.9 G5

| | | | |
|---|---|---|---|
| 1902.80 | 52°.2±1°.3 | 0″.46±0″.01 | 3n |
| 1916.30 | 41.4±0.2 | 0.47±0.02 | 2 |
| 1924.05 | 36.6±2.8 | 0.45±0.04 | 2 |

The BD magnitude is 9.3, and my estimates of the magnitudes of the components are in accord with this value. The angular motion shown by the measures is rapid for so faint and wide a pair.

13060 **A 1034**

$5^h\ 26^m\ 31^s$; +70° 46′ +70°.355

8.3−8.8 8.0 G5

| | | | |
|---|---|---|---|
| 1905.77 | 271°.4±1°.4 | 0″.32±0″.02 | 2n |
| 1919.28 | 242.1±1.8 | 0.26±0.00 | 2 |
| 1921.83 | 237.4±3.1 | 0.28±0.01 | 2 |

*929* **A 1563**

$5^h\ 34^m\ 50^s$; +42° 59′ +42°.1354

8.7−8.7 8.5 G0

| | | | |
|---|---|---|---|
| 1907.77 | 272°.8±0°.3 | 0″.18±0″.00 | 2n |
| 1919.91 | 259.9±1.7 | 0.19±0.02 | 2 |
| 1924.74 | 255.4 | 0.16 | 1 |

2928 **A 494**

$5^h\ 40^m\ 28^s$; −6° 49′ −6°.1293

A and B, 6.5−7.0 5.98 F5

| | | | |
|---|---|---|---|
| 1903.80 | 121°.4±2°.9 | 0″.14±0″.01 | 4n |
| 1915.78 | 279.2±1.3 | 0.19±0.01 | 2 |
| 1916.84 | 293.8±1.4 | 0.15±0.00 | 2 |
| 1918.11 | 340.6 | 0.10 | 1 |
| 1920.95 | 88.1 | 0.16 | 1 |
| 1924.05 | 114.4±2.9 | 0.15±0.00 | 2 |

AB and C, −10.0

| | | | |
|---|---|---|---|
| 1903.98 | 231°.8 | 99″.00 | 1n |
| 1915.82 | 232.6 | 99.74 | 1 |

C and D, −13.5

| | | | |
|---|---|---|---|
| 1903.99 | 199.0±0°.1 | 1″.03±0″.04 | 2n |
| 1915.82 | 199.6 | 1.00 | 1 |

The lack of measures in the period from 1903 to 1915 prevents the computation of a satisfactory orbit at this time. Except in 1918, the quadrant was never in doubt, and it appears that the revolution period is approximately 21 years.

13076 **A 1040**

$5^h\ 41^m\ 28^s$; +31° 17′ +31°.1072

8.5−9.3 8.2 G0

| | | | |
|---|---|---|---|
| 1905.52 | 130°.7±2°.5 | 0″.69±0″.03 | 3n |
| 1916.88 | 121.4±0.9 | 0.76±0.01 | 2 |
| 1921.80 | 120.8±0.2 | 0.69±0.00 | 2 |

*976* **A 2711**

$5^h\ 43^m\ 14^s$; +10° 47′ +10°.876

9.0−9.0 8.3 F0

| | | | |
|---|---|---|---|
| 1914.74 | 93°.3±1°.1 | 0″.19±0″.01 | 3n |
| 1920.80 | 76.8±1.9 | 0.18±0.02 | 2 |

*989* **A 2657**

$5^h\ 45^m\ 39^s$; +1° 36′ +1°.1137

8.8−8.8 8.1 F0

| | | | |
|---|---|---|---|
| 1913.90 | 126°.0±3°.6 | 0″.16±0.03 | 2n |
| 1920.76 | 148.4 | 0.15 | 1 |
| 1921.72 | 158.5±2.8 | 0.17±0.00 | 2 |

*1023*

**A 1569**

$5^h\ 54^m\ 00^s$; +55° 54′ +55°.1033

8.5−9.2 8.2 F8

| | | | |
|---|---|---|---|
| 1907.39 | 105°.6±1°.4 | 0″.38±0″.03 | 2n |
| 1917.69 | 84.2 | 0.39 | 1 |
| 1921.85 | 83.0±2.7 | 0.38±0.02 | 2 |

*1039*

**A 2440**

$5^h\ 55^m\ 20^s$; +16° 06′ +16°.935

9.0−10.5 9.0−

| | | | |
|---|---|---|---|
| 1912.82 | 121°.2±2°.4 | 0″.27±0″.01 | 3n |
| 1918.98 | 110.3 | 0.36 | 1 |
| 1921.94 | 105.6±1.0 | 0.26±0.02 | 2 |

*1057*

**A 1315**

$5^h\ 58^m\ 48^s$; +58° 13′ +58°.887

9.5−9.6 9.0−

| | | | |
|---|---|---|---|
| 1906.90 | 350°.0±2°.5 | 0″.28±0″.02 | 2n |
| 1916.26 | 1.6±4.3 | 0.29±0.01 | 2 |
| 1919.77 | 9.8 | 0.26 | 1 |
| 1921.85 | 12.6±0.5 | 0.30±0.03 | 2 |

*1072*

**A 1729**

$6^h\ 00^m\ 52^s$; +45° 36′ +45°.1235

7.2−9.2 7.18 A2

| | | | |
|---|---|---|---|
| 1908.25 | 8°.6±1°.5 | 0″.28±0″.03 | 2n |
| 1920.00 | 30.6±0.6 | 0.32±0.03 | 2 |

**3217**

**A 56**

$6^h\ 10^m\ 06^s$; +29° 03′ +29°.1147

8.2−11.7 8.2 A5

| | | | |
|---|---|---|---|
| 1900.20 | 48°.0±0°.4 | 1″.03±0″.09 | 3n |
| 1904.17 | 49.5±0.8 | 1.01±0.03 | 2 |
| 1919.92 | 57.6±2.8 | 1.02±0.06 | 2 |

**3261**

**A 668**

$6^h\ 13^m\ 03^s$; −9° 01′ −8°.1368

6.6−6.6 6.03 B9

| | | | |
|---|---|---|---|
| 1904.05 | 338°.2±1°.0 | 0″.23±0″.01 | 4n |
| 1905.91 | 341.0±0.3 | 0.21±0.01 | 2 |
| 1916.18 | 351.7±0.8 | 0.27±0.02 | 3 |
| 1922.79 | 354.7 | 0.30 | 1 |

*1199*

**A 2667**

$6^h\ 18^m\ 50^s$; +2° 18′ +2°.1197

7.5−8.0 6.25 A5

| | | | |
|---|---|---|---|
| 1913.86 | 269°.8±2°.4 | 0″.37±0″.01 | 2n |
| 1920.75 | 285.8±0.7 | 0.39±0.02 | 2 |

*1265*

**A 2817**

$6^h\ 28^m\ 43^s$; +7° 52′ +7°.1327

9.3−9.3 8.5 G5

| | | | |
|---|---|---|---|
| 1914.83 | 94°.6±0°.7 | 0″.18±0″.01 | 2n |
| 1921.94 | 40.2±0.5 | 0.20±0.04 | 2 |

**3472**

**A 217**

$6^h\ 33^m\ 10^s$; +30° 09′ +30°.1275

9.1−9.1 8.5 F5

| | | | |
|---|---|---|---|
| 1901.83 | 44°.7±2.9 | 0″.17±0″.01 | 4n |
| 1904.08 | 50.4 | 0.14 | 1 |
| 1917.86 | 63.2 | 0.20 | 1 |
| 1919.92 | 68.8±0.2 | 0.15±0.01 | 2 |

**3554**

**A 218**

$6^h\ 38^m\ 35^s$; +30° 44′ +30°.1303

8.6−8.6 8.0 F5

| | | | |
|---|---|---|---|
| 1901.83 | 246°.8±0°.7 | 0″.17±0″.01 | 4n |
| 1904.08 | 244.0 | 0.20 | 1 |
| 1919.22 | 226.4±0.6 | 0.24±0.01 | 3 |

*1345*

**A 1735**

$6^h\ 41^m\ 31^s$; +54° 24′ +54°.1070

8.5−10.0 8.6 F2

| | | | |
|---|---|---|---|
| 1908.15 | 164°.9±3°.4 | 0″.37±0″.06 | 3n |
| 1917.69 | 155.0 | 0.52 | 1 |
| 1919.91 | 150.2±0.7 | 0.48±0.03 | 2 |

**3583**

**A 122**

$6^h\ 42^m\ 17^s$; +29° 25′ +29°.1332

8.8−9.3 8.6 G0

| | | | |
|---|---|---|---|
| 1900.93 | 30°.0±1°.7 | 0″.43±0″.06 | 2 |
| 1904.08 | 27.9 | 0.42 | 1 |
| 1917.99 | 22.4 | 0.43 | 1 |
| 1919.61 | 19.5±2.1 | 0.40±0.00 | 2 |

*1384*

**A 1955**

$6^h\ 45^m\ 51^s$; +3° 54′ +4°.1452

9.6−9.6 8.9 A5

| | | | |
|---|---|---|---|
| 1909.04 | 10°.9±0°.1 | 0″.25±0″.01 | 2n |
| 1922.30 | 355.3±0.9 | 0.21±0.00 | 2 |

——

**A 2935**

$6^h\ 46^m\ 12^s$; −12° 23′ −12°.1635

9.6−9.6 9.1 G5

| | | | |
|---|---|---|---|
| 1915.14 | 304°.6±4°.0 | 0″.27±0″.01 | 2n |
| 1921.82 | 230.2±6.4 | 0.21±0.00 | 2 |
| 1922.92 | 226.4±0.2 | 0.23±0.04 | 2 |

**3648**

**A 512**

$6^h\ 48^m\ 40^s$; +25° 07′ +25°.1465

9.4−9.6 8.9−

| | | | |
|---|---|---|---|
| 1903.89 | 142°.4±1°.5 | 0″.26±0″.01 | 2n |
| 1917.77 | 151.2±1.4 | 0.26±0.01 | 2 |
| 1919.23 | 160.1 | 0.27 | 1 |
| 1921.85 | 157.6 | 0.21 | 1 |

*1402*

**A 1956**

$6^h\ 48^m\ 52^s$; +4° 02′ +4°.1474

9.1−9.8 8.8 G5

| | | | |
|---|---|---|---|
| 1909.04 | 298°.6±0°.7 | 0″.18±0″.01 | 2n |
| 1922.30 | 268.0±1.5 | 0.39±0.00 | 2 |

**3662** **A 513**

$6^h\ 49^m\ 41^s$; +25° 02′ +25°.1478

9.0−9.1 8.91 G5

| | | | |
|---|---|---|---|
| 1903.89 | 345°.0±0°.2 | 0.″42±0.″03 | 2n |
| 1918.26 | 316.0±3.1 | 0.42±0.03 | 3 |
| 1921.85 | 307.6 | 0.42 | 1 |

*1439* **A 2681**

$6^h\ 54^m\ 52^s$; +2° 57′ +3°.1477

8.5−8.7 7.8 A5

| | | | |
|---|---|---|---|
| 1913.97 | 293°.5±0°.9 | 0.″23±0.″01 | 2n |
| 1921.28 | 279.8±3.8 | 0.21±0.01 | 3 |

*1446* **A 1737**

$6^h\ 55^m\ 50^s$; −0° 17′ −0°.1517

9.2−10.0 8.9 A2

| | | | |
|---|---|---|---|
| 1908.20 | 210°.2±0°.9 | 0.″31±0.″01 | 2n |
| 1919.64 | 190.5±2.8 | 0.29±0.03 | 3 |

**13117** **A 1061**

$6^h\ 56^m\ 17^s$; +25° 18′ +25°.1524

A and B, 8.7−8.9 8.2 G

| | | | |
|---|---|---|---|
| 1904.95 | 313°.8±0°.7 | 0.″27±0.″00 | 2n |
| 1906.22 | 319.1±0.9 | 0.32±0.01 | 2 |
| 1917.77 | 325.0±1.1 | 0.32±0.02 | 2 |
| | AB and C=Σ1000, −9.0 | | |
| 1904.94 | 66°.8 | 22.″29 | 1n |
| 1917.74 | 66.6 | 22.13 | 1 |

*1466* **A 2462**

$6^h\ 59^m\ 46^s$; +16° 03′ +16°.1365

9.5−9.5 8.7 F8

| | | | |
|---|---|---|---|
| 1912.87 | 175°.2±3°.9 | 0.″22±0.″03 | 2n |
| 1919.86 | 106.7±2.2 | 0.15±0.00 | 2 |
| 1921.85 | 98.7 | 0.15 | 1 |

**3762** **A 1324**

$6^h\ 59^m\ 59^s$; +56° 31′ +56°.1173

A and B, 9.2−9.2 8.6 G

| | | | |
|---|---|---|---|
| 1906.83 | 316°.4±3°.2 | 0.″25 | 3n |
| 1916.70 | 325.1 | 0.30 | 1 |
| 1919.77 | 327.3 | 0.30 | 1 |
| | AB and C=Σ1002, −9.5 | | |
| 1906.82 | 317°.2 | 30.″20 | 1n |
| 1916.70 | 317.6 | 30.22 | 1 |

*1467* **A 1959**

$7^h\ 00^m\ 08^s$; +39° 45′ +39°.1828

9.5−9.5 8.8−

| | | | |
|---|---|---|---|
| 1909.12 | 157°.2±2°.0 | 0.″27±0.″00 | 2n |
| 1922.75 | 143.4±1.1 | 0.25±0.02 | 2 |

**3815** **A 519**

$7^h\ 01^m\ 51^s$; −2° 58′ −2°.1908

9.0−9.1 8.6 G5

| | | | |
|---|---|---|---|
| 1903.04 | 273°.2±1°.5 | 0.″39±0.″03 | 3n |
| 1916.20 | Round | | 1 |
| 1917.06 | Too close | | 1 |
| 1917.81 | 125.4 | 0.13 | 1 |
| 1918.99 | 76.4 | 0.14 | 1 |
| 1922.84 | Too close | | 2 |
| 1923.20 | 1.1±11.4 | 0.10 | 3 |
| 1923.88 | 323.6 | 0.10±0.01 | 1 |
| 1927.06 | 295.0 | 0.14 | 1 |
| 1929.03 | 281.4 | 0.27 | 1 |

The period is evidently short, but lack of measures in the years from 1903 to 1917 prevents satisfactory orbit computation.

*1532* **A 2855**

$7^h\ 14^m\ 13^s$; +1° 04′ +1°.1757

9.1−9.1 8.2 K0

| | | | |
|---|---|---|---|
| 1914.45 | 313°.0±3°.8 | 0.″32±0.″02 | 2n |
| 1920.93 | 281.1±0.5 | 0.36±0.02 | 2 |

*1550* **A 1963**

$7^h\ 18^m\ 04^s$; −1° 30′ −1°.1663

8.7−8.7 7.9 B9

| | | | |
|---|---|---|---|
| 1909.28 | 266°.0±0°.8 | 0.″18±0.″02 | 2n |
| 1920.03 | 19.2±1.4 | 0.14±0.01 | 2 |
| 1923.88 | 292.8 | 0.25 | 1 |

*1628* **A 2875**

$7^h\ 35^m\ 53^s$; +13° 09′ +13°.1722

A and B, 9.6−9.6 9.0−

| | | | |
|---|---|---|---|
| 1914.50 | 316°.0±0°.4 | 0.″82±0.″09 | 2n |
| 1921.90 | 304.2±0.8 | 0.74±0.01 | 2 |
| | A and C, −11.0 | | |
| 1921.87 | 308°.7 | 79.″16 | 1n |
| | C and D, −11.5 | | |
| 1921.87 | 346°.8 | 1.″21 | 1n |

*1640* **A 1968**

$7^h\ 38^m\ 31^s$; −1° 18′ −1°.1798

9.5−9.6 9.0 G0

| | | | |
|---|---|---|---|
| 1909.25 | 69°.1±3°.1 | 0.″29±0.″01 | 3n |
| 1919.92 | 45.4±1.0 | 0.28±0.02 | 2 |

*1695* **A 1578**

$7^h\ 54^m\ 57^s$; −9° 41′ −9°.2298

9.0−9.7 8.7 F8

| | | | |
|---|---|---|---|
| 1907.21 | 321°.7±1°.3 | 0.″31±0.″01 | 2n |
| 1917.49 | 309.4±4.7 | 0.31±0.01 | 2 |
| 1920.18 | 306.9±7.0 | 0.33±0.05 | 2 |

*1713* **A 1581**

$8^h\ 00^m\ 40^s$; −6° 16′ −6°.2423

9.7−9.7 9.0−

| | | | |
|---|---|---|---|
| 1907.21 | 189°.7±1°.4 | 0″.28±0″.02 | 2n |
| 1918.14 | 222.6±2.7 | 0.29±0.01 | 2 |
| 1920.18 | 233.4±2.9 | 0.32±0.02 | 2 |

**13147** **A 1073**

$8^h\ 00^m\ 51^s$; +58° 34′ +58°.1090

8.8−9.1 8.6 F8

| | | | |
|---|---|---|---|
| 1905.90 | 128°.3±1°.1 | 0″.36 | 3n |
| 1916.29 | 117.2±0.8 | 0.36 | 2 |
| 1928.33 | 105.9±3.2 | 0.28 | 2 |

*1721* **A 1333**

$8^h\ 03^m\ 12^s$; +54° 16′ +54°.1200

8.6−8.6 8.0 A0

| | | | |
|---|---|---|---|
| 1906.83 | 248°.4±4°.0 | 0″.19±0″.01 | 3n |
| 1919.79 | 226.2±0.7 | 0.19±0.01 | 2 |
| 1928.35 | 225.2±1.4 | 0.24±0.01 | 2 |

*1729* **A 1971**

$8^h\ 03^m\ 30^s$; −0° 38′ −0°.1904

9.1−9.1 8.4 G5

| | | | |
|---|---|---|---|
| 1908.97 | 104°.6±1°.1 | 0″.35±0″.03 | 3n |
| 1919.92 | 71.4±0.9 | 0.51±0.02 | 2 |

*1737* **A 1335**

$8^h\ 06^m\ 16^s$; +55° 57′ +56°.1269

7.9−11.2 7.9 A2

| | | | |
|---|---|---|---|
| 1906.00 | 198°.4±1°.4 | 1″.43±0″.03 | 3n |
| 1916.55 | 205.9±0.2 | 1.53±0.15 | 2 |
| 1928.33 | 209.0±2.5 | 1.46±0.06 | 2 |

**4543** **A 337**

$8^h\ 14^m\ 47^s$; −5° 13′ −4°.2288

8.0−8.3 7.75 F2

| | | | |
|---|---|---|---|
| 1902.22 | 64°.0±2°.2 | 0″.27±0″.02 | 4n |
| 1917.12 | 209.4±1.5 | 0.34±0.02 | 3 |

*1763* **A 2889**

$8^h\ 15^m\ 16^s$; +2° 37′ +2°.1926

8.5−9.0 7.8 A0

| | | | |
|---|---|---|---|
| 1914.51 | 46°.0±2°.2 | 0″.24±0″.01 | 2n |
| 1921.00 | 31.2±3.3 | 0.22±0.02 | 2 |

*1768* **A 1745**

$8^h\ 17^m\ 35^s$; +47° 34′ +47°.1566

C and c, 9.5−10.0 9.3−

| | | | |
|---|---|---|---|
| 1907.86 | 29°.0±0°.1 | 0″.37±0″.01 | 2n |
| 1918.34 | 59.0 | 0.24 | 1 |
| 1920.15 | 68.0±1.5 | 0.22±0.00 | 2 |

This close pair is the star C of the wide triple OΣ190. Hussey found a faint companion to the star A of the same triple, but failed to see the close pair, probably because the distance, in 1898, was smaller. My measures of Hussey's pair (Hu224) and of OΣ190AC follow:

Hu224=OΣ190 AB, 8.2−12.0

| | | | |
|---|---|---|---|
| 1907.86 | 316°.7 | 4″.09 | 1n |
| 1920.12 | 316.2 | 4.21 | 1 |

OΣ190, AC

| | | | |
|---|---|---|---|
| 1907.86 | 168°.0 | 38″.63 | 1n |
| 1920.12 | 168.6 | 38.59 | 1 |

The star A is +47°.1567, for which the *HDC* has $8^m.2$, F8.

*1781* **A 1746**

$8^h\ 23^m\ 42^s$; +24° 42′ +25°.1920

B and C, 8.2−8.2 7.64 G

| | | | |
|---|---|---|---|
| 1908.29 | 140°.7±3°.0 | 0″.18±0″.01 | 3n |
| 1919.99 | 350.7±1.8 | 0.18±0.01 | 2 |
| 1924.35 | 66.4 | 0.18 | 1 |

A and BC=Σ1224, 7.4−

| | | | |
|---|---|---|---|
| 1908.30 | 43°.0±0°.2 | 5″.68±0″.06 | 2n |
| 1918.99 | 44.4±0.2 | 5.69±0.02 | 2 |

This triple is $\upsilon^1$ *Cancri*. The *Henry Draper Catalogue* gives for the star A, $7^m.18$, A3. Auwers assigns proper motions 0″.108 in 222°.0 and 0″.087 in 210°.2, respectively, to the components A and B, but it is practically certain that the three stars form a single physical system. The quadrant, in the close pair, is in doubt for all the measures.

**4627** **A 550**

$8^h\ 25^m\ 11^s$; −4° 15′ −3°.2356

7.3−7.3 6.68 F0

| | | | |
|---|---|---|---|
| 1903.04 | 189°.7±0°.3 | 0″.16±0″.00 | 2n |
| 1909.29 | 180.3 | 0.12 | 1 |
| 1911.21 | 167.7 | 0.20 | 1 |
| 1916.96 | Too close | | 1 |
| 1920.14 | 23.7±3.8 | 0.14±0.01 | 3 |
| 1923.33 | 0.4±2.4 | 0.15±0.01 | 3 |

**4634** **A 551**

$8^h\ 25^m\ 58^s$; −2° 21′ −2°.2581

A and B, 7.2−7.2 6.29 F0

| | | | |
|---|---|---|---|
| 1903.04 | 59°.7±2°.2 | 0″.24±0″.01 | 3n |
| 1904.08 | 61.5 | 0.27 | 1 |
| 1910.25 | 63.1±2.8 | 0.37±0.02 | 2 |
| 1917.05 | 67.8±1.0 | 0.37±0.01 | 3 |
| 1924.27 | 68.0 | 0.30 | 1 |

AB and C=Σ1233, −11.0

| | | | |
|---|---|---|---|
| 1903.03 | 328°.8±0°.1 | 18.16±0.05 | 2 |
| 1911.03 | 328.8 | 18.4: | 1 |

*1832* **A 2551**

$8^h\ 46^m\ 01^s$; +2° 48′ +3°.2059

9.3−10.0 9.1 F8

| | | | |
|---|---|---|---|
| 1913.11 | 46°.4±1°.5 | 0″.20±0″.03 | 2n |
| 1923.14 | 59.5±1.6 | 0.22±0.02 | 2 |

*1833* **A 2552**

$8^h\ 46^m\ 06^s$; +1° 08′ +1°.2174

8.4−8.4 7.7 F5

| | | | |
|---|---|---|---|
| 1913.11 | 79°.2±1°.5 | 0″.26±0″.02 | 2n |
| 1921.05 | 52.8±0.5 | 0.25±0.01 | 2 |
| 1923.13 | 46.5±0.4 | 0.24±0.01 | 2 |

*1837* **A 2473**

$8^h\ 47^m\ 51^s$; +18° 11′ +18°.2057

7.5−7.5 6.72 G5

| | | | |
|---|---|---|---|
| 1912.87 | 324°.8±1°.7 | 0″.27±0″.00 | 2n |
| 1920.01 | 335.2±0.5 | 0.32±0.04 | 2 |

*1840* **A 1584**

$8^h\ 49^m\ 24^s$; +55° 08′ +55°.1297

8.0−8.0 7.35 G0

| | | | |
|---|---|---|---|
| 1907.13 | 348°.0±1°.5 | 0″.28±0″.03 | 2n |
| 1917.90 | 31.9 | 0.24 | 1 |
| 1919.80 | 44.8 | 0.30 | 1 |
| 1921.33 | 53.4±1.6 | 0.30±0.01 | 2 |
| 1928.35 | 69.7±0.3 | 0.49±0.01 | 2 |

*1851* **A 2131**

$8^h\ 51^m\ 58^s$; +26° 24′ +26°.1865

A and B, 7.0−8.3 6.67 G0

| | | | |
|---|---|---|---|
| 1910.15 | 254°.6±1°.1 | 0″.32±0″.01 | 3n |
| 1918.79 | 299.2±0.9 | 0.31±0.01 | 3 |
| 1921.05 | 313.6±3.2 | 0.30±0.03 | 2 |
| 1924.35 | 341.6 | 0.31 | 1 |
| | AB and C=Ho357, −12.5 | | |
| 1910.13 | 4°.2±0°.3 | 38″.79±0″.01 | 2n |
| 1919.19 | 3.2±0.1 | 42.76±0.04 | 2 |

The proper motion of AB is 0″.44 in 173°.1, and this accounts for the relative motion of C.

*1869* **A 1585**

$9^h\ 00^m\ 13^s$; +47° 21′ +47°.1633

4.2−4.2 3.68 A0

| | | | |
|---|---|---|---|
| 1907.83 | 283°.2±0°.5 | 0″.21±0″.00 | 3n |
| 1908.34 | 286.3 | 0.23 | 1 |
| 1909.33 | 281.4±0.7 | 0.25±0.01 | 3 |
| 1910.17 | 283.4 | 0.20 | 1 |
| 1916.27 | 270.8 | 0.20 | 1 |
| 1918.11 | 264.9 | 0.16 | 1 |
| 1919.29 | 264.9 | 0.15 | 1 |
| 1928.32 | Too close | | 1 |

*κ Ursae Majoris* is the brightest double star in my catalogue. It is quite probable that the angular distance was at or near its maximum about the time of my discovery, otherwise it would have been discovered by Burnham in his repeated examinations of the naked eye stars. Merrill, with the interferometer attached to the 100-inch reflector at Mount Wilson, obtained the following measures, after he and Anderson had independently discovered the pair with the interferometer:

| | | | |
|---|---|---|---|
| 1921.17 | 252°.20±0°.17 | 0″.0836±0″.0004 | 2n |
| 1921.25 | 250.90±0.21 | 0.0801±0.0003 | 2 |

*1871* **A 2971**

$9^h\ 01^m\ 33^s$; +5° 59′ +6°.2095

9.5−9.5 8.8−

| | | | |
|---|---|---|---|
| 1914.82 | 341°.7±0°.3 | 0″.33±0″.00 | 2n |
| 1922.00 | 330.0±0.1 | 0.30±0.03 | 2 |

*1879* **A 2475**

$9^h\ 06^m\ 51^s$; +16° 18′ +16°.1913

9.5−9.5 9.0 A2

| | | | |
|---|---|---|---|
| 1912.22 | 242°.3±1°.2 | 0″.22±0″.03 | 2n |
| 1920.01 | 228.1±0.7 | 0.24±0.02 | 2 |

*1901* **A 1984**

$9^h\ 16^m\ 44^s$; +24° 28′ +24°.2070

9.1−9.6 9.21 A0

| | | | |
|---|---|---|---|
| 1909.30 | 318°.0±0°.4 | 0″.30 | 2n |
| 1921.23 | 286.6±0.6 | 0.28 | 2 |

*1910* **A 1342**

$9^h\ 20^m\ 25^s$; −9° 37′ −9°.2816

7.3−7.3 6.53 A2

| | | | |
|---|---|---|---|
| 1905.99 | 18°.0 | 0″.14 | 1n |
| 1906.34 | 33.8±1.0 | 0.16±0″.02 | 2 |
| 1916.31 | 213.8±10.9 | 0.14±0.01 | 2 |
| 1918.12 | 335.8±10.4 | 0.12±0.02 | 2 |
| 1922.98 | 4.8 | 0.14 | 1 |

The mean of the first three measures was published in *Lick Obs. Bull.* No. 109, but, in view of the very rapid motion shown in the later measures, it seems better to record the first measure separately. The angle measures in 1916 to 1922 are, of course, affected with a large error of measure because the angular separation is so small.

**5077** **A 222**

$9^h\ 23^m\ 07^s$; +28° 48′ +29°.1900

8.5−8.7 8.1 F8

| | | | |
|---|---|---|---|
| 1901.93 | 325°.8±2°.0 | 0″.23±0″.01 | 3n |
| 1904.31 | 332.0 | 0.28 | 1 |
| 1912.08 | 310.3 | 0.27 | 1 |
| 1918.16 | 294.1±0.3 | 0.22±0.01 | 2 |

*1927* **A 1764**

$9^h\ 27^m\ 23^s$; +46° 17′ +46°.1517

9.0–10.2 9.0 F8

| | | | |
|---|---|---|---|
| 1908.33 | 199°.6±1°.8 | 0″.32±0″.04 | 3n |
| 1920.12 | 193.0 | 0.35 | 1 |
| 1923.18 | 189.6±0.7 | 0.45±0.01 | 2 |

*1933* **A 2757**

$9^h\ 32^m\ 14^s$; +5° 28′ +5°.2198

9.0–10.0 8.7 K0

| | | | |
|---|---|---|---|
| 1914.35 | 176°.5±1°.9 | 0″.30±0″.01 | 2n |
| 1921.13 | 161.4±0.5 | 0.30±0.03 | 2 |

*1937* **A 1765**

$9^h\ 34^m\ 38^s$; +46° 08′ +46°.1528

8.4–8.4 7.9 F5

| | | | |
|---|---|---|---|
| 1908.27 | 303°.9±3°.8 | 0″.16±0″.00 | 3n |
| 1919.70 | 269.9±1.0 | 0.15±0.01 | 2 |

*1938* **A 2479**

$9^h\ 34^m\ 54^s$; +15° 41′ +16°.2000

9.0–9.8 8.7 F8

| | | | |
|---|---|---|---|
| 1912.28 | 257°.6±0°.4 | 0″.28 | 2n |
| 1919.58 | 265.1±0.0 | 0.30 | 2 |
| 1926.35 | 280.9 | 0.23 | 1 |

*1949* **A 2481**

$9^h\ 38^m\ 45^s$; +18° 07′ +18°.2247

9.0–9.6 8.7 F2

| | | | |
|---|---|---|---|
| 1912.29 | 49°.0±0°.1 | 0″.32±0″.01 | 2n |
| 1920.01 | 41.0±0.2 | 0.36±0.01 | 2 |
| 1926.35 | 32.3 | 0.30 | 1 |

*1976* **A 1346**

$9^h\ 55^m\ 47^s$; +53° 30′ +53°.1375

8.6–9.3 8.5 F8

| | | | |
|---|---|---|---|
| 1906.96 | 150°.7±3°.0 | 0″.42±0″.01 | 2n |
| 1917.90 | 145.8 | 0.37 | 1 |
| 1921.33 | 141.8±0.2 | 0.40±0.02 | 2 |
| 1928.35 | 136.5±1.4 | 0.31±0.02 | 2 |

*1980* **A 1347**

$10^h\ 00^m\ 01^s$; +57° 47′ +58°.1232

8.7–9.4 8.9 G5

| | | | |
|---|---|---|---|
| 1906.85 | 23°.0±0°.5 | 0″.39±0″.01 | 3n |
| 1916.28 | 16.4±1.9 | 0.36±0.02 | 2 |
| 1928.33 | 5.4±0.4 | 0.28±0.01 | 2 |

This star is rated 8.7 magnitude in the BD, but I found it brighter than the neighboring star +58°.1233, which is called 8.1 in the BD and 8.2 in *HDC*.

*1990* **A 2145**

$10^h\ 06^m\ 34^s$; +20° 35′ +21°.2156

7.2–7.2 6.65, F5, A2

| | | | |
|---|---|---|---|
| 1910.36 | 195°.3±1.9 | 0″.15±0″.01 | 3n |
| 1918.33 | Round | | 1 |
| 1919.98 | Too close | | 1 |
| 1921.18 | 89.2 | 0.14 | 1 |
| 1923.16 | 76.4±0.2 | 0.14±0.00 | 2 |
| 1924.35 | 78.1 | 0.16 | 1 |
| 1925.34 | 66.6±3.5 | 0.12±0.01 | 2 |

Though of equal brightness, the components are of different spectral class, the spectrum being noted as "composite" in the *Henry Draper Catalogue*. The measures from 1921 on were all made with difficulty, but the elongation in the direction noted was real.

*1995* **A 2146**

$10^h\ 09^m\ 20^s$; +21° 33′ +22°.2190

9.4–9.4 9.0 F8

| | | | |
|---|---|---|---|
| 1910.36 | 180°.1±5°.4 | 0″.19±0″.01 | 3n |
| 1918.33 | 213.1 | 0.22 | 1 |
| 1921.18 | 232.0 | 0.20 | 1 |
| 1923.16 | 237.8±0.1 | 0.22±0.02 | 2 |
| 1925.30 | 241.2±3.6 | 0.22±0.01 | 3 |

**5499** **A 556**

$10^h\ 34^m\ 31^s$; −8° 35′ −8°.2963

6.8–10.0 7.12 G0

| | | | |
|---|---|---|---|
| 1903.04 | 54°.0±1°.0 | 1″.34±0″.05 | 2n |
| 1916.21 | 70.0±0.6 | 1.70±0.05 | 3 |
| 1921.36 | 73.1 | 1.61 | 1 |
| 1925.34 | 76.8±2.1 | 1.68±0.02 | 2 |
| 1926.26 | 77.0±0.5 | 1.55±0.11 | 2 |

*2052* **A 2768**

$10^h\ 40^m\ 03^s$; +3° 51′ +4°.2375

7.0–8.5 6.63 F5

| | | | |
|---|---|---|---|
| 1914.36 | 270°.6±0°.3 | 0″.38±0″.05 | 2n |
| 1921.18 | 259.8±1.4 | 0.50±0.01 | 2 |
| 1926.33 | 248.3 | 0.52 | 1 |

*2066* **A 2373**

$10^h\ 49^m\ 24^s$; +16° 22′ +16°.2180

8.7–8.7 7.9 F5

| | | | |
|---|---|---|---|
| 1911.37 | 222°.8±1°.2 | 0″.19±0″.01 | 3n |
| 1919.96 | 216.7±3.2 | 0.22±0.01 | 3 |
| 1923.16 | 212.4±1.2 | 0.24±0.00 | 2 |
| 1926.35 | 205.3 | 0.24 | 1 |

*2082* **A 1590**

$11^h\ 00^m\ 33^s$; +54° 49′ +55°.1439

8.8–9.3 8.7 G0

| | | | |
|---|---|---|---|
| 1907.19 | 262°.3±1°.4 | 0″.42 | 3n |
| 1923.20 | 253.6±0.0 | 0.36 | 2 |
| 1928.35 | 221.2±0.1 | 0.52 | 2 |

5666 **A 676**

$11^h\ 02^m\ 10^s$; −3° 57′ −3°.3040

A and BC=Σ1506, 7.3−9.5 7.53 G5

| | | | |
|---|---|---|---|
| 1904.08 | 216°.1 | 11″.12 | 1n |
| 1917.24 | 217.2±0°.1 | 11.22±0″.08 | 2 |

B and C, 9.5−9.5

| | | | |
|---|---|---|---|
| 1904.18 | 236°.8±1°.3 | 0″.22±0″.02 | 2 |
| 1917.10 | 68.3 | 0.24 | 1 |
| 1918.14 | 70.8±0.1 | 0.25±0.01 | 2 |
| 1925.35 | 210.0 | 0.18 | 1 |
| 1926.33 | 220.0 | 0.25 | 1 |

The quadrant in the close pair is always in doubt. Later measures are needed to determine whether or not the arrangement here given is correct.

*2087* **A 1591**

$11^h\ 02^m\ 38^s$; +55° 05′ +55°.1440

9.0−9.0 8.5 G0

| | | | |
|---|---|---|---|
| 1907.26 | 298°.2±3°.3 | 0″.17±0″.01 | 3n |
| 1923.20 | 259.8±0.3 | 0.27±0.01 | 2 |
| 1928.35 | 259.2±1.0 | 0.26±0.02 | 2 |

*2098* **A 1353**

$11^h\ 10^m\ 41^s$; +55° 41′ +56°.1508

7.7−8.5 7.48 F2

| | | | |
|---|---|---|---|
| 1906.16 | 199°.6±2°.0 | 0″.49±0″.03 | 3n |
| 1916.26 | 194.6±0.5 | 0.40±0.01 | 2 |
| 1928.34 | 179.0±0.8 | 0.35±0.00 | 2 |

5776 **A 137**

$11^h\ 22^m\ 59^s$; −7° 35′ −7°.3233

8.7−9.0 8.5 F8

| | | | |
|---|---|---|---|
| 1901.27 | 60°.6±1°.1 | 0″.30±0″.02 | 3n |
| 1902.33 | 65.6 | 0.20 | 1 |
| 1916.21 | 70.4±1.7 | 0.34±0.03 | 2 |
| 1926.30 | 78.2±1.8 | 0.32±0.01 | 2 |

—— **A 2579**

$11^h\ 50^m\ 40^s$; −15° 24′ −14°.3427

8.5−8.7 8.06 F0

| | | | |
|---|---|---|---|
| 1911.27 | 182°.8±4°.8 | 0″.22±0″.02 | 2n |
| 1913.37 | 174.7±2.3 | 0.25±0.00 | 2 |
| 1920.30 | 157.0±2.6 | 0.26±0.03 | 2 |
| 1926.33 | 151.0 | 0.24 | 1 |

5957 **A 75**

$11^h\ 51^m\ 26^s$; +72° 12′ +72°.550

7.3−8.0 7.54 F5

| | | | |
|---|---|---|---|
| 1900.30 | 208°.5±3°.0 | 0″.28±0″.02 | 2n |
| 1902.21 | 205.3±0.8 | 0.24±0.02 | 4 |
| 1905.28 | 204.9±0.7 | 0.24±0.01 | 2 |
| 1907.79 | 197.3±0.5 | 0.21±0.02 | 3 |
| 1911.46 | 195.3±5.8 | 0.19±0.01 | 2 |
| 1916.31 | 160.0±10.0 | 0.15±0.02 | 2 |
| 1921.32 | 97.0±6.0 | 0.14±0.02 | 2 |
| 1923.49 | Too close | | 1 |
| 1924.38 | 60.0 | 0.15 | 1 |
| 1925.35 | 37.4 | 0.20 | 1 |

*2159* **A 2163**

$11^h\ 59^m\ 39^s$; +21° 24′ +21°.2383

9.7−9.7 9.0−

| | | | |
|---|---|---|---|
| 1910.38 | 313°.9±3°.4 | 0″.29±0″.01 | 3n |
| 1920.26 | 333.2 | 0.26 | 1 |
| 1921.27 | 337.6±0.2 | 0.26±0.03 | 2 |

6015 **A 681**

$12^h\ 01^m\ 11^s$; +25° 23′ +25°.2453

9.0−9.5 9.0 G5

| | | | |
|---|---|---|---|
| 1904.38 | 134°.2±2°.8 | 0″.41±0″.02 | 3n |
| 1918.31 | 126.8±0.2 | 0.36±0.03 | 2 |
| 1924.46 | 114.8±1.0 | 0.45±0.00 | 2 |

6016 **A 682**

$12^h\ 01^m\ 39^s$; +29° 24′ +24°.2422

8.0−9.2 8.41 F5

| | | | |
|---|---|---|---|
| 1904.37 | 335°.7±2°.4 | 0″.38±0″.01 | 2n |
| 1918.93 | 326.0±1.5 | 0.40±0.02 | 2 |
| 1924.46 | 321.2±3.0 | 0.40±0.01 | 2 |

*2176* **A 1849**

$12^h\ 11^m\ 18^s$; +46° 59′ +47°.1937

9.5−9.5 8.8−

| | | | |
|---|---|---|---|
| 1908.32 | 76°.6±3°.0 | 0″.25±0″.01 | 3n |
| 1919.48 | 70.6 | 0.22 | 1 |
| 1921.31 | 63.1±1.1 | 0.22±0.01 | 2 |

6170 **A 78**

$12^h\ 24^m\ 13^s$; −5° 19′ −4°.3281

8.0−8.5 7.50 F2

| | | | |
|---|---|---|---|
| 1900.37 | 83°.3±3°.9 | 0″.29±0″.02 | 4n |
| 1901.36 | 77.7±0.8 | 0.26±0.02 | 3 |
| 1903.41 | 83.4 | 0.30 | 1 |
| 1914.44 | 83.1 | 0.34 | 1 |
| 1918.21 | 89.0±0.1 | 0.35±0.04 | 2 |
| 1926.39 | 94.4±0.4 | 0.40±0.02 | 2 |

*2205* **A 1851**

$12^h\ 39^m\ 43^s$; +26° 39′ +27°.2163

9.4−9.8 9.6 K0

| | | | |
|---|---|---|---|
| 1908.38 | 177°.7±0°.9 | 0″.43±0″.04 | 2n |
| 1918.44 | 229.3±1.3 | 0.42±0.02 | 2 |
| 1921.34 | 243.8±0.1 | 0.42±0.02 | 2 |
| 1924.46 | 253.8±1.2 | 0.46±0.01 | 2 |

13235 **A 1092**

$12^h\ 53^m\ 32^s$; +70° 10′ +70°.720

9.5−9.7 9.0−

| | | | |
|---|---|---|---|
| 1905.26 | 156°.6±3°.6 | 0″.28±0″.01 | 3n |
| 1918.48 | 133.9 | 0.32 | 1 |
| 1919.43 | 128.2±0.1 | 0.33±0.04 | 2 |

*2230* **A 1786**

$13^h\ 06^m\ 42^s$; $+9°\ 04'$ $+9°.2722$

9.0−10.5 9.0−

| | | | |
|---|---|---|---|
| 1908.27 | 95°.0±0°.4 | 4″.15±0″.16 | 2n |
| 1916.32 | 95.4±0.1 | 4.95±0.01 | 2 |
| 1921.36 | 95.7±0.1 | 5.40±0.05 | 2 |
| 1924.42 | 95.4±0.5 | 5.72±0.17 | 2 |

This star is not given in any proper motion catalogue, but the measures indicate uniform rectilinear motion of one or the other component amounting to 0″.097 a year. It is probably simply a perspective pair.

*2231* **A 1359**

$13^h\ 08^m\ 51^s$; $+31°\ 06'$ $+31°.2462$

A and B, 9.0−10.5 9.2 G5

| | | | |
|---|---|---|---|
| 1906.50 | 120°.8±1°.7 | 0″.29±0″.02 | 3n |
| 1918.43 | 108.0±0.8 | 0.30±0.02 | 2 |
| | AB and C, =Σ1729, −11.0 | | |
| 1906.47 | 274°.8 | 8″.46 | 1n |

*2249* **A 2489**

$13^h\ 21^m\ 10^s$; $-0°\ 28'$ $+0°.3050$

9.3−9.3 8.78 G5

| | | | |
|---|---|---|---|
| 1912.35 | 190°.6±3°.7 | 0″.33±0″.02 | 3n |
| 1918.94 | 183.4±1.4 | 0.28±0.01 | 2 |
| 1926.41 | 173.4±4.1 | 0.22±0.01 | 3 |

*2254* **A 1609**

$13^h\ 23^m\ 38^s$; $+44°\ 45'$ $+45°.2108$

A and B, 8.5−8.5 8.17 K0

| | | | |
|---|---|---|---|
| 1907.54 | 355°.3±1°.1 | 0″.40±0″.02 | 3n |
| 1918.57 | 25.1 | 0.41 | 1 |
| 1919.44 | 29.9±0.9 | 0.45±0.04 | 2 |
| 1921.31 | 35.4±1.7 | 0.44±0.04 | 2 |
| 1924.46 | 43.4±0.1 | 0.48±0.03 | 2 |
| | AB and C −13.0 | | |
| 1907.53 | 178°.6±0°.2 | 2″.14±0″.08 | 2n |
| 1919.48 | 184.6 | 2.42 | 1 |
| 1921.34 | 186.1 | 2.25 | 1 |
| 1924.46 | 187.8 | 2.42 | 1 |

*2261* **A 1789**

$13^h\ 27^m\ 59^s$; $+7°\ 45'$ $+8°.2721$

9.2−9.2 8.5 A5

| | | | |
|---|---|---|---|
| 1908.28 | 123°.2±0°.6 | 0″.26±0″.01 | 2n |
| 1914.47 | 119.0±4.1 | 0.20±0.00 | 2 |
| 1916.32 | 106.4±1.1 | 0.26±0.01 | 2 |
| 1921.41 | 92.6±2.1 | 0.24±0.03 | 2 |
| 1926.42 | 88.2±4.8 | 0.19±0.00 | 2 |

**13248** **A 1095**

$13^h\ 31^m\ 16^s$; $+29°\ 59'$ $+30°.2420$

8.6−9.2 8.4 G0

| | | | |
|---|---|---|---|
| 1905.54 | 176°.5±0°.3 | 0″.31±0″.01 | 3n |
| 1918.43 | 195.2±0.5 | 0.35±0.00 | 2 |
| 1928.50 | 206.2±0.6 | 0.34±0.02 | 2 |

*2266* **A 1792**

$13^h\ 31^m\ 30^s$; $+9°\ 02'$ $+9°.2777$

8.7−9.2 8.3 F2

| | | | |
|---|---|---|---|
| 1908.25 | 346°.0±3°.1 | 0″.17±0″.01 | 3n |
| 1914.45 | 340.0 | 0.19 | 1 |
| 1918.46 | 336.0 | 0.24 | 1 |
| 1926.41 | 326.0±1.6 | 0.32±0.01 | 2 |

*2284* **A 1614**

$13^h\ 55^m\ 45^s$; $+52°\ 14'$ $+52°.1757$

8.9−9.0 8.7 G5

| | | | |
|---|---|---|---|
| 1907.46 | 1°.7±1°.0 | 0″.29±0″.01 | 3n |
| 1919.42 | 227.2±3.4 | 0.36±0.02 | 2 |
| 1921.26 | 213.0±0.2 | 0.35±0.00 | 2 |
| 1923.52 | 204.8 | 0.38 | 1 |
| 1924.52 | 198.1±1.1 | 0.43±0.03 | 2 |
| 1928.55 | 186.0±0.9 | 0.48±0.05 | 2 |

*2283* **A 2167**

$13^h\ 55^m\ 48^s$; $+2°\ 28'$ $+2°.2752$

9.0−9.0 8.5 G0

| | | | |
|---|---|---|---|
| 1910.42 | 80°.1±2°.5 | 0″.21±0″.01 | 3n |
| 1918.49 | 45.4 | 0.16 | 1 |
| 1921.27 | 30.4±0.6 | 0.18±0.02 | 2 |
| 1926.38 | Too close | | 1 |

**13257** **A 1097**

$14^h\ 00^m\ 20^s$; $+57°\ 28'$ $+57°.1478$

8.0−8.3 7.8 F5

| | | | |
|---|---|---|---|
| 1905.57 | 71°.5±0°.9 | 0″.28±0″.02 | 2n |
| 1916.27 | 85.2 | 0.35 | 1 |
| 1919.48 | 91.0 | 0.34 | 1 |
| 1921.26 | 96.5±1.3 | 0.32±0.04 | 2 |
| 1924.52 | 99.8±0.7 | 0.36±0.01 | 2 |

**6728** **A 346**

$14^h\ 05^m\ 33^s$; $+24°\ 58'$ $+25°.2732$

9.0−10.0 9.0 G

| | | | |
|---|---|---|---|
| 1902.56 | 337°.4±1°.6 | 0″.62±0″.02 | 2n |
| 1911.44 | 331.4±0.2 | 0.62±0.02 | 2 |
| 1919.41 | 330.1±0.5 | 0.61±0.06 | 2 |
| 1924.46 | 328.8±1.8 | 0.72±0.05 | 2 |
| 1926.31 | 325.2 | 0.62 | 1 |

*2301* **A 1797**

$14^h\ 09^m\ 51^s$; $+4°\ 06'$ $+4°.2837$

9.4−9.7 8.9 G0

| | | | |
|---|---|---|---|
| 1908.28 | 154°.1±0°.4 | 0″.30±0″.02 | 2n |
| 1918.45 | 159.7±2.2 | 0.32±0.02 | 2 |
| 1926.49 | 164.8±0.7 | 0.25±0.01 | 2 |

**13262** **A 1099**

$14^h\ 10^m\ 24^s$; $+11°\ 18'$ $+11°.2650$

8.8−9.1 8.4 A3

| | | | |
|---|---|---|---|
| 1905.37 | 99°.4±2°.4 | 0″.18±0″.01 | 3n |
| 1915.38 | 92.6±1.2 | 0.20±0.01 | 2 |
| 1921.41 | 88.6±1.2 | 0.20±0.01 | 2 |
| 1926.36 | 88.2±1.8 | 0.20±0.01 | 2 |

**13264** **A 1101**

$14^h\ 13^m\ 23^s$; $+10°\ 32'$ $+10°.2657$

A and B, 9.2–9.7 8.7 K0

| | | | |
|---|---|---|---|
| 1905.37 | 240°.9±2°.2 | 0″.22±0″.02 | 3n |
| 1914.48 | 220.5±1.4 | 0.28±0.01 | 2 |
| 1916.31 | 219.2 | 0.31 | 1 |
| 1918.42 | 217.0 | 0.33 | 1 |
| 1921.41 | 203.2±2.0 | 0.28±0.01 | 2 |
| 1924.53 | 197.1 | 0.32 | 1 |
| 1926.36 | 192.4±0.1 | 0.27±0.04 | 2 |
| | AB and C=Σ1823, −9.7 | | |
| 1905.34 | 149°.6±0°.2 | 3″.49±0″.06 | 2n |
| 1921.40 | 149.8 | 3.56 | 1 |

**13267** **A 1102**

$14^h\ 16^m\ 59^s$; $+69°\ 28'$ $+69°.743$

8.2–8.4 8.19 F2

| | | | |
|---|---|---|---|
| 1905.37 | 293°.0±0°.7 | 0″.26±0″.02 | 3n |
| 1915.37 | 272.1±6.0 | 0.25±0.01 | 2 |
| 1921.26 | 257.3±1.5 | 0.26±0.01 | 2 |

**6845** **A 148**

$14^h\ 20^m\ 15^s$; $+51°\ 20'$ $+51°.1913$

8.8–8.8 8.4 F5

| | | | |
|---|---|---|---|
| 1901.32 | 347°.8±1°.3 | 0″.30±0″.01 | 3n |
| 1903.60 | 350.4±0.2 | 0.33±0.02 | 2 |
| 1916.43 | 354.8±0.7 | 0.46±0.05 | 2 |
| 1924.51 | 355.4±1.1 | 0.51±0.01 | 2 |

*2317* **A 2069**

$14^h\ 24^m\ 22^s$; $+16°\ 38'$ $+17°.2737$

8.3–8.5 7.7 F8

| | | | |
|---|---|---|---|
| 1909.42 | 229°.6±0°.2 | 0″.24±0″.01 | 3n |
| 1920.32 | 191.3±2.1 | 0.22±0.01 | 3 |
| 1923.56 | 175.5 | 0.16 | 1 |
| 1927.54 | 129.8 | 0.16 | 1 |

**6913** **A 570**

$14^h\ 30^m\ 08^s$; $+26°\ 54'$ $+27°.2388$

6.4–6.6 5.90 A2

| | | | |
|---|---|---|---|
| 1903.40 | 198°.6±0°.8 | 0″.20±0″.01 | 4n |
| 1906.41 | 173.0±1.0 | 0.24±0.01 | 3 |
| 1907.52 | 166.8±0.5 | 0.23±0.01 | 2 |
| 1911.45 | 128.3±2.0 | 0.22±0.01 | 3 |
| 1914.58 | 88.5±2.9 | 0.16±0.01 | 2 |
| 1915.59 | 79.0±2.1 | 0.18±0.00 | 2 |
| 1916.49 | 70.0 | 0.18 | 1 |
| 1918.27 | 54.2±5.9 | 0.18±0.02 | 2 |
| 1919.41 | 38.6±1.4 | 0.18±0.01 | 2 |
| 1920.37 | 23.1±1.2 | 0.19±0.01 | 3 |
| 1921.39 | 10.2±1.9 | 0.22±0.01 | 3 |
| 1922.61 | 356.7±0.7 | 0.17±0.02 | 3 |
| 1923.49 | 340.0±0.7 | 0.17±0.02 | 4 |
| 1924.46 | 325.8±0.8 | 0.19±0.01 | 2 |
| 1926.33 | 285.5±2.5 | 0.14±0.01 | 3 |
| 1927.38 | 272.0±2.1 | 0.16±0.01 | 3 |
| 1928.50 | 252.3±2.0 | 0.21±0.03 | 3 |

Dr. Jessica M. Young computed the orbit of this pair, under my direction, using the observations to 1923.49 inclusive. The elements are: $P=28.45$ years, $T=1924.88$, $e=0.171$, $a=0″.202$, $\omega=219°.9$, $i=35°.8$, $\Omega=170°.9$.

The representation of the last four positions (O−C) is:

| | | |
|---|---|---|
| 1924.46 | +2°.9 | +0″.03 |
| 1926.33 | −3.6 | ±0.00 |
| 1927.38 | +3.9 | +0.02 |
| 1928.50 | +6.1 | +0.06 |

The proper motion of the system is 0″.088 in 236°.9 (Boss). From the measures Jackson and Furner derived the dynamic parallax 0″.027 (1919) and Russell and Miss Moore, 0″.020 (1928).

**6944** **A 347**

$14^h\ 35^m\ 10^s$; $+48°\ 26'$ $+48°.2222$

8.0–8.2 7.8 F2

| | | | |
|---|---|---|---|
| 1902.66 | 72°.9±0°.2 | 0″.34±0″.02 | 2n |
| 1906.45 | 65.5±1.3 | 0.37±0.02 | 2 |
| 1911.51 | 55.0 | 0.38 | 1 |
| 1914.58 | 52.2±0.3 | 0.38±0.02 | 2 |
| 1916.43 | 44.7±0.8 | 0.36±0.02 | 2 |
| 1921.31 | 36.2±0.6 | 0.40±0.03 | 2 |
| 1923.49 | 28.2±1.4 | 0.42±0.01 | 2 |

**13279** **A 1109**

$14^h\ 40^m\ 20^s$; $+6°\ 48'$ $+7°.2830$

7.5–10.0 7.5 F8

| | | | |
|---|---|---|---|
| 1905.48 | 32°.4±0°.3 | 0″.63±0″.05 | 3n |
| 1916.40 | 45.9 | 0.77 | 1 |
| 1918.43 | 50.8±2.9 | 0.73±0.08 | 2 |
| 1921.36 | 52.0±1.1 | 0.79±0.03 | 2 |
| 1924.93 | 56.0±1.4 | 0.89±0.08 | 2 |
| 1926.49 | 53.8±0.7 | 1.05±0.01 | 2 |

—— **A 2983**

$14^h\ 46^m\ 49^s$; $+10°\ 26'$ $+10°.2747$

9.2–9.2 8.5 G5

| | | | |
|---|---|---|---|
| 1916.40 | 7°.4±4°.3 | 0″.19±0″.01 | 3n |
| 1920.47 | 79.6±1.8 | 0.16±0.02 | 2 |
| 1923.45 | 128.3±0.3 | 0.16±0.01 | 3 |
| 1924.54 | 143.4±2.2 | 0.20±0.01 | 2 |
| 1926.36 | 167.8±2.3 | 0.16±0.02 | 2 |
| 1927.46 | Too close | | 2 |

**13282** **A 1110**

$14^h\ 47^m\ 15^s$; $+8°\ 11'$ $+8°.2925$

A and B, 7.5–7.7 6.95 F5

| | | | |
|---|---|---|---|
| 1905.32 | 274°.5±2°.3 | 0″.22±0″.01 | 3n |
| 1914.34 | 266.8±1.5 | 0.27±0.01 | 3 |
| 1923.40 | 260.7±0.5 | 0.34±0.02 | 2 |
| 1926.36 | 262.9±0.1 | 0.39±0.05 | 2 |
| | AB and C, −12.0 | | |
| 1905.29 | 202°.6 | 19″.84 | 1n |
| 1916.31 | 203.4 | 19.51 | 1 |
| | AB and D, −12.5 | | |
| 1905.29 | 327°.1 | 22″.67 | 1n |
| 1916.31 | 327.2 | 23.26 | 1 |

2336 **A 2171**

$14^h\ 52^m\ 51^s$; +0° 22′ +0″.3273

9.5–9.6 9.0 F8

| | | | |
|---|---|---|---|
| 1910.43 | 120°.9±1°.4 | 0″.29±0″.01 | 3n |
| 1918.52 | 111.8±0.8 | 0.27±0.01 | 2 |
| 1923.42 | 108.0±0.8 | 0.27±0.00 | 2 |
| 1926.52 | 105.0±1.3 | 0.30±0.01 | 2 |

2340 **A 1627**

$14^h\ 53^m\ 51^s$; +39° 51′ +40°.2829

8.5–8.5 7.92 F0

| | | | |
|---|---|---|---|
| 1907.55 | 182°.5±1°.2 | 0″.24±0″.01 | 3n |
| 1918.54 | 202.8±0.8 | 0.24±0.03 | 2 |
| 1923.49 | 209.9±0.9 | 0.24±0.01 | 2 |
| 1928.53 | 227.8±0.2 | 0.20±0.01 | 2 |

2339 **A 2172**

$14^h\ 54^m\ 00^s$; +3° 07′ +3°.2957

8.4–9.0 8.1 K0

| | | | |
|---|---|---|---|
| 1910.45 | 339°.1±1°.9 | 0″.19±0″.01 | 3n |
| 1918.52 | 326.3±1.9 | 0.19±0.00 | 2 |
| 1923.43 | 317.3±2.4 | 0.23±0.01 | 3 |
| 1926.52 | 318.3±1.6 | 0.23±0.03 | 2 |

2343 **A 2072**

$14^h\ 56^m\ 40^s$; +17° 44′ +18°.2965

9.7–9.7 9.1–

| | | | |
|---|---|---|---|
| 1909.42 | 304°.3±0°.9 | 0″.78±0″.05 | 3n |
| 1919.83 | 297.5±1.4 | 0.78±0.02 | 2 |
| 1923.49 | 294.8±0.0 | 0.70±0.03 | 2 |
| 1927.38 | 290.6±0.3 | 0.67±0.00 | 2 |

7090 **A 14**

$14^h\ 58^m\ 33^s$; −3° 54′ −3°.3707

9.0–12.0 9.2 K2

| | | | |
|---|---|---|---|
| 1899.43 | 16°.9±0°.3 | 3″.76±0″.08 | 3n |
| 1915.42 | 25.2±1.8 | 3.07±0.01 | 3 |
| 1922.60 | 28.4±0.8 | 2.76±0.04 | 2 |
| 1925.46 | 30.9±0.2 | 2.96±0.04 | 2 |

7133 **A 81**

$15^h\ 06^m\ 16^s$; −6° 23′ −6°.4141

9.0–9.5 8.6 F2

| | | | |
|---|---|---|---|
| 1900.53 | 66°.0±3°.3 | 0″.54±0″.05 | 4n |
| 1901.50 | 66.4±4.3 | 0.39±0.04 | 2 |
| 1902.33 | 65.8 | 0.39 | 1 |
| 1915.50 | 60.8±0.1 | 0.50±0.01 | 2 |
| 1923.43 | 56.3±0.3 | 0.52±0.03 | 2 |

13308 **A 1120**

$15^h\ 24^m\ 53^s$; +9° 53′ +10°.2854

8.3–9.0 7.87 G0

| | | | |
|---|---|---|---|
| 1905.47 | 329°.7±0°.8 | 0″.28±0″.01 | 3n |
| 1917.06 | 337.8±2.0 | 0.27±0.02 | 3 |
| 1923.46 | 344.8±3.6 | 0.18±0.02 | 3 |
| 1924.54 | 346.7±1.0 | 0.20±0.02 | 2 |
| 1926.40 | 350.4±3.9 | 0.14±0.01 | 2 |

2374 **A 2175**

$15^h\ 25^m\ 42^s$; +3° 02′ +3°.3034

8.8–8.8 8.1 A5

| | | | |
|---|---|---|---|
| 1910.47 | 349°.1±4°.8 | 0″.17±0″.00 | 4n |
| 1918.50 | 3.0±0.4 | 0.18±0.03 | 2 |
| 1921.36 | 14.8±1.4 | 0.18±0.00 | 2 |
| 1923.43 | 6.2±0.2 | 0.16±0.01 | 2 |
| 1925.58 | 25.2±1.5 | 0.15±0.01 | 2 |

In 1925 the seeing was very good and the components were clearly separated; in 1923, conditions were less favorable and measures were difficult.

2378 **A 1633**

$15^h\ 27^m\ 49^s$; +47° 01′ +47°.2229

9.2–10.2 9.4 F8

| | | | |
|---|---|---|---|
| 1907.55 | 139°.9±1°.8 | 0″.28±0″.00 | 2n |
| 1918.54 | 127.2±3.5 | 0.34±0.04 | 2 |
| 1924.52 | 121.8 | 0.37 | 1 |
| 1928.39 | 120.5 | 0.34 | 1 |

2381 **A 1634**

$15^h\ 29^m\ 59^s$; +41° 04′ +41°.2611

5.5–5.5 4.98 A2

| | | | |
|---|---|---|---|
| 1907.55 | 236°.9±6°.9 | 0″.09±0.01 | 2n |
| 1910.55 | 225.5 | 0.11 | 1 |
| 1911.47 | 212: | <0.10 | 1 |
| 1914.59 | 215: | <0.10 | 1 |
| 1920.38 | Too close | | 1 |
| 1921.61 | 140: | <0.10 | 1 |
| 1928.53 | 33.6±12.4 | 0.10±0.01 | 2 |

$\nu^2$ Bootis is certainly double but the measures here recorded are all quite uncertain. The angles, however, represent the general directions of elongation.

2383 **A 2076**

$15^h\ 38^m\ 13^s$; +18° 50′ +19°.3000

8.2–8.2 7.62 A2

| | | | |
|---|---|---|---|
| 1909.38 | 146°.1±2°.0 | 0″.24±0″.02 | 3n |
| 1919.02 | 156.4±1.1 | 0.34±0.01 | 2 |
| 1923.44 | 159.6±1.7 | 0.34±0.04 | 2 |
| 1927.46 | 163.3±2.8 | 0.36±0.03 | 2 |

2385 **A 2176**

$15^h\ 39^m\ 28^s$; +0° 37′ +0°.3389

8.2–8.2 7.39 A0

| | | | |
|---|---|---|---|
| 1910.43 | 205°.7±1°.0 | 0″.22±0″.02 | 3n |
| 1918.50 | 215.6±1.4 | 0.19±0.01 | 2 |
| 1923.41 | 231.2±0.8 | 0.19±0.01 | 2 |
| 1925.58 | 240.7±3.1 | 0.17±0.02 | 3 |

2386 **A 2230**

$15^h\ 41^m\ 31^s$; +2° 41′ +2°.2989

6.1–13.0 5.80 G5

| | | | |
|---|---|---|---|
| 1910.51 | 102°.6±1°.0 | 3″.58±0″.09 | 3n |
| 1918.50 | 95.2±0.9 | 3.69±0.08 | 2 |
| 1923.41 | 89.5±0.5 | 3.68±0.02 | 2 |
| 1925.57 | 90.0±1.1 | 3.49±0.08 | 2 |

The parallax of $\psi$ *Serpentis* is $+0''.049$, the proper motion $0''.170$ in $287°.9$ (*Yale Cat.*). The motion shown is probably real and orbital, though the pair is always a difficult one to measure.

**7405** **A 21**

$15^h\ 49^m\ 01^s$; $-5°\ 50'$ $-5°.4182$

8.5–10.5 8.7 K0

| | | | |
|---|---|---|---|
| 1899.57 | 181°.2±2°.0 | 0″.54±0″.05 | 3n |
| 1915.42 | 186.2±0.8 | 0.49±0.01 | 2 |
| 1925.51 | 191.6±1.6 | 0.44±0.01 | 2 |

*2398* **A 2080**

$15^h\ 51^m\ 54^s$; $+17°\ 08'$ $+17°.2929$

8.5–8.5 7.7 F2

| | | | |
|---|---|---|---|
| 1909.43 | 319°.0±3°.0 | 0″.24±0″.02 | 4n |
| 1918.86 | 334.9±1.6 | 0.21±0.01 | 3 |
| 1921.35 | 335.4±1.3 | 0.21±0.01 | 3 |
| 1923.51 | 341.4±1.7 | 0.17±0.00 | 2 |
| 1927.37 | 343.5 | 0.17 | 1 |

*2400* **A 1639**

$15^h\ 57^m\ 54^s$; $+11°\ 49'$ $+12°.2930$

A and B, 9.0–11.0 8.3 G0

| | | | |
|---|---|---|---|
| 1907.63 | 337°.4±0°.6 | 0″.26±0″.01 | 2n |
| 1917.08 | 345.1±1.5 | 0.34±0.02 | 2 |
| 1927.04 | 356.0±1.0 | 0.30±0.01 | 2 |
| | A B and C=Σ1992, −9.0 | | |
| 1907.62 | 325°.2 | 5″.81 | 1n |
| 1916.58 | 326.0 | 5.86 | 1 |

*2410* **A 1798**

$16^h\ 05^m\ 37^s$; $+14°\ 33'$ $+14°.2999$

8.4–8.9 7.9 F0

| | | | |
|---|---|---|---|
| 1908.30 | 111°.4±2°.2 | 0″.27±0″.01 | 3n |
| 1916.58 | 104.5 | 0.23 | 1 |
| 1917.64 | 97.4±6.0 | 0.24±0.01 | 2 |
| 1926.56 | 93.9 | 0.19 | 1 |
| 1927.53 | 87.8 | 0.24 | 1 |

*2411* **A 1799**

$16^h\ 09^m\ 11^s$; $+15°\ 15'$ $+15°.2964$

9.1–9.2 8.5 G5

| | | | |
|---|---|---|---|
| 1908.30 | 170°.9±0°.8 | 0″.31±0″.02 | 3n |
| 1917.08 | 166.4±1.3 | 0.37±0.03 | 2 |
| 1927.04 | 157.4±0.1 | 0.42±0.02 | 2 |

*2419* **A 2181**

$16^h\ 14^m\ 19^s$; $+1°\ 18'$ $+1°.3191$

10.0–10.0 9.5 G0

| | | | |
|---|---|---|---|
| 1910.44 | 299°.6±1°.2 | 0″.43±0″.00 | 2n |
| 1918.59 | 308.6±0.6 | 0.38±0.04 | 2 |
| 1923.43 | 316.8±0.5 | 0.43±0.05 | 2 |
| 1925.63 | 334.3 | 0.45 | 1 |
| 1926.51 | 323.4 | 0.41 | 1 |

The measure in 1925 was made under poor observing conditions and deserves very little weight.

*2428* **A 1859**

$16^h\ 26^m\ 27^s$; $+12°\ 10'$ $+12°.3016$

9.0–9.0 8.3–

| | | | |
|---|---|---|---|
| 1908.40 | 51°.6±3°.4 | 0″.25±0″.02 | 3n |
| 1916.58 | 48.0 | 0.20 | 1 |
| 1917.58 | 56.4 | 0.22 | 1 |
| 1926.56 | 70.4 | 0.17 | 1 |
| 1927.53 | 80.1 | 0.15 | 1 |

The measure in 1927 was made under good conditions and has full weight; the seeing was not so good for the 1926 measure.

**13349** **A 693**

$16^h\ 29^m\ 09^s$; $-2°\ 09'$ $-1°.3206$

9.2–9.2 8.52 F5

| | | | |
|---|---|---|---|
| 1904.52 | 188°.3±1°.3 | 0″.19±0″.01 | 3n |
| 1915.86 | 138.9±8.6 | 0.17±0.02 | 6 |
| 1919.98 | 102.5±13.6 | 0.18±0.01 | 4 |
| 1923.02 | 90.9±1.6 | 0.18±0.02 | 4 |
| 1925.55 | 87.8±2.0 | 0.17±0.01 | 2 |

**7716** **A 349**

$16^h\ 39^m\ 21^s$; $+30°\ 15'$ $+30°.2860$

9.5–10.0 9.4–

| | | | |
|---|---|---|---|
| 1902.68 | 111°.8±2°.6 | 0″.56±0″.02 | 3n |
| 1910.36 | 97.3 | 0.55 | 1 |
| 1916.37 | 93.7 | 0.55 | 1 |
| 1917.67 | 89.9±1.5 | 0.50±0.03 | 2 |
| 1920.60 | 84.6±0.5 | 0.48±0.03 | 2 |
| 1923.59 | 78.2±1.0 | 0.43±0.03 | 2 |

**13353** **A 1141**

$16^h\ 43^m\ 11^s$; $-0°\ 39'$ $-0°.3178$

8.7–8.7 7.9 F8

| | | | |
|---|---|---|---|
| 1905.45 | 18°.8±3°.2 | 0″.18±0″.01 | 3n |
| 1915.49 | 4.0±0.7 | 0.18±0.02 | 3 |
| 1921.48 | 352.3±3.5 | 0.16±0.01 | 3 |
| 1925.55 | Too close. | | 2 |

*2454* **A 1866**

$16^h\ 47^m\ 46^s$; $+46°\ 04'$ $+46°.2220$

B and C, 9.5–9.7 4.86 A2p

| | | | |
|---|---|---|---|
| 1908.67 | 311°.7±2°.9 | 0″.25±0″.02 | 3n |
| 1917.66 | 11.8±1.5 | 0.30±0.04 | 2 |
| 1920.67 | 28.5 | 0.28 | 1 |
| 1921.59 | 34.2±1.3 | 0.23±0.01 | 2 |
| 1928.52 | 57.0±2.4 | 0.31±0.04 | 3 |
| | A and BC=$\beta$ 627, 5.0– | | |
| 1908.66 | 319°.7±0°.5 | 1″.54±0″.02 | 2n |
| 1917.66 | 328.8±1.3 | 1.53±0.02 | 2 |
| 1921.59 | 327.3 | 1.55 | 1 |
| 1928.43 | 333.9 | 1.62 | 1 |

The bright star is 52 *Herculis*. Burnham discovered the wide pair, listed as 7779 in his *General Catalogue*, in 1878, his measures being: 1878.38 309°.4 1″.83 5n.

13357 **A 1143**

16h 55m 40s; +57° 16′ +57°.1716

A and B, 9.0–9.1 9.1 G0

| | | | |
|---|---|---|---|
| 1905.66 | 252°.4±0°.9 | 0″.39±0″.02 | 3n |
| 1917.03 | 236.7±1.7 | 0.36±0.02 | 2 |
| 1921.38 | 233.6±0.8 | 0.38±0.01 | 4 |

C and D, 9.4–9.6

| | | | |
|---|---|---|---|
| 1905.66 | 148°.2±1°.3 | 0″.76±0″.03 | 3n |
| 1917.03 | 145.4±2.8 | 0.82±0.04 | 2 |
| 1921.62 | 146.1±2.5 | 0.76±0.02 | 2 |

AB and CD

| | | | |
|---|---|---|---|
| 1905.61 | 5°.0 | 96″.50 | 1n |
| 1916.34 | 4.8 | 96.50 | 1 |

AB and E, −13.5

| | | | |
|---|---|---|---|
| 1905.64 | 351°.4±0°.1 | 45″.80±0″.10 | 2n |
| 1916.34 | 352.0 | 46.23 | 1 |

13363 **A 1146**

17h 04m 02s; +69° 52′ +69°.891

8.2–8.5 7.89 F5

| | | | |
|---|---|---|---|
| 1905.51 | 316°.6±0°.5 | 0″.27±0″.01 | 3n |
| 1915.55 | 307.7±0.5 | 0.32±0.02 | 2 |
| 1923.70 | 297.0 | 0.42 | 1 |

13362 **A 1145**

17h 05m 39s; −1° 01′ −0°.3230

6.2–8.2 6.02 A0

| | | | |
|---|---|---|---|
| 1905.41 | 240°.8±3°.8 | 0″.44±0″.01 | 3n |
| 1915.51 | 227.5±1.8 | 0.46±0.03 | 4 |
| 1920.55 | 221.7±1.2 | 0.38±0.01 | 3 |
| 1922.60 | 222.9 | 0.40 | 1 |
| 1925.61 | 215.2±0.1 | 0.45±0.05 | 2 |

*2486* **A 2087**

17h 11m 25s; +17° 20′ +17°.3190

9.7–9.7 9.0–

| | | | |
|---|---|---|---|
| 1909.63 | 193°.0±0°.4 | 0″.39±0″.03 | 2n |
| 1918.58 | 196.4±2.0 | 0.34±0.01 | 2 |
| 1927.52 | 208.9±3.1 | 0.32±0.04 | 2 |

*2497* **A 2592**

17h 12m 56s; −9° 45′ −9°.4525

7.5–8.0 7.06 F5

| | | | |
|---|---|---|---|
| 1913.66 | 343°.8±1°.0 | 0″.28±0″.03 | 2n |
| 1920.47 | 327.5±0.7 | 0.32±0.02 | 2 |
| 1925.53 | 320.9±0.7 | 0.40±0.05 | 2 |

*2488* **A 2984**

17h 14m 03s; −0° 23′ −0°.3255

4.5–7.5 4.82 K0

| | | | |
|---|---|---|---|
| 1915.54 | 298°.3±1°.9 | 0″.52±0″.04 | 3n |
| 1920.55 | 302.4±5.1 | 0.54±0.05 | 3 |
| 1923.60 | 309.4±0.4 | 0.60±0.05 | 2 |
| 1925.62 | 311.5±2.2 | 0.70±0.01 | 2 |

*2493* **A 2593**

17h 18m 01s; −7° 03′ −6°.4581

9.2–9.2 8.8 G5

| | | | |
|---|---|---|---|
| 1913.66 | 137°.6±3°.3 | 0″.19±0″.01 | 2n |
| 1920.54 | Uncertain, too close | | 4 |
| 1925.55 | 45.2±1.7 | 0.20±0.02 | 2 |

On two of the four nights in 1920, the position angle was estimated at about 75°, the distance at 0″.14, but the seeing was not good enough to permit accurate measures.

—— **A 2244**

17h 25m 17s, −20° 56′ −20°.4775

8.2–8.4 8.0 F8

| | | | |
|---|---|---|---|
| 1910.60 | 130°.6±1°.5 | 0″.27±0″.01 | 2n |
| 1917.57 | 154.5±1.5 | 0.20±0.02 | 2 |
| 1920.63 | 171.0 | 0.18 | 1 |
| 1921.66 | 200.1 | 0.19 | 1 |
| 1926.61 | 331.4 | 0.18 | 1 |
| 1927.54 | 340: | <0.15 | 1 |

*2506* **A 2184**

17h 25m 19s, +16° 30′ +16°.3188

7.0–10.5 7.16 F0

| | | | |
|---|---|---|---|
| 1910.47 | 357°.9±1°.4 | 0″.90±0″.06 | 3n |
| 1918.64 | 4.4±0.8 | 0.99±0.05 | 2 |
| 1925.70 | 3.4±0.1 | 1.03±0.01 | 2 |
| 1927.51 | 12.6±4.0 | 1.10±0.04 | 2 |

13379 **A 1152**

17h 26m 59s; +56° 12′ +56°.1986

9.3–9.5 8.8–

| | | | |
|---|---|---|---|
| 1905.66 | 355°.3±1°.3 | 0″.19±0″.01 | 3n |
| 1918.15 | 61.4±5.1 | 0.18±0.02 | 2 |
| 1920.62 | 69.7±4.2 | 0.20±0.02 | 3 |
| 1921.59 | 73.0 | 0.21 | 1 |

8044 **A 351**

17h 27m 36s; +29° 26′ +29°.3029

9.5–9.9 9.3 A3

| | | | |
|---|---|---|---|
| 1902.48 | 65°.4±1°.7 | 0″.55±0″.06 | 4n |
| 1910.31 | 75.8±1.6 | 0.38±0.01 | 2 |
| 1916.58 | 220.3 | 0.19 | 1 |
| 1917.67 | 227.2±2.0 | 0.28±0.06 | 2 |
| 1919.45 | 234.3±0.4 | 0.35±0.01 | 2 |
| 1920.60 | 233.4±0.8 | 0.42±0.01 | 3 |
| 1924.62 | 241.6±1.6 | 0.54±0.02 | 4 |

13383 **A 1155**

17h 30m 04s; +74° 32′ +74°.713

7.7–10.3 8.17 F2

| | | | |
|---|---|---|---|
| 1905.54 | 334°.4±1°.1 | 0″.43±0″.05 | 3n |
| 1915.57 | 318.0±3.8 | 0.43±0.04 | 3 |
| 1920.67 | 314.9 | 0.45 | 1 |
| 1921.76 | 315.6±4.0 | 0.48±0.06 | 3 |

8061 **A 352**

$17^h\ 30^m\ 06^s$; +28° 50′ +28°.2771

8.7−8.7 8.1 F2

| | | | |
|---|---|---|---|
| 1902.73 | 183°.0±4°.1 | 0″.18±0″.01 | 2n |
| 1910.31 | 159.6±5.1 | 0.18±0.01 | 2 |
| 1916.58 | Too close | | 1 |
| 1917.67 | Uncertain; too close | | 2 |
| 1919.45 | 34.6±2.3 | 0.11±0.01 | 2 |
| 1920.60 | 25.1 | 0.15±0.01 | 3 |
| 1924.65 | 10.8 | 0.15±0.01 | 3 |

*2519* **A 1879**

$17^h\ 33^m\ 09^s$; +13° 24′ +13°.3404

7.8−10.0 7.7 A3

| | | | |
|---|---|---|---|
| 1908.61 | 36°.2±1°.1 | 0″.34±0″.01 | 2n |
| 1917.60 | 43.7±2.4 | 0.25±0.02 | 2 |
| 1921.49 | 50.0±1.3 | 0.30±0.01 | 3 |
| 1926.61 | 54.8 | 0.28 | 1 |
| 1927.53 | 60.4 | 0.30 | 1 |

8230 **A 235**

$17^h\ 51^m\ 17^s$; +25° 00′ +25°.3368

8.3−8.5 8.06 K0

| | | | |
|---|---|---|---|
| 1901.60 | 65°.2±1°.6 | 0″.20±0″.02 | 4n |
| 1906.49 | 75.2±1.7 | 0.34±0.02 | 2 |
| 1911.41 | 84.7±0.9 | 0.42±0.03 | 3 |
| 1915.80 | 85.4 | 0.37 | 1 |
| 1916.51 | 90.7±1.8 | 0.31±0.02 | 2 |
| 1920.62 | 98.9±2.1 | 0.24±0.01 | 2 |
| 1921.58 | 102.8±1.0 | 0.25±0.02 | 3 |
| 1923.70 | 119.0±1.1 | 0.17±0.00 | 2 |
| 1924.67 | 140: | 0.12: | 1 |
| 1925.31 | Too close | | 1 |
| 1926.33 | Too close | | 1 |
| 1927.50 | Too close | | 1 |
| 1928.59 | 246.4±4.3 | 0.16±0.03 | 2 |

—— **A 2258**

$18^h\ 03^m\ 59^s$; −19° 33′ −19°.4842

9.7−9.7 9.4 G0

| | | | |
|---|---|---|---|
| 1910.61 | 261°.8±0°.1 | 0″.36±0″.03 | 2n |
| 1917.60 | 252.6±0.0 | 0.36±0.02 | 2 |
| 1926.61 | 240.5 | 0.32 | 1 |

8491 **A 578**

$18^h\ 16^m\ 20^s$; +43° 49′ +43°.2938

A and B, 8.7−9.4 8.5−

| | | | |
|---|---|---|---|
| 1903.60 | 28°.4±2°.9 | 0″.22±0″.03 | 4n |
| 1917.66 | 5.2±3.9 | 0.27±0.01 | 3 |
| 1921.52 | 352.6±3.4 | 0.30±0.01 | 2 |
| 1928.51 | 348.1±1.1 | 0.32±0.02 | 2 |
| | AB and C, −13.5 | | |
| 1903.60 | 176°.0±1°.5 | 1″.72±0″.04 | 4n |
| 1917.64 | 171.8±1.5 | 1.86±0.10 | 2 |
| 1921.52 | 167.4±0.3 | 1.77±0.06 | 2 |
| 1928.51 | 168.7±0.5 | 1.76±0.07 | 2 |

8587 **A 581**

$18^h\ 26^m\ 35^s$; +4° 06′ +4°.3760

A and B, 8.8−9.3 8.4−

| | | | |
|---|---|---|---|
| 1903.41 | 80°.0±2°.5 | 0″.31±0″.03 | 4n |
| 1916.55 | 89.2±2.0 | 0.35±0.00 | 2 |
| 1923.60 | 87.4 | 0.33 | 1 |
| 1927.05 | 91.8±3.7 | 0.34±0.04 | 2 |
| | AB and C, −15.0 | | |
| 1903.43 | 233°.8±0°.6 | 8″.48±0″.20 | 2n |

*2657* **A 1377**

$18^h\ 32^m\ 49^s$; +52° 19′ +52°.2238

A and B, 6.2−6.2 5.42 K0

| | | | |
|---|---|---|---|
| 1906.42 | 269°.3±0°.8 | 0″.15±0″.01 | 3n |
| 1908.45 | 268.2 | 0.14 | 1 |
| 1911.51 | 271.6 | 0.16 | 1 |
| 1914.67 | 304.6±15.0 | 0.12±0.02 | 2 |
| 1917.32 | 316.8±1.5 | 0.15±0.02 | 2 |
| 1918.59 | 328.9 | 0.12 | 1 |
| 1919.48 | 339.2 | 0.15 | 1 |
| 1920.60 | 347.8±0.1 | 0.15±0.00 | 2 |
| 1921.56 | 345.2±8.3 | 0.12±0.02 | 3 |
| 1923.65 | 359.0±3.9 | 0.13±0.00 | 2 |
| 1924.71 | 11.4 | 0.11 | 1 |
| | AB and C=Σ2348, −8.5 | | |
| 1906.38 | 271°.9 | 25″.60 | 1n |

The Struve pair is recorded as number 8669 in Burnham's *General Catalogue.*

8679 **A 88**

$18^h\ 35^m\ 46^s$; −3° 14′ −3°.4331

A and B, 7.0−7.1 6.47 F8

| | | | |
|---|---|---|---|
| 1900.46 | 353°.2±1°.0 | 0″.14±0″.01 | 3n |
| 1901.56 | 338.3±2.8 | 0.14±0.01 | 3 |
| 1902.66 | 318.1±3.9 | 0.12±0.01 | 3 |
| 1903.44 | 298.4±1.3 | 0.11±0.01 | 3 |
| 1904.53 | 278.4±4.1 | 0.14±0.01 | 4 |
| 1905.53 | 224.8±3.7 | 0.12±0.01 | 4 |
| 1906.48 | 199.1±3.2 | 0.11±0.02 | 4 |
| 1907.30 | 193.5 | 0.14 | 1 |
| 1908.39 | 181.4±5.9 | 0.15±0.02 | 3 |
| 1909.67 | 150.4±14.0 | 0.10±0.01 | 2 |
| 1910.56 | 47.0±14.9 | 0.11±0.01 | 2 |
| 1911.55 | 18.7 | 0.15 | 1 |
| 1912.57 | 356.1±3.2 | 0.15±0.00 | 3 |
| 1914.55 | 331.2±4.2 | 0.15±0.01 | 4 |
| 1915.52 | 306.4±3.4 | 0.15±0.01 | 3 |
| 1916.76 | 248.4±5.4 | 0.14±0.00 | 2 |
| 1917.64 | 228.1±3.4 | 0.14±0.01 | 2 |
| 1918.52 | 200.4±1.9 | 0.14±0.00 | 3 |
| 1919.62 | 188.4±4.2 | 0.15±0.00 | 3 |
| 1920.37 | 173.6±1.4 | 0.16±0.01 | 2 |
| 1921.53 | 144.4±6.6 | 0.12±0.01 | 4 |
| 1922.62 | Too close | | 1 |
| 1923.57 | 10.9±2.8 | 0.14±0.01 | 4 |
| 1924.65 | 344.2 | 0.12 | 1 |
| 1925.61 | 340.6±1.4 | 0.15±0.01 | 3 |
| 1927.75 | Too close | | 1 |
| 1928.63 | 272.2±10.2 | 0.14±0.01 | 2 |
| | AB and C, −14.0 | | |
| 1908.44 | 113°.4±0°.1 | 15″.55±0″.05 | 2n |
| 1915.50 | 112.0 | 15.72 | 1 |
| 1921.35 | 113.2 | 15.97 | 1 |

The measures show that this pair has made more than two complete revolutions since its discovery in 1900, and that the angular separation even at maximum is very small. I computed an orbit in 1912, just as the first revolution had been completed. This still represents the motion fairly well, indicating that the period found is nearly correct. It is hardly profitable at this time to undertake a correction to the other elements. The elements are: $P=12.12$ years, $T=1910.10$, $e=0.273$, $a=0''.176$, $\omega=270°.0$, $i=62°.4$, $\Omega=2°.4$. The representation of the positions to 1912, inclusive, is given in *Publ. Lick Obs.*, **12**, 129. The residuals (O−C) for the later dates are:

| | $\Delta\theta$ | $\Delta\rho$ |
|---|---|---|
| 1914.55 | + 8°.3 | +0''.01 |
| 1915.52 | +10.6 | + .04 |
| 1916.76 | − 2.0 | + .03 |
| 1917.64 | + 3.2 | + .01 |
| 1918.52 | − 7.2 | − .02 |
| 1919.62 | − 2.8 | − .02 |
| 1920.37 | −11.2 | − .01 |
| 1921.53 | − 5.7 | + .03 |
| 1923.57 | − 3.0 | ± .00 |
| 1924.65 | −11.4 | − .03 |
| 1925.61 | − 1.6 | − .01 |
| 1928.63 | +13.0 | + .03 |

The largest angle residual (+13°.0), reduced to arc measurement, is only 0''.03. In 1922 the computed angular separation was only 0''.07, in 1927, 0''.11. The general direction of elongation could be estimated in both years. The trigonometric parallax is +0''.024 (*Yale Cat.*), the spectroscopic parallax 0''.032 (Mt. W.), the dynamic parallax, from my orbit, 0''.025 (Russell and Miss Moore). From the elements and the trigonometric parallax the mass is $2.7\odot$.

—— **A 2988**

$18^h\ 38^m\ 56^s$; +24° 47′ +24°.3493

8.5−8.5 7.79 A5

| | | | |
|---|---|---|---|
| 1916.43 | 192°.4±1°.9 | 0''.19±0''.01 | 3n |
| 1920.54 | 186.0±2.0 | 0.19±0.01 | 2 |
| 1923.70 | 181.0±0.5 | 0.20±0.01 | 2 |
| 1925.55 | 177.3 | 0.21 | 1 |
| 1927.49 | 176.9±2.5 | 0.20±0.00 | 2 |

8883 **A 93**

$18^h\ 51^m\ 02^s$; −5° 36′ −5°.4798

9.0−9.6 8.9 G5

| | | | |
|---|---|---|---|
| 1900.54 | 325°.0±3°.1 | 0''.28±0''.02 | 3n |
| 1901.54 | 323.2 | 0.30 | 1 |
| 1914.24 | 21.8±3.5 | 0.24±0.02 | 3 |
| 1915.60 | 21.8±1.4 | 0.25±0.00 | 2 |
| 1919.46 | 37.2 | 0.23 | 1 |
| 1920.52 | 43.6±0.1 | 0.24±0.01 | 2 |
| 1923.53 | 54.9±2.4 | 0.28±0.02 | 3 |
| 1925.58 | 64.7±0.8 | 0.34±0.05 | 2 |
| 1928.60 | 71.4±1.0 | 0.32±0.04 | 2 |

2747 **A 2192**

$18^h\ 53^m\ 19^s$; +3° 23′ +3°.3836

7.8−7.8 6.94 A2

| | | | |
|---|---|---|---|
| 1910.48 | 241°.4±3°.1 | 0''.27±0''.02 | 3n |
| 1918.51 | 230.3±0.7 | 0.30±0.02 | 2 |
| 1921.45 | 224.6±2.7 | 0.28±0.02 | 4 |
| 1926.70 | 216.9 | 0.18 | 1 |

—— **A 2992**

$19^h\ 01^m\ 50^s$; +26° 37′ +26°.3437

9.5−9.5 8.8−

| | | | |
|---|---|---|---|
| 1916.44 | 59°.7±2°.5 | 0''.23±0''.01 | 3n |
| 1920.57 | 43.5±4.7 | 0.21±0.02 | 4 |
| 1921.83 | 41.2 | 0.18 | 1 |
| 1923.70 | 42.0±2.3 | 0.16±0.02 | 2 |
| 1927.51 | 32.8±4.4 | 0.19±0.01 | 2 |

9040 **A 361**

$19^h\ 05^m\ 01^s$; +8° 06′ +7°.3971

9.5−9.7 9.07 G0

| | | | |
|---|---|---|---|
| 1902.66 | 24°.5±1°.9 | 0''.31±0''.04 | 3n |
| 1916.55 | 12.5±0.2 | 0.34±0.01 | 2 |
| 1922.65 | 5.2±0.3 | 0.32±0.02 | 2 |
| 1927.77 | 2.1±0.3 | 0.28±0.01 | 2 |

9077 **A 95**

$19^h\ 08^m\ 19^s$; −7° 31′ −7°.4876

7.2−7.6 6.76 G0

| | | | |
|---|---|---|---|
| 1900.46 | 36°.8±2°.1 | 0''.23±0''.02 | 3n |
| 1901.79 | 33.7±1.8 | 0.20±0.01 | 4 |
| 1910.74 | 19.9 | 0.19 | 1 |
| 1914.53 | 17.0±1.9 | 0.19±0.01 | 2 |
| 1915.57 | 14.2±1.7 | 0.26±0.06 | 2 |
| 1918.89 | 4.3±1.3 | 0.22±0.01 | 3 |
| 1921.42 | 354.4±1.4 | 0.19±0.02 | 2 |
| 1923.53 | 351.4±2.6 | 0.18±0.02 | 3 |
| 1925.67 | 349.9±3.3 | 0.17±0.02 | 2 |
| 1927.76 | 346.3±0.4 | 0.17±0.00 | 2 |
| 1928.60 | 343.2±1.0 | 0.18±0.04 | 2 |

9097 **A 151**

$19^h\ 09^m\ 14^s$; +21° 11′ +21°.3686

7.8−9.0 7.6 A2

| | | | |
|---|---|---|---|
| 1900.57 | 125°.5±2°.0 | 0''.54±0''.03 | 4n |
| 1914.56 | 130.0 | 0.51 | 1 |
| 1917.64 | 133.8±1.3 | 0.48±0.02 | 2 |
| 1924.81 | 135.6 | 0.55 | 1 |

2830 **A 1391**

$19^h\ 09^m\ 32^s$; +54° 24′ +54°.2095

8.8−9.3 8.3−

| | | | |
|---|---|---|---|
| 1906.75 | 60°.3±1°.2 | 0''.32±0''.02 | 3n |
| 1918.56 | 67.1±0.9 | 0.30±0.01 | 2 |
| 1923.65 | 71.0±0.5 | 0.32±0.01 | 2 |

*2859* **A 2269**

$19^h\ 17^m\ 25^s$; +4° 08′ +3°3976

9.3−9.8 9.0 A0

| | | | |
|---|---|---|---|
| 1910.58 | 38°.4±1°.7 | 0″.28±0″.01 | 3n |
| 1918.53 | 33.6±0.4 | 0.28±0.04 | 2 |
| 1923.74 | 30.2 | 0.32 | 1 |
| 1926.72 | 27.4 | 0.28 | 1 |

**9272** **A 592**

$19^h\ 20^m\ 47^s$; +42° 00′ +41°3327

8.9−9.9 8.9−

| | | | |
|---|---|---|---|
| 1903.85 | 217°.3±2°.1 | 0″.27±0″.02 | 2n |
| 1917.30 | 222.6±2.5 | 0.32±0.05 | 3 |
| 1928.80 | 228.6±1.3 | 0.33±0.02 | 2 |

*2914* **A 1653**

$19^h\ 27^m\ 16^s$; +12° 18′ +12°3929

7.8−10.0 7.67 A3

| | | | |
|---|---|---|---|
| 1907.57 | 302°.1±2°.1 | 0″.22±0″.02 | 3n |
| 1917.56 | 280.8±1.7 | 0.22±0.01 | 2 |
| 1921.54 | 277.4±3.4 | 0.25±0.01 | 3 |
| 1927.79 | 269.8 | 0.22 | 1 |

**13476** **A 713**

$19^h\ 29^m\ 50^s$; +47° 22′ +47°2854

7.2−7.7 7.18 A3

| | | | |
|---|---|---|---|
| 1904.58 | 210°.8±0°.8 | 0″.27±0″.01 | 3n |
| 1906.63 | 215.2±0.8 | 0.27±0.01 | 2 |
| 1909.80 | 221.8±1.7 | 0.27±0.01 | 2 |
| 1911.85 | 219.8±0.9 | 0.30±0.01 | 3 |
| 1914.64 | 220.5 | 0.30 | 1 |
| 1917.05 | 226.1±0.5 | 0.32±0.03 | 2 |
| 1921.56 | 227.8±1.6 | 0.33±0.00 | 2 |

**9392** **A 367**

$19^h\ 30^m\ 31^s$; +5° 39′ +5°4184

8.8−10.0 8.7 F8

| | | | |
|---|---|---|---|
| 1902.71 | 336°.2±1°.7 | 0″.53±0″.01 | 3n |
| 1916.21 | 330.7±0.1 | 0.56±0.07 | 2 |
| 1923.58 | 325.2±1.3 | 0.62±0.00 | 2 |

**9426** **A 597**

$19^h\ 32^m\ 08^s$; +42° 15′ +42°3382

8.2−10.7 8.3 G0

| | | | |
|---|---|---|---|
| 1903.83 | 154°.3±0°.3 | 1″.15±0″.01 | 3n |
| 1917.63 | 140.6±2.0 | 1.24±0.04 | 2 |
| 1921.56 | 136.9±0.7 | 1.18±0.02 | 2 |
| 1928.80 | 135.5 | 1.37 | 1 |

**9430** **A 162**

$19^h\ 32^m\ 54^s$; +23° 22′ +23°3699

8.7−8.7 8.1 A0

| | | | |
|---|---|---|---|
| 1900.66 | 144°.6±0°.9 | 0″.21±0″.02 | 3n |
| 1914.57 | 166.5±1.0 | 0.19±0.01 | 3 |
| 1917.64 | 172.9±2.3 | 0.22±0.04 | 2 |
| 1921.30 | 178.1±1.4 | 0.21±0.01 | 3 |
| 1924.27 | 178.1±1.3 | 0.20±0.02 | 2 |

**9443** **A 163**

$19^h\ 33^m\ 38^s$; +23° 09′ +22°3738

9.4−9.5 8.9−

| | | | |
|---|---|---|---|
| 1900.65 | 233°.4±2°.4 | 0″.30±0″.05 | 4n |
| 1917.64 | 219.5±1.0 | 0.29±0.01 | 2 |
| 1921.62 | 215.4±0.8 | 0.32±0.00 | 2 |
| 1924.80 | 211.4 | 0.30 | 1 |

*2957* **A 1400**

$19^h\ 33^m\ 54^s$; +39° 55′ +39°3831

8.0−10.0 7.90 A3

| | | | |
|---|---|---|---|
| 1906.53 | 289°.7±1°.4 | 0″.31±0″.01 | 3n |
| 1919.23 | 302.2±2.2 | 0.34±0.03 | 2 |
| 1921.66 | 303.9±0.8 | 0.36±0.05 | 2 |
| 1927.58 | 316.9 | 0.37 | 1 |

*3024* **A 1404**

$19^h\ 44^m\ 12^s$; +39° 46′ +39°3905

A and B, 7.5−8.0 7.12 B9

| | | | |
|---|---|---|---|
| 1906.54 | 322°.1±1°.8 | 0″.21±0″.01 | 3n |
| 1918.21 | 306.3±0.3 | 0.18±0.02 | 2 |
| 1927.58 | 234: | <0.10 | 1 |
| | A and C, −13.0 | | |
| 1906.51 | 272°.6 | 19″.70 | 1n |
| 1918.76 | 272.8 | 19.50 | 1 |
| | A and D, −12.0 | | |
| 1906.51 | 79°.2 | 21″.90 | 1n |
| 1918.76 | 78.9 | 21.82 | 1 |

**9596** **A 108**

$19^h\ 44^m\ 26^s$; −8° 17′ −8°5103

8.1−8.5 7.9 K0

| | | | |
|---|---|---|---|
| 1900.55 | 182°.7±4°.0 | 0″.27±0″.03 | 3n |
| 1901.57 | 186.7 | 0.25 | 1 |
| 1915.09 | 197.8±2.5 | 0.22±0.02 | 4 |
| 1921.68 | 201.4±0.6 | 0.22±0.00 | 2 |
| 1925.65 | 207.4±2.5 | 0.22±0.01 | 2 |

*3035* **A 1658**

$19^h\ 46^m\ 22^s$; +14° 56′ +14°4048

8.1−8.3 7.64 F5

| | | | |
|---|---|---|---|
| 1907.51 | 161°.0±3°.6 | 0″.23±0″.02 | 4n |
| 1917.56 | 115.9±1.6 | 0.23±0.03 | 2 |
| 1919.46 | 112.3 | 0.23 | 1 |
| 1920.34 | 103.3 | 0.21 | 1 |
| 1921.52 | 98.1±0.7 | 0.23±0.02 | 2 |
| 1922.65 | 94.9±2.3 | 0.21±0.01 | 3 |
| 1927.79 | 71.9 | 0.17 | 1 |

**9727** **A 275**

$19^h\ 52^m\ 50^s$; +25° 05′ +24°3926

A and B, 9.4−9.6 8.8−

| | | | |
|---|---|---|---|
| 1901.94 | 86°.8±1°.4 | 0″.18±0″.02 | 3n |
| 1916.46 | 97.6±0.5 | 0.30±0.03 | 2 |
| 1923.72 | 101.1±0.3 | 0.34±0.01 | 2 |
| 1927.49 | 102.6 | 0.27 | 1 |
| | AB and C, −13.8 | | |
| 1916.46 | 217°.1±0°.2 | 12″.10±0″.13 | 2n |
| 1927.49 | 217.5 | 11.76 | 1 |

**9742** **A 604**

$19^h\ 54^m\ 51^s$; $+5°\ 05'$ $+4°.4286$

8.9–9.0 8.55 F8

| | | | |
|---|---|---|---|
| 1903.52 | 263°.3±0°.5 | 0″.20±0″.01 | 3n |
| 1914.50 | 229.0±1.6 | 0.13±0.01 | 2 |
| 1915.80 | 220.2±14.3 | 0.16±0.02 | 2 |
| 1918.60 | 210.6±7.2 | 0.13±0.01 | 2 |
| 1920.62 | 191.8±15.8 | 0.13±0.01 | 2 |
| 1921.55 | 187.0±0.9 | 0.12±0.00 | 2 |
| 1923.64 | Too close | | 3 |
| 1927.76 | Too close | | 2 |

**9754** **A 606**

$19^h\ 55^m\ 34^s$; $+4°\ 48'$ $+4°.4292$

9.3–9.3 8.7 G5

| | | | |
|---|---|---|---|
| 1903.51 | 105°.9±0°.9 | 0″.28±0″.01 | 3n |
| 1914.50 | 145.2±0.4 | 0.32±0.01 | 2 |
| 1915.80 | 147.0±0.3 | 0.30±0.01 | 2 |
| 1918.60 | 153.6±0.1 | 0.30±0.00 | 2 |
| 1920.62 | 165.4±0.9 | 0.28±0.01 | 2 |
| 1921.55 | 168.6±2.2 | 0.30±0.01 | 2 |
| 1923.64 | 171.7±1.8 | 0.26±0.02 | 3 |
| 1924.67 | 176.1 | 0.25 | 1 |
| 1927.75 | 191.8±0.8 | 0.25±0.02 | 2 |

*3126* **A 2275**

$19^h\ 58^m\ 41^s$; $+1°\ 12'$ $+0°.4385$

9.0–9.6 9.09 F8

| | | | |
|---|---|---|---|
| 1910.68 | 63°.1±0°.4 | 0″.40±0″.01 | 2n |
| 1918.54 | 69.5±3.2 | 0.39±0.03 | 3 |
| 1928.69 | 79.8±1.2 | 0.35±0.01 | 2 |

**13519** **A 1205**

$20^h\ 16^m\ 11^s$; $+29°\ 03'$ $+28°.3698$

8.9–9.7 8.7–

| | | | |
|---|---|---|---|
| 1905.81 | 345°.6±1°.0 | 0″.34±0″.01 | 3n |
| 1918.66 | 328.2±1.5 | 0.34±0.04 | 2 |
| 1921.56 | 325.0±1.6 | 0.42±0.03 | 2 |

**10099** **A 1672**

$20^h\ 17^m\ 01^s$; $+14°\ 13'$ $+13°.4356$

B and C, 9.5–10.0 6.81 A0, G

| | | | |
|---|---|---|---|
| 1907.66 | 266°.5±0°.6 | 0″.24±0″.01 | 3n |
| 1917.57 | 256.9±0.6 | 0.22±0.01 | 2 |
| 1921.63 | 256.1±2.3 | 0.26±0.00 | 3 |
| | A and BC=Σ2665, 7.0– | | |
| 1907.64 | 17°.0 | 3″.27 | 1n |
| 1917.57 | 15.2±0°.5 | 3.16±0″.06 | 2 |

The spectrum of the primary of the Struve pair is marked "Composite" in the *Henry Draper Catalogue*.

*3251* **A 1427**

$20^h\ 18^m\ 27^s$; $+39°\ 15'$ $+38°.4021$

A and B, 7.0–9.0 6.07 A0

| | | | |
|---|---|---|---|
| 1906.55 | 111°.9±1°.1 | 0″.25±0″.01 | 3n |
| 1918.66 | 117.2 | 0.26 | 1 |
| 1921.52 | 119.9±0.1 | 0.24±0.03 | 2 |
| 1927.57 | 129.1±1.5 | 0.22±0.02 | 2 |
| | AB and C=Σ2668, –9.0 | | |
| 1906.54 | 286°.3±0°.8 | 3″.26±0″.07 | 2n |
| 1918.66 | 286.1 | 3.36 | 1 |
| 1921.51 | 285.0 | 3.31 | 1 |

**10167** **A 288**

$20^h\ 20^m\ 56^s$; $+20°\ 42'$ $+20°.4541$

8.5–8.7 8.0 F2

| | | | |
|---|---|---|---|
| 1901.39 | 351°.6±1°.7 | 0″.26±0″.01 | 3n |
| 1917.68 | 359.3±1.9 | 0.31±0.03 | 2 |
| 1924.80 | 5.0 | 0.27 | 1 |

**10237** **A 730**

$20^h\ 24^m\ 03^s$; $+59°\ 26'$ $+59°.2228$

6.8–7.0 6.48 A0

| | | | |
|---|---|---|---|
| 1904.52 | 313°.9±0°.8 | 0″.21±0″.01 | 3n |
| 1908.65 | 306.8±0.6 | 0.20±0.03 | 2 |
| 1911.42 | 306.4 | 0.19 | 1 |
| 1915.61 | 297.1±4.7 | 0.17±0.02 | 7 |
| 1921.12 | 281.3±1.5 | 0.14±0.01 | 2 |

**10249** **A 610**

$20^h\ 26^m\ 36^s$; $+6°\ 59'$ $+6°.4543$

8.9–9.1 8.5 G0

| | | | |
|---|---|---|---|
| 1903.63 | 189°.6±1°.0 | 0″.50±0″.01 | 3n |
| 1915.79 | 208.6±0.8 | 0.52±0.02 | 2 |
| 1920.63 | 210.8±0.5 | 0.53±0.03 | 2 |
| 1923.61 | 216.0±3.9 | 0.52±0.01 | 2 |
| 1928.76 | 220.8±1.1 | 0.50±0.01 | 2 |

*3301* **A 1675**

$20^h\ 28^m\ 47^s$; $+15°\ 38'$ $+15°.4181$

7.2–7.2 6.80 A2

| | | | |
|---|---|---|---|
| 1907.58 | 327°.4±2.2 | 0″.16±0″.01 | 3n |
| 1912.57 | 316.1±0.1 | 0.18±0.02 | 2 |
| 1914.65 | 315.2 | 0.19 | 1 |
| 1917.56 | 302.0±1.8 | 0.16±0.02 | 2 |
| 1920.63 | 299.9±0.7 | 0.15±0.01 | 2 |
| 1923.65 | 288.6±5.6 | 0.14±0.01 | 2 |
| 1927.79 | 271.2 | 0.14 | 1 |

**13551** **A 750**

$20^h\ 51^m\ 02^s$; $+45°\ 55'$ $+45°.3310$

8.5–9.3 8.3–

| | | | |
|---|---|---|---|
| 1904.56 | 249°.7±0°.8 | 0″.30±0″.02 | 3n |
| 1917.59 | 227.2±2.0 | 0.17±0.00 | 2 |
| 1921.54 | Too close | | 2 |
| 1922.79 | 107? | 0.14 | 1 |
| 1927.60 | 268.7 | 0.23 | 1 |
| 1928.64 | 268.2 | 0.26 | 1 |

**10618** **A 751**

$20^h\ 52^m\ 28^s$; $+59°\ 07'$ $+58°.2187$

7.0–7.3 6.82 F2, A2

| | | | |
|---|---|---|---|
| 1904.53 | 33°.9±1°.2 | 0″.16±0″.01 | 3n |
| 1908.65 | 24.4±2.8 | 0.17±0.00 | 2 |
| 1911.51 | 16.1 | 0.20 | 1 |
| 1914.71 | 336.4±15.8 | 0.13±0.01 | 2 |
| 1915.49 | Too close | | 1 |
| 1916.64 | Too close | | 2 |
| 1920.68 | 212.5 | 0.15 | 1 |
| 1921.54 | 195.9±1.3 | 0.13±0.01 | 2 |
| 1922.79 | 157.1 | 0.14 | 1 |

3444 **A 1688**

$20^h\ 59^m\ 49^s$; +14° 14′ +13°.4600

9.1−9.2 8.6 A5

| | | | |
|---|---|---|---|
| 1907.65 | 75°.9±3°.5 | 0″.24±0″.01 | 3n |
| 1917.66 | 96.0±0.0 | 0.24±0.01 | 2 |
| 1921.61 | Too close | | 3 |
| 1927.79 | 272.9 | 0.23 | 1 |

10759 **A 758**

$21^h\ 04^m\ 22^s$; +60° 17′ +59°.2315

A and B, 9.0−10.0 8.8−

| | | | |
|---|---|---|---|
| 1904.53 | 353°.1±2°.0 | 0″.28±0″.02 | 3n |
| 1916.33 | 11.0±3.4 | 0.26±0.01 | 3 |
| 1921.25 | 17.5±3.4 | 0.25±0.03 | 3 |
| | AB and C, −14.0 | | |
| 1904.48 | 189°.0 | 8″.95 | 1n |
| 1915.70 | 190.2 | 8.67 | 1 |
| 1921.54 | 188.4 | 8.57 | 1 |

13567 **A 884**

$21^h\ 11^m\ 34^s$; +46° 42′ +46°.3231

9.1−9.2 8.7 F0

| | | | |
|---|---|---|---|
| 1904.62 | 186°.4±0°.5 | 0″.37 | 3n |
| 1917.61 | 179.9±1.8 | 0.42 | 2 |
| 1928.13 | 166.7±2.8 | 0.44±0.02 | 2 |

13566 **A 883**

$21^h\ 12^m\ 07^s$; −1° 02′ −1°.4131

A and B, 7.6−7.8 7.31 A0

| | | | |
|---|---|---|---|
| 1904.71 | 43°.8±2°.4 | 0″.14±0″.01 | 3n |
| 1906.49 | 30.0±3.9 | 0.14±0.01 | 2 |
| 1908.94 | 23.5±5.6 | 0.15±0.02 | 4 |
| 1915.67 | 357.5±7.1 | 0.14±0.02 | 4 |
| 1917.99 | 341.9±9.1 | 0.14±0.01 | 3 |
| 1921.64 | 340.4±2.3 | 0.15±0.01 | 2 |
| 1922.67 | 325.1±2.2 | 0.15±0.02 | 3 |
| 1924.08 | 319.6±3.4 | 0.16±0.02 | 2 |
| 1926.68 | 318.2±3.0 | 0.16±0.01 | 2 |
| | AB and C=Σ2775, −10.5 | | |
| 1904.67 | 177°.7 | 21″.27 | 1n |

3501 **A 1441**

$21^h\ 15^m\ 08^s$; +39° 49′ +39°.4507

9.3−9.4 9.0 A0

| | | | |
|---|---|---|---|
| 1906.51 | 353°.3±0°.9 | 0″.27±0″.01 | 3n |
| 1917.66 | 346.0±0.6 | 0.26±0.01 | 2 |
| 1927.58 | 342.4 | 0.24 | 1 |

10694 **A 764**

$21^h\ 20^m\ 52^s$; +57° 21′ +56°.2564

8.0−10.0 8.1 G5

| | | | |
|---|---|---|---|
| 1904.48 | 254°.4±1°.3 | 0″.41±0″.02 | 3n |
| 1908.85 | 262.4 | 0.43 | 1 |
| 1916.26 | 276.3±1.7 | 0.46±0.02 | 2 |
| 1920.66 | 288.7±1.5 | 0.52±0.02 | 2 |
| 1921.86 | 294.7 | 0.57 | 1 |

3520 **A 2288**

$21^h\ 21^m\ 49^s$; +3° 28′ +3°.4562

9.5−9.6 9.2 G5

| | | | |
|---|---|---|---|
| 1910.59 | 214°.5±0°.8 | 0″.33±0″.01 | 3n |
| 1918.58 | 200.3±0.6 | 0.30±0.02 | 2 |
| 1921.64 | 195.8±0.2 | 0.32±0.03 | 2 |
| 1927.78 | 182.4 | 0.29 | 1 |

13574 **A 887**

$21^h\ 23^m\ 02^s$; +11° 08′ +10°.4542

8.5−9.5 8.3 F8

| | | | |
|---|---|---|---|
| 1904.71 | 110°.8±3°.7 | 0″.25±0″.01 | 3n |
| 1912.56 | 119.3 | 0.25 | 1 |
| 1915.23 | 121.0±3.4 | 0.24±0.02 | 5 |
| 1924.55 | 126.9 | 0.26 | 1 |
| 1926.75 | 123.0±1.0 | 0.22±0.01 | 2 |

11140 **A 772**

$21^h\ 37^m\ 19^s$; +29° 56′ +29°.4474

8.9−9.3 8.6 A5

| | | | |
|---|---|---|---|
| 1904.54 | 27°.2±1°.8 | 0″.25±0″.01 | 3n |
| 1918.73 | 5.6±1.6 | 0.28±0.02 | 4 |
| 1921.54 | 358.8±1.9 | 0.31±0.01 | 2 |

13585 **A 1223**

$21^h\ 43^m\ 29^s$; +11° 39′ +11°.4653

A and B, 8.9−9.2 8.5 G0

| | | | |
|---|---|---|---|
| 1905.53 | 48°.8±0°.8 | 0″.13±0″.01 | 3n |
| 1915.79 | 9.0 | 0.16 | 1 |
| 1916.54 | 5.6 | 0.15 | 1 |
| 1918.60 | 344.0±3.2 | 0.16±0.02 | 2 |
| 1921.64 | Uncertain; too close | | 1 |
| 1926.75 | 243.8±7.1 | 0.14±0.02 | 2 |
| | AB and C, −14.7 | | |
| 1905.53 | 348°.8±0°.5 | 1″.60±0″.02 | 3n |
| 1918.60 | 13.0±0.2 | 3.08±0.09 | 2 |
| 1921.64 | 13.7 | 3.19 | 1 |
| 1926.75 | 23.4±3.2 | 3.52±0.06 | 2 |

The measures of AB and C indicate an annual proper motion of about 0″.10, but the brighter star is not listed in Charlier's lists of proper motion stars.

13591 **A 889**

$21^h\ 48^m\ 46^s$; +28° 56′ +28°.4210

9.0−10.0 9.3 K0

| | | | |
|---|---|---|---|
| 1904.60 | 57°.5±1°.7 | 0″.24±0″.01 | 3n |
| 1918.72 | 10.8±3.1 | 0.21±0.01 | 2 |
| 1921.56 | 34.4±3.2 | 0.24±0.02 | 2 |

It is assumed that a double reversal of quadrant occurred in the interval between 1904 and 1918, for the angle measures in all three years were carefully checked, and the quadrant was at no time in doubt.

**11348** **A 620**

$21^h\ 52^m\ 32^s$; +43° 49′ +43°.4084

8.3−8.4 7.9 G5

| | | | |
|---|---|---|---|
| 1903.68 | 247°.3±0°.3 | 0″.20±0″.01 | 3n |
| 1916.54 | 253.2±1.2 | 0.22±0.02 | 2 |
| 1928.60 | 257.9±0.4 | 0.24±0.03 | 2 |

*3617* **A 1449**

$21^h\ 53^m\ 28^s$; +38° 35′ +38°.4638

9.4−9.4 8.8 G0

| | | | |
|---|---|---|---|
| 1906.55 | 24°.0±1°.1 | 0″.40±0″.01 | 3n |
| 1917.67 | 15.1±0.7 | 0.40±0.01 | 2 |
| 1921.48 | 10.7±0.2 | 0.40±0.04 | 2 |
| 1927.58 | 10.1 | 0.39 | 1 |

**13609** **A 893**

$22^h\ 02^m\ 34^s$; +29° 37′ +29°.4578

8.7−9.7 8.7 G5

| | | | |
|---|---|---|---|
| 1904.81 | 243°.0±0°.6 | 0″.20±0″.01 | 3n |
| 1918.72 | 205.6±2.3 | 0.24±0.00 | 2 |
| 1921.54 | 205.9±0.3 | 0.26±0.00 | 2 |

**11507** **A 308**

$22^h\ 05^m\ 11^s$; +25° 23′ +24°.4536

8.5−9.0 8.26 A2

| | | | |
|---|---|---|---|
| 1901.74 | 118°.0±2°.4 | 0″.19±0″.01 | 4n |
| 1917.64 | 131.5±3.4 | 0.22±0.01 | 3 |
| 1921.57 | 127.3±1.4 | 0.18±0.01 | 2 |
| 1928.72 | 136.6±3.0 | 0.17±0.01 | 2 |

**11652** **A 782**

$22^h\ 16^m\ 20^s$; +71° 42′ +71°.1120

9.4−10.0 9.2−

| | | | |
|---|---|---|---|
| 1904.56 | 260°.2±0°.7 | 0″.35±0″.02 | 3n |
| 1915.57 | 270.2±1.0 | 0.41±0.04 | 2 |
| 1921.61 | 271.2±1.3 | 0.44±0.01 | 2 |

*3748* **A 1470**

$22^h\ 33^m\ 40^s$; +52° 57′ +52°.3247

9.0−9.0 8.4 G0

| | | | |
|---|---|---|---|
| 1906.55 | 269°.3±1°.5 | 0″.17±0″.01 | 3n |
| 1918.54 | 278.4±8.2 | 0.15±0.01 | 2 |
| 1921.54 | Too close | | 2 |
| 1928.77 | 88.7 | 0.14 | 1 |

*3757* **A 2099**

$22^h\ 37^m\ 27^s$; +0° 57′ +0°.4905

8.7−9.2 8.4 G5

| | | | |
|---|---|---|---|
| 1909.84 | 343°.4±1°.9 | 0″.30±0″.00 | 2n |
| 1918.58 | 349.6±1.0 | 0.31±0.01 | 2 |
| 1926.69 | 359.6±0.5 | 0.31±0.04 | 2 |

*3771* **A 2696**

$22^h\ 42^m\ 30^s$; −2° 55′ −3°.5501

B and C, 9.2−9.7 8.7 F5

| | | | |
|---|---|---|---|
| 1913.76 | 261°.8±1°.4 | 0″.20±0″.02 | 2n |
| 1920.55 | 216.2 | 0.23 | 1 |
| 1921.57 | 208.9±0.1 | 0.25±0.00 | 2 |
| 1922.64 | 208.0±6.2 | 0.21±0.02 | 3 |
| 1923.61 | Too close | | 1 |

A and BC=Σ2938, 8.5−

| | | | |
|---|---|---|---|
| 1913.74 | 342°.0 | 19″.32 | 1n |
| 1921.54 | 342.4 | 19.85 | 1 |

**12025** **A 632**

$22^h\ 50^m\ 00^s$; +57° 27′ +56°.2890

8.0−8.8 7.8 K0

| | | | |
|---|---|---|---|
| 1903.72 | 135°.6±0°.7 | 0″.46±0″.01 | 2n |
| 1915.66 | 84.8±2.5 | 0.34±0.02 | 3 |
| 1916.82 | 80.8 | 0.29 | 1 |
| 1920.56 | 64.8±1.4 | 0.34±0.02 | 2 |
| 1921.55 | 59.9±0.3 | 0.38±0.01 | 2 |

**12143** **A 417**

$23^h\ 02^m\ 33^s$; −7° 58′ −8°.6018

6.0−6.0 5.56 F0

| | | | |
|---|---|---|---|
| 1902.64 | 61°.0±2°.6 | 0″.19±0″.02 | 4n |
| 1903.66 | 73.5±1.6 | 0.19±0.01 | 5 |
| 1904.58 | 84.8±1.5 | 0.22±0.02 | 2 |
| 1905.59 | 94.6±0.4 | 0.20±0.01 | 3 |
| 1906.51 | 106.1±1.4 | 0.20±0.01 | 3 |
| 1907.70 | 118.0 | 0.20 | 1 |
| 1908.66 | 130.6±1.9 | 0.19±0.01 | 3 |
| 1909.72 | 144.9±3.0 | 0.17±0.01 | 3 |
| 1910.72 | 156.9±2.5 | 0.20±0.01 | 3 |
| 1911.84 | 169.4±3.2 | 0.22±0.01 | 3 |
| 1912.64 | 177.5±1.7 | 0.19±0.01 | 3 |
| 1914.60 | 196.0±2.9 | 0.18±0.01 | 4 |
| 1915.62 | 207.5±0.8 | 0.16±0.01 | 3 |
| 1916.68 | 230.0±2.5 | 0.14±0.01 | 3 |
| 1917.70 | 290.5±23.3 | 0.12±0.01 | 4 |
| 1918.56 | 330.9±3.6 | 0.13±0.01 | 3 |
| 1919.66 | 355.2±2.6 | 0.15±0.01 | 2 |
| 1920.54 | 16.4±1.7 | 0.18±0.00 | 2 |
| 1921.56 | 29.3±1.9 | 0.19±0.02 | 3 |
| 1922.62 | 41.5±1.5 | 0.19±0.00 | 3 |
| 1923.64 | 50.2±1.1 | 0.22±0.04 | 2 |
| 1924.55 | 64.3 | 0.22 | 1 |
| 1925.66 | 72.2±0.5 | 0.19±0.01 | 3 |
| 1927.78 | 94.4 | 0.19 | 1 |
| 1928.83 | 113.5±2.1 | 0.22±0.02 | 2 |

An orbit for this pair, which is 83 *Aquarii*, was computed in 1918. The elements follow, with the residuals (O−C) for the last five measures. The residuals in angle indicate that the period is not much greater than 22 years and that the other elements also require correction. A reinvestigation of the orbit is, however, reserved for a later paper.

| | | | |
|---|---|---|---|
| $P$= 23.82 years | Date | $\Delta\theta$ | $\Delta\rho$ |
| $T$=1917.68 | 1923.64 | + 8°.4 | −0″.03 |
| $e$= 0.404 | 1924.55 | +15.8 | −0.03 |
| $a$= 0″.245 | 1925.66 | +15.1 | −0.05 |
| $\omega$= 261°.3 | 1927.78 | +18.3 | −0.02 |
| $i$= 56.35 | 1928.83 | +26.3 | +0.02 |
| $\Omega$= 21.6 | | | |

**13643** **A 1238** +10°.4887

23h 06m 17s; +10° 41′ 7.51 F5

A and B, 7.4–7.9

| | | | |
|---|---|---|---|
| 1905.57 | 228°.1±0°.6 | 0″.25±0″.01 | 3n |
| 1915.76 | 181.4±0.9 | 0.26±0.01 | 2 |
| 1916.74 | 177.8 | 0.28 | 1 |
| 1917.71 | 173.4±1.0 | 0.25±0.02 | 3 |
| 1921.34 | 162.0±1.6 | 0.29±0.04 | 4 |
| 1923.00 | 158.2±1.1 | 0.32±0.02 | 3 |
| 1926.74 | 142.9±0.6 | 0.30±0.02 | 2 |

AB and CD

| | | | |
|---|---|---|---|
| 1905.57 | 295°.0 | 70.3: | 1n |
| 1915.75 | 293.8 | 70.92 | 1 |
| 1922.63 | 293.6 | 71.61 | 1 |
| 1926.73 | 293.6 | 70.72 | 1 |

C and D, 11.7–11.7

| | | | |
|---|---|---|---|
| 1905.58 | 119°.2±0°.7 | 1″.10±0″.01 | 2n |
| 1915.75 | 120.5 | 1.17 | 1 |
| 1922.65 | 120.6±2.3 | 1.23±0.06 | 2 |
| 1926.74 | 120.0±0.1 | 1.24±0.05 | 2 |

*3828* **A 2298** +1°.4694

23h 10m 05s; +2° 25′ 8.2 G5

8.8–8.9

| | | | |
|---|---|---|---|
| 1910.67 | 120°.9±2°.6 | 0″.24±0″.01 | 3n |
| 1918.60 | 102.7±3.7 | 0.25±0.00 | 2 |
| 1921.61 | 82.0±1.0 | 0.16±0.01 | 2 |
| 1923.67 | Too close | | 1 |
| 1926.70 | 291.2±1.5 | 0.17±0.01 | 2 |

**12226** **A 418** −9°.6146

23h 10m 52s; −9° 11′ 8.2 F0

8.2–9.2

| | | | |
|---|---|---|---|
| 1902.65 | 23°.8±4°.1 | 0″.21±0″.01 | 3n |
| 1903.77 | 21.7±1.7 | 0.19±0.01 | 2 |
| 1915.67 | 26.6±1.2 | 0.30±0.03 | 2 |
| 1925.67 | 37.2±0.3 | 0.30±0.01 | 2 |

Rejecting one discordant measure (29°.9), the mean for 1902 would be 1902.64, 20°.8±0°.8, 0″.20±0.01, 2n. The measures of distance obviously do not accord with the rate of change in angle.

*3892* **A 1493** +54°.3017

23h 35m 46s; +54° 47′ 8.6 A0

8.9–9.2

| | | | |
|---|---|---|---|
| 1906.62 | 214°.1±0°.4 | 0″.20±0″.01 | 3n |
| 1917.69 | 186.8±0.1 | 0.21±0.01 | 2 |
| 1921.57 | 173.2±6.9 | 0.18±0.01 | 3 |
| 1928.77 | 145.6 | 0.17 | 1 |

**12488** **A 643** +44°.4464

23h 36m 54s; +45° 27′ 7.77 A2

7.9–8.0

| | | | |
|---|---|---|---|
| 1903.93 | 264°.4±2°.4 | 0″.21±0″.01 | 3n |
| 1911.63 | 251.6 | 0.19 | 1 |
| 1916.62 | 248.1±2.0 | 0.23±0.00 | 2 |
| 1917.59 | 246.3 | 0.24 | 1 |
| 1921.55 | 241.6±0.7 | 0.22±0.03 | 2 |
| 1928.16 | 231.6±1.1 | 0.24±0.01 | 4 |

**13655** **A 1242** +11°.5045

23h 40m 33s; +11° 33′ 8.8 G5

9.2–9.2

| | | | |
|---|---|---|---|
| 1905.62 | 265°.7±0°.5 | 0″.52±0″.01 | 3n |
| 1916.21 | 276.8±1.7 | 0.60±0.01 | 2 |
| 1921.08 | 281.5±1.0 | 0.57±0.02 | 2 |
| 1923.68 | 285.4 | 0.67 | 1 |
| 1926.74 | 288.2±0.8 | 0.59±0.02 | 2 |

**12583** **A 424** +26°.4698

23h 47m 18s; +27° 24′ 6.94 F0

7.3–7.8

| | | | |
|---|---|---|---|
| 1902.86 | 229°.9±1°.3 | 0″.20±0″.01 | 3n |
| 1904.46 | 232.1 | 0.19 | 1 |
| 1907.57 | 236.6±1.0 | 0.22±0.01 | 2 |
| 1910.61 | 239.1±0.3 | 0.19±0.01 | 2 |
| 1914.76 | 243.0 | 0.19 | 1 |
| 1916.51 | 245.4±4.1 | 0.18±0.01 | 4 |
| 1921.58 | 251.4±4.0 | 0.15±0.01 | 2 |
| 1928.72 | 283.7±6.7 | 0.10±0.00 | 2 |

**12589** **A 793** +46°.4185

23h 48m 06s; +46° 49′ 8.1–

8.6–8.7

| | | | |
|---|---|---|---|
| 1904.59 | 296°.0±1°.5 | 0″.16±0″.01 | 3n |
| 1917.20 | 267.1±2.2 | 0.14±0.01 | 2n |
| 1921.64 | Too close | | 3 |
| 1927.73 | Too close | | 1 |
| 1928.63 | Too close | | 2 |

*3918* **A 2700** −7°.6095

23h 49m 12s; −6° 53′ 8.9 G0

9.4–9.4

| | | | |
|---|---|---|---|
| 1913.70 | 217°.5±1°.7 | 0″.21±0″.00 | 3n |
| 1921.61 | 165.6±1.4 | 0.22±0.02 | 2 |
| 1925.67 | 129.9±1.3 | 0.20±0.02 | 2 |

**13661** **A 1247** +12°.5034

23h 49m 28s; +12° 36′ 8.9–

9.4–9.7

| | | | |
|---|---|---|---|
| 1905.71 | 321°.8±0°.3 | 0″.24±0″.01 | 3n |
| 1917.31 | 309.3±4.0 | 0.29±0.01 | 3 |
| 1921.72 | 305.7±1.9 | 0.26±0.02 | 2 |
| 1926.74 | 296.3 | 0.29 | 1 |

**12638** **A 426** +24°.4862

23h 53m 38s; +25° 04′ 8.9 G0

9.2–9.3

| | | | |
|---|---|---|---|
| 1902.86 | 93°.9±3°.7 | 0″.24±0″.03 | 3n |
| 1908.75 | 92.7 | 0.28 | 1 |
| 1915.84 | 73.0±0.1 | 0.25±0.01 | 2 |
| 1920.67 | 65.6±0.3 | 0.26±0.01 | 2 |
| 1928.71 | 54.7±3.4 | 0.27±0.03 | 3 |

*3935* **A 2100** +3°.4909

23h 54m 13s; +4° 27′ 6.87 F0

7.5–8.0

| | | | |
|---|---|---|---|
| 1909.81 | 288°.9±1°.5 | 0″.21±0″.02 | 3n |
| 1918.60 | 267.7±1.2 | 0.32±0.02 | 2 |
| 1921.98 | 263.4±1.2 | 0.39±0.03 | 3 |
| 1926.71 | 256.8 | 0.38 | 1 |
| 1927.76 | 256.4±0.5 | 0.39±0.01 | 3 |

**13664** **A 1249** +9°.5314

23h 59m 52s; +10° 30′ 8.67 G0

9.2–9.9

| | | | |
|---|---|---|---|
| 1905.55 | 239°.9±0°.0 | 0″.33±0″.02 | 3n |
| 1916.21 | 236.2±1.4 | 0.34±0.05 | 2 |
| 1926.74 | 227.4±1.9 | 0.28±0.01 | 2 |

May 1, 1929.

# EIGHTEEN NEW DOUBLE STARS

By Robert G. Aitken

In publishing my Twenty-fifth List of New Double Stars, in *Lick Observatory Bulletin* Number 383, I stated that about 400 stars still remained to be examined, to complete the additional survey undertaken by agreement with the observers in South Africa. These have now all been examined and the new pairs discovered have been measured. The results are given in the following tabulation, which, in all details, follows the form of the Twenty-fifth List.

| | | R.A. 1950 | Decl. 1950 | Date | Angle | Dist. | Mag. | BD Mag. | n | |
|---|---|---|---|---|---|---|---|---|---|---|
| A3086 | − 5°.1207 | 5h 13m 37s | − 5° 38′ | 1929.10 | 344°.6 | 1″.56 | 9.0−13.8 | 8.3 | 2 | AB |
| | | | | 1929.10 | 5.4 | 9.38 | − 9.2 | ..... | 2 | AC=Σ 675 |
| A3087 | −16.1887 | 7 16 29 | −16 11 | 1927.71 | 195.6 | 2.74 | 8.6−12.0 | 8.6 | 2 | AB |
| | | | | 1927.71 | 29.8 | 10.22 | −13.2 | ..... | 2 | AC |
| A3088 | −14.1873 | 7 20 58 | −14 46 | 1927.71 | 287.0 | 0.90 | 9.2−11.5 | 9.2 | 2 | |
| A3089 | −17.1981 | 7 25 09 | −17 26 | 1927.89 | 292.7 | 0.41 | 8.7−10.2 | 8.7 | 3 | |
| A3090 | −14.1939 | 7 28 39 | −15 07 | 1928.22 | 141.5 | 1.40 | 9.1−11.0 | 9.1 | 2 | A14m.5 star nf 10″± |
| A3091 | −14.1970 | 7 31 24 | −15 03 | 1928.22 | 290.0 | 1.46 | 8.7−11.0 | 8.7 | 2 | |
| A3092 | −14.1999 | 7 33 46 | −14 23 | 1928.22 | 176.5 | 5.25 | 6.9−13.5 | 6.9 | 2 | This is AB of Σ 1120 |
| A3093 | −14.2066 | 7 36 38 | −14 14 | 1928.23 | 39.1 | 0.54 | 9.7− 9.7 | 9.0 | 2 | AB |
| | | | | 1928.23 | 242.4 | 11.48 | −13.5 | ..... | 2 | AB and C |
| A3094 | −14.2084 | 7 38 15 | −14 32 | 1928.23 | 321.6 | 0.64 | 8.5−10.0 | 8.5 | 2 | |
| A3095 | −16.2105 | 7 42 22 | −17 10 | 1929.15 | 36.9 | 1.46 | 9.7−14.0 | ..... | 2 | BC |
| | | | | 1929.15 | 278.1 | 18.72 | 8.1− | 8.1 | 2 | AB |
| A3096 | −17.2138 | 7 43 04 | −18 06 | 1929.15 | 234.4 | 0.44 | 9.5− 9.5 | 8.9 | 2 | |
| A3097 | −14.2245 | 7 49 30 | −15 08 | 1929.08 | 278.0 | 1.44 | 8.2−10.5 | 8.2 | 2 | |
| A3098 | −14.2258 | 7 50 44 | −14 53 | 1929.08 | 48.6 | 3.46 | 9.0−12.5 | 9.0 | 2 | |
| A3099 | −14.2264 | 7 51 26 | −14 30 | 1929.11 | 145.3 | 0.86 | 9.7− 9.9 | 9.2 | 3 | |
| A3100 | −16.2236 | 7 57 24 | −17 07 | 1929.23 | 130.2 | 0.44 | 9.3−11.2 | 9.3 | 2 | |
| A3101 | −16.2323 | 8 07 42 | −16 49 | 1928.19 | 223.9 | 3.33 | 9.0−13.0 | 9.0 | 2 | |
| A3102 | −15.2297 | 8 08 41 | −15 44 | 1929.20 | 356.9 | 1.50 | 9.3−10.0 | 9.0 | 2 | |
| A3103 | −15.2365 | 8 15 28 | −15 37 | 1928.19 | 266.0 | 0.94 | 8.5−10.2 | 8.5 | 2 | |

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ASTRONOMY

# LICK OBSERVATORY BULLETIN

NUMBER 414

## ON THE USE OF A HALF COLOR FILTER FOR THE DETERMINATION OF COLOR INDICES

BY

ROBERT J. TRUMPLER

The determination of magnitudes and color indices of stars in certain sample areas and the statistical discussion of such data furnishes at present the most promising method for studying the space distribution of the stars. In the Milky Way in particular it is necessary to extend the observations to faint stars in order to reach those distant star clouds. The determination of color indices of faint stars is therefore of considerable importance, and the method here described is intended to facilitate this difficult task.

Direct photographs taken through color filters in the primary focus of large reflectors are practically the only means for observing color equivalents of faint stars and exposures of an hour or more will be required. In such cases the principle of using two successive exposures of the same duration as being of identical effect must be abandoned. It is practically never possible to obtain uniform conditions for transparency of the atmosphere, for seeing and guiding, and the focus of the mirror as well as its optical shape will generally change during long periods of observation. Such changes not only affect the zero point of the magnitude scale but also the rate of increase in the size of the star images for successive magnitudes, and it becomes imperative to use for every long exposure a separate reduction curve for converting the estimates of image intensity into stellar magnitudes.

It is proposed to employ a filter of which one half transmits violet and ultra-violet light, the other half green and yellow light. On a photograph taken through such a filter the images in one half of the field measure the photographic magnitudes of the stars, in the other half the photovisual. A second exposure with the filter turned 180° will have the reversed arrangement; but the two exposures are quite independent and may be taken on different nights. The idea is, that, if the photographic magnitudes in the area to be investigated are known, the stars photographed through the violet half serve to establish the relation between image intensity and magnitude (reduction curve) for that peculiar exposure, and that this relation can be used also to reduce the image measures of the yellow half. By such procedure, it is true, a systematic error is introduced similar to the Purkinje effect of visual observations. This error is due to the well known fact that the contrast of a plate, or the increase in the star images for a given increase in the light intensity is not the same for all wavelengths. It stands to reason, however, that this Purkinje error, which is of the order of a few tenths of a magnitude, is independent of the conditions peculiar to each exposure such as telescopic image definition, guiding, etc. For a given filter combination, plate emulsion, and development this error can be evaluated once for all, and we shall show that if desirable it can even be determined for each plate.

With Wratten gelatine filters cemented between two thin glass plates it is easy to make a combination filter as described above. Clear glass may be used for the violet half of the filter (both halves must have the same optical thickness on account of the change in focus setting), but then, for a given exposure, the violet images would be much stronger than the yellow ones for all but extremely red stars. It seems desirable that the violet and yellow images of a star of average color (F-G type) should be of nearly equal intensity. This is accomplished by using for the violet half a filter of suitable transmission; it is, moreover, advantageous to choose this filter so that the effective wavelength of the photographic magnitudes is shifted farther toward

the ultra-violet. This will increase the scale of the measured color equivalents and thus improve the accuracy of the color determination.

Experiments made by C. J. Krieger have shown that a K2 filter for the yellow half and a 35D filter (Wratten color filters of the Eastman Kodak Company) for the violet half form a favorable combination when used with Cramer Instantaneous Iso plates. Equal image intensity is obtained for a star of type G0, the color indices are 35% larger than those of the Mount Wilson North Polar Sequence, and the exposure required for this combination is only 3 to 4 times longer than for a direct photograph without filter on Eastman 40 plates.

The method requires in the first place that the photographic magnitudes in the area to be investigated should be known. If that is not the case they may be determined for a well distributed sequence of stars in the usual way by comparison with the North Polar Sequence. The photographs required for these comparisons may be made without filter in order to shorten the exposure time. Two photographs of the area are then obtained with the combination filter: one with the violet half east (south), and one with the violet half west (north). The two should have approximately the same exposure, but do not need to be taken on the same night.

All the images yellow or violet are measured by comparison with an arbitrary scale of images; these scale readings will be designated by S. According to Schwarzschild's law we have then:

$$S=f(I.t^p)=F(m-m_o-2.5\ p\ logt)$$

where $I$ is the brightness of the star, $m$ its magnitude, $t$ the exposure time, $p$ the Schwarzschild exponent, a constant which depends on the plate emulsion and development but seems to be independent of the wavelength of the light.[1] Inverting the arbitrary function $F$ we have:

$$(1)\quad m-m_o-2.5\ p\ logt=\varphi(S)$$

The function $\varphi(S)$ is the equation of the reduction curve which relates the scale reading of each image to the magnitude of the star. For the reasons already stated, this function will be slightly different for each exposure and will differ further for the yellow and the violet images of any given half filter exposure. Using the subscript $v$ for the images taken through the yellow filter which measure the photovisual magnitude, and the subscript $p$ for the violet filter which furnishes photographic magnitudes, we have thus

$$(2)\quad \begin{aligned} m_p&=\varphi_p(S_p)+m_{op}+2.5\ p\ logt=\Phi(S_p)\\ m_v&=\varphi_v(S_v)+m_{ov}+2.5\ p\ logt \end{aligned}$$

[1] See N. W. Storer, *L. O. Bull.* **14**, 48, 1929.

The function $\Phi(S_p)$ is readily found by plotting the observed scale readings against the known photographic magnitudes for those stars of the area which were photographed through the violet filter. The reduction curve drawn through the plotted points will permit us to convert the measures $S_p$ into photographic magnitudes $m_p$. If we use the same curve for the reduction of the measures $S_v$ of the yellow images of the same exposure, we do not obtain true photovisual magnitudes, but quantities which we shall call quasi-visual magnitudes $\mu$:

$$(3)\quad \mu=\Phi(S_v)$$

From the equations (2) we find the corrections which have to be applied to these quasi-visual magnitudes in order to convert them into true photovisual magnitudes $m_v$:

$$(4)\quad m_v=\mu+[\varphi_v(S_v)-\varphi_p(S_v)]+[m_{ov}-m_{op}]$$

The first correction term is a function of the image intensity $S_v$ only; it removes the photographic Purkinje effect, and we shall designate it by $n(S_v)$. The last term is a constant $c$ which depends on the transmission of the two filters, the sensitivity of the plate, and the selective absorption of the atmosphere; it has the character of a zero correction. We can then write equation (4) in the following form:

$$(4^1)\quad m_v=\mu+n(S_v)+c$$

For the determination of the color index we need two photographs of the area with the half filter; the first, with the violet half east, designated by $'$; the second, with the yellow half east, by $''$. The color index $C$ is calculated from the measures $S'$ and $S''$ according to the following formulae:

(5) Eastern half:

$$C=m_p-m_v=\Phi'(S'_p)-\Phi''(S''_v)-n''(S''_v)-c.$$

Western half:

$$C=m_p-m_v=\Phi''(S''_p)-\Phi'(S'_v)-n'(S'_v)-c$$

$\Phi'$ and $\Phi''$ are the reduction curves of the two exposures. Both are established by means of the known photographic magnitudes of the sequence stars. If the effective wavelength of the latter is not the same as that of the violet filter images, a correction for color equation has first to be applied. This requires a knowledge of the approximate color-indices of the sequence stars; it is thus necessary to proceed in successive approximations. In the first approximation the color equation and the small Purkinje corrections $n$ are neglected and approximate values of the color indices of the sequence stars derived. By means of the latter the known photographic magnitudes of the sequence stars are reduced to the color system of the violet filter. The

reduction curves $\Phi'$ and $\Phi''$ are then redetermined in second approximation and used for the reduction of all measures according to formulae (5).

We have now to take up the determination of the Purkinje correction $n$ which removes a systematic error depending on the brightness of the star. It has been pointed out already that the function $n(S_v)$ is conditioned by the plate emulsion and its development and must be identical for two exposures taken on the same plate. Suppose we have a plate on which two exposures of the same area were made with the half filter: a long exposure which we shall designate with the subscript $l$, and a short exposure of about one tenth the duration of the former, designated with the subscript $s$. From the known photographic magnitudes of the sequence stars photographed with the violet filter the reduction curves $\Phi_l$ and $\Phi_s$ are derived separately for the two exposures, and the quasi-visual magnitudes are then determined for the stars photographed through the yellow filter according to formula (3). For each star which can be measured on both exposures we thus obtain two values for the quasi-visual magnitude:

$$\mu_l = \Phi_l(S_{vl}), \text{ and } \mu_s = \Phi_s(S_{vs})$$

while according to equation (4[1]):

$$m_v = \mu_l + n(S_{vl}) + c = \mu_s + n(S_{vs}) + c$$

Each pair of images furnishes for the determination of $n$ an equation of the form

$$n(S_{vl}) - n(S_{vs}) = \mu_s - \mu_l \tag{6}$$

From the differences in the values of the function $n$ for given arguments, the function itself can be reconstructed, except for an arbitrary constant.

If the scale readings $S$ are so chosen that they are approximately proportional to the magnitudes, then the differences $S_{vl} - S_{vs}$ are nearly equal for all stars and we can divide equation (6) by this quantity:

$$\frac{n(S_{vl}) - n(S_{vs})}{S_{vl} - S_{vs}} = \frac{\mu_s - \mu_l}{S_{vl} - S_{vs}}$$

For infinitely small intervals $S_{vl} - S_{vs}$ the left hand side would represent the differential quotient of the function $n$, and the function itself would result from numerical integration.

As $n$ is a small correction and its relation to the image intensity is not likely to be complicated, it will generally be sufficient to use for $n$ an interpolation formula of the second order:

$$n(S_v) = a.(S_v - S_o) + b.(S_v - S_o)^2 \tag{7}$$

No constant term needs to be included, as such is already taken care of by the constant $c$ in formula (4).

It is convenient to use for $S_o$ a medium value of the scale interval, but the same value should be adopted for all plates, as otherwise the zero correction $c$ of the observed color indices will slightly vary. The problem of finding the function $n$ then resolves itself into the determination of the two numerical coefficients $a$ and $b$. For this purpose we insert the interpolation formula (7) into equation (6):

$$a(S_{vl} - S_{vs}) + b.[(S_{vl} - S_o)^2 - (S_{vs} - S_o)^2] = \mu_s - \mu_l, \text{ or}$$
$$(a - 2bS_o)\ (S_{vl} - S_{vs}) + b.(S_{vl} + S_{vs})\ .\ (S_{vl} - S_{vs}) = \mu_s - \mu_l$$

Again, if the scale readings $S$ are approximately proportional to the magnitudes, we can divide by $S_{vl} - S_{vs}$, and our equation takes the simple form:

$$A + B\,\frac{S_{vl} + S_{vs}}{2} = \frac{\mu_s - \mu_l}{S_{vl} - S_{vs}} \tag{8}$$

where $A = a - 2bS_o$, $B = 2b$.

Every star photographed through the yellow half of the filter for which both the long and short exposure images have been measured furnishes such an equation, and the constants $A$ and $B$ are found either by a least squares solution or by the graphical method, and from these $a$ and $b$ are computed. While the determination of the correction $n(S_v)$ for each plate from an additional short exposure is simple enough, it will generally be found that this correction remains constant for the same instrument, material, and development. In densely crowded star fields or clusters it may be preferable to take the long and short exposure on separate plates of the same emulsion number, developing them together in the same tray.

After the corrections $n$ have been applied according to formulae (5) there remains the task of calibrating the observed color equivalents against spectral types or other color-index systems. This can only be done by observing a number of stars of known spectral type or color-index and it implies the determination of the zero correction $c$ which has to be chosen so that the color-indices of stars of spectral type $A_o$ become zero.

The star images falling into a narrow strip of the plate along the dividing line of the filter can not be utilized as it remains uncertain with which filter they were taken. If two determinations of the color indices are made, it is advisable to take the first pair of plates with the dividing line oriented north-south, the second with the dividing line east-west. It is also possible to combine a pair of half filter exposures with photographs taken through full filters of the same transmission as those selected for the half filter. The reduction curves of the full filter photographs are then established by comparison with the results of the half filter exposures.

The method here described for the determination of color-indices from photographs taken through a half color filter has the following advantages:

1. The method is differential.
2. It is economical in exposure time.
3. A separate reduction curve is derived for each individual exposure, which permits the use of long exposures.
4. The color equivalents measured are larger than the usual color-indices and are thus less affected by errors of observation.
5. Only one image of each star needs to be taken on each plate which is of special advantage in densely crowded star fields or clusters.

July 1929.

Issued August 29, 1929.

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ASTRONOMY

# LICK OBSERVATORY BULLETIN

NUMBER 415

## OBSERVATIONS OF COMET *a* 1929 (SCHWASSMANN-WACHMANN)

By S. Einarsson

The following positions were obtained from photographs with the Crossley Reflector:

| 1929 U. T. | α(1929.0) | δ(1929.0) | $\alpha^{\log p\Delta}$ | $\delta^{\log p\Delta}$ | Comparison stars |
|---|---|---|---|---|---|
| Feb. $28^d$ $6^h$ $17^m$ $30^s$ | $5^h$ $53^m$ $29^s.55$ | +22° 28′ 05″.0 | 9.529 | 0.460 | A. G. Berlin B: 2066, 2070, 2082. |
| Mar. 2 4 47 30 | 5 55 30.63 | +22 32 51.0 | 9.255 | 0.379 | Ibid.: 2099, 2101, 2115. |
| 31 5 31 30* | 6 36 17.49 | +23 14 39.2 | 9.591 | 0.492 | Ibid.: 2512, 2515, 2522. |
| Apr. 27 5 53 30 | 7 26 37.61 | +22 39 53.0 | 9.677 | 0.599 | Paris ph. +22°; $7^h$ $28^m$: 40, 70, 94. |

* Image of the comet is very faint and difficult to measure.

May, 1929.

## PHOTOGRAPHIC POSITIONS OF COMETS

By Louis Berman

The following photographic positions were obtained from plates secured with the Crossley reflector:

Comet 1927 IV (Stearns)

| 1928 U. T. | α(1928.0) | δ(1928.0) | log pαΔ | log pδΔ | Comparison stars |
|---|---|---|---|---|---|
| July $6^d$ $6^h$ $0^m$ $14^s$ | $16^h$ $24^m$ $01^s.33$ | +68° 10′ 09″.7 | 9.114 | 0.534n | Grnwch ph. +68°; $16^h$ $20^m$: 5749, 5750, 5752 |

Comet 1925 II (Schwassmann-Wachmann)

| | | | | | |
|---|---|---|---|---|---|
| Oct. $19^d$ $7^h$ $59^m$ $13^s$ | $3^h$ $41^m$ $58^s.20$ | +30° 48′ 57″.0 | 9.432n | 0.160 | Oxford ph. +30°; $3^h$ $36^m$: 6363, 6364, 6376; $3^h$ $45^m$: 6572 |
| Oct. 21 12 18 00 | 3 41 6.96 | +30 48 32.8 | 9.518 | 0.232 | Ibid. +30°; $3^h$ $36^m$: 6368, 6372, 6375, 6376 |
| Dec. 9 7 10 10 | 3 18 03.08 | +29 36 07.0 | 9.119 | 0.101 | Ibid. +29°; $3^h$ $13^m$: 7857, 7867, 7880 |

Comet 1928 III (Forbes)

| | | | | | |
|---|---|---|---|---|---|
| Nov. $23^d$ $13^h$ $35^m$ $35^s$* | $12^h$ $15^m$ $17^s.44$: | −23° 32′ 21″.2: | 9.509n | 0.847 | Cordoba A. G. −23°: 9299, 9311, 9315 |
| Nov. 24 13 43 14 | 12 17 59.66 | −24 13 32.4 | 9.598n | 0.853 | Ibid.: 9324, 9337, 9342 |
| Nov. 25 13 25 56 | 12 20 39.92 | −24 53 03.6 | 9.530n | 0.848 | Ibid.: 9357, 9364, 9366 |

* Photographed with the 4-inch Ross lens; image hardly visible and difficult to measure.

# MICROMETRICAL MEASURES OF DOUBLE STARS

By Louis Berman

The measures which follow were made during the month of July 1927 with the 12-inch refractor under good conditions of seeing. The letters "n" or "p" after the position angle readings indicate in what position the micrometer wires were held—normal or parallel respectively to the line joining the eyes. In some instances eye estimates of the relative brightness of the two stars have also been made. The right ascensions and declinations are for the equinox of 1950.0.

ξ Ursae Majoris

$\alpha = 11^h\,15^m.6$ $\delta = +31°\,49'$

4.41 4.87 G0

| | | | | | |
|---|---|---|---|---|---|
| 1927.503 | 79°.1n | 1".94 | | | |
| .506 | 79.5n | 1.78 | | | |
| .514 | 78.1n | 1.82 | | | |
| .522 | 77.7n | 1.83 | | | |
| .525 | 77.7n | 1.86 | | | |
| .530 | 78.2n | 1.86 | | | |
| 1927.52 | 78.4 | 1.85 | | | 6n |

Σ 1757

$\alpha = 13^h\,31^m.7$ $\delta = -0°\,04'$

7.8 8.9 K0

| | | | | | |
|---|---|---|---|---|---|
| 1927.506 | 90.8n | 2.61 | 7.8 | 8.6 | |
| .514 | 89.6n | 2.87 | 7.8 | 8.2 | |
| .525 | 90.2p | 2.73 | 7.8 | 8.7 | |
| 1927.52 | 90.2 | 2.74 | 7.8 | 8.5 | 3n |

Σ 1785

$\alpha = 13^h\,46^m.8$ $\delta = +27°\,14'$

7.2 7.5 K2

| | | | | | |
|---|---|---|---|---|---|
| 1927.503 | 55.4n | 1.08 | | | |
| .525 | 56.5n | 0.90 | 7.2 | 7.4 | |
| .530 | 55.8n | 1.13 | | | |
| 1927.52 | 55.9 | 1.04 | 7.2 | 7.4 | 3n |

Σ 1788

$\alpha = 13^h\,52^m.4$ $\delta = -7°\,49'$

6.7 7.9 F8

| | | | | | |
|---|---|---|---|---|---|
| 1927.506 | 85°.4p | 3".07 | 6.7 | 8.2 | |
| .525 | 85.5p | 3.03 | 6.7 | 8.0 | |
| 1927.52 | 85.5 | 3.05 | 6.7 | 8.1 | 2n |

β 106

$\alpha = 14^h\,46^m.6$ $\delta = -13°\,57'$

5.4 6.3 A2p

| | | | | | |
|---|---|---|---|---|---|
| 1927.506 | 348.5n | 1.46 | 5.4 | 5.9 | |
| .514 | 348.4n | 1.79 | | | |
| .530 | 347.8n | 1.81 | 5.4 | 6.0 | |
| 1927.52 | 348.2 | 1.69 | 5.4 | 6.0 | 3n |

Σ 1888

$\alpha = 14^h\,49^m.1$ $\delta = +19°\,19'$

4.7 6.6 G5

| | | | | | |
|---|---|---|---|---|---|
| 1927.506 | 35.5n | 3.67 | | | |
| .514 | 34.5n | 3.51 | | | |
| .530 | 33.8 | 3.57 | | | |
| 1927.52 | 34.6 | 3.58 | | | 3n |

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ASTRONOMY

# LICK OBSERVATORY BULLETIN

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# A DETERMINATION OF MAGNITUDES, SPECTRAL TYPES AND COLOR INDICES IN THE SCUTUM CLOUD WITH A STATISTICAL DISCUSSION

BY

C. J. KRIEGER

## I. INTRODUCTION

The methods which are employed for the detailed study of the density and luminosity functions vary greatly with the distances from the solar system. While the closer neighborhood, within about 100 parsecs, can be analyzed with respect to star distribution by means of parallax determinations, statistical methods are employed in the more distant parts of the local system, up to about 1,000 parsecs, and are based principally on star counts, as a function of both galactic latitude and longitude. For objects which are not members of the local system, as clusters or clouds, other methods are required, because it is outside the scope of a "Typical Stellar System" to include such formations, which are now believed to be of the same general order as our local system and to arrange themselves beyond the limits of the latter. Easton[1] pointed out the peculiar position of the galactic clouds and likened them to the condensations in the spiral nebulae. However, the distances of these clouds were not known. Pannekoek,[2] under the assumption that the luminosity function in the clouds is of the form of a normal frequency function, determined a distance of 40,000 parsecs for the bright *Cygnus* patch, and 60,000 parsecs for the *Aquila* stream. A. Kopff[3] pointed out why this assumption cannot be made, and that it leads to values which are too large. His estimate for the *Cygnus* cloud was 4,000 to 6,000 parsecs.

Evidently, different methods are required and one in particular has been applied which is based on the relatively small dispersion in absolute magnitude of a given spectral class. Malmquist[4] drew attention to the necessity of including the spectral type in statistical discussions of the distribution of the stars, and he also developed a general method. Seares[5] derives these provisional results:

| | |
|---|---|
| *Taurus* cloud | 7,000 parsecs |
| *Scutum* cloud[6] | 7,000 parsecs |
| *Perseus* cloud | 8,000 parsecs |
| *Aquila* cloud | 14,000 parsecs |

A. Kopff[7] suggested approximately equal distances for the *Scutum* and *Cygnus* clouds and, based upon observations by Shapley[8] of the color indices of faint stars in *Scutum*, assigned an apparent magnitude of 14.0 to the stars of color-type *a* and *f*, and with $M=-4$ to $-5$ finds $d=4{,}000$ to 6,000 parsecs. E. A. Kreiken[9], by the same general method, after having determined the colors by means of effective wavelengths, found the distance of the *Scutum* cloud as 1,500 parsecs. K. G. Malmquist[10] deduced from the magnitudes and colors of the white stars that it extends from 1,800 to 5,000 parsecs with maximum density at 3,400 parsecs. In view of this wide range in the value of the distance of an individual cloud (from 1,500 to 7,000 parsecs) it was considered desirable to undertake a detailed study of the characteristics of the *Scutum* cloud with

[1] *Ap. J.* **12**, 136, 1900.
[2] *M. N.* **79**, 500, 1919.
[3] *A. N.* **216**, 325, 1922.
[4] Charlier Festskrift, *Lund. Medd. II*, Nr. 46, 1927.
[5] Report, Mt. Wilson, 1921, pp. 228 and 264 of Carnegie Inst. Year Book No. 20.
[6] Scutum or Clypeus Sobieskii, Sobieski's shield. One of the constellations introduced by Johannes Hevelius at Danzig, who lived 1611–1687.
[7] *A. N.* **216**, 331, 1922.
[8] *Ap. J.* **46**, 64, 1917.
[9] *Obs.* **47**, 130, 1924.
[10] *Lund. Medd.* Ser. II, Nr. 46, 1927.

the instruments at the Lick Observatory. As a result of the study it was expected to obtain the relation between color index and magnitude, distance and dimensions of the cloud, and material for a discussion of the luminosity and density functions.

Briefly, the method consists in the determination of the photographic magnitudes and spectral types of all stars in a given area up to a certain limiting magnitude. While the spectral types of the brighter stars up to about 13th magnitude can be obtained directly from slitless spectrograms, those of the fainter stars are determined by means of color indices secured with color filters. These data will permit a detailed statistical discussion.

Accordingly, six circular areas, 15′ in diameter and well distributed in the *Scutum* cloud (see figure 1) were selected, and a comparison of each area with the standard stars of the North Polar Sequence, a four hour slitless spectrogram, and exposures of the field with color filters for obtaining color indices were secured. The comparison with the North Polar Sequence furnishes the photographic magnitudes of a number of standard field stars, by means of which the photographic magnitudes of all stars can be determined. For most areas only one Crossley reflector comparison with the North Polar Sequence is available, and a Crocker telescope comparison of the entire *Scutum* cloud with the North Polar Sequence affords a check.

The long exposure spectrograms furnish the spectra of a comparatively small number of bright stars to about 13th magnitude, and the extension of the color determination to fainter magnitudes was made by color filter exposures. The method is differential and consists in measuring the intensity of images obtained with a violet and yellow filter in quick succession with full-filters, or simultaneously with a half-filter, in terms of the same gradation curve. The difference in intensity of the two images is an indication of the spectral type, and the calibration of the color indices is made by means of those stars of which we have both spectral types and color indices. Precautions will be necessary regarding the dependency of the color index on the brightness of the image, the effect of different material and instruments on the color equation, consistency of magnitudes within themselves, zero point corrections, etc. Photographic magnitudes to $15^{m}.5$ can be reached in ten minutes with the Crossley Reflector and with the plates and filter selected. The study was extended to stars of 18th photographic magnitude, requiring two-hour exposures. For the stars from $11^{m}.0$ to $15^{m}.5$ six areas of 15′ diameter were used, while on account of the increasing star numbers a small portion of one standard area furnished sufficient star material from $14^{m}.0$ to $18^{m}.0$.

## II. Instruments, Material and Observations

### (a) *Instruments*

The principal instrument employed for securing the observations was the 36½-inch Crossley reflector of the Lick Observatory, the focal length being 17½ feet. Ordinary photographs with or without filters were taken with the plate in the primary focus. The plate holder attachment has a carriage movable in both coordinates for guiding and the guide wires are connected to the movable carriage. Spectrograms were obtained with the two-prism slitless quartz spectrograph[11] also attachable to the Crossley reflector. Minimum deviation is at λ3600 and the dispersion is 7.4 mm between $H\beta$ and $H\epsilon$. The useful field is roughly circular and about 20′ in diameter. The Crocker refractor with the 4-inch Ross lens was used for obtaining two plates of the *Scutum* cloud. The focal length is 60 inches.

### (b) *Material*

For the determination of color indices of faint stars it is desirable to have the intensities of images in light of two widely separated regions of the spectrum, preferably the ultraviolet and extreme red, in order to obtain large color indices. This would, however, necessitate very long exposure times due to the low sensitivity of the photographic plate at the extreme end of the color range, and it was decided to employ filters which would place one point in the violet region, near λ4000, and the second point near λ5500 in the yellow-green region. The transmission properties of the 35D, and K2 Filters are shown in figures 8 and 35 of the Filter Catalogue of the Eastman Kodak Co.

As the method contemplated the use of both whole and half filters, the following material was secured:

(1) K2 full filter, 3x3 in., cemented in glass (was available at the observatory).

(2) 35D full filter, 3x3 in., cemented in glass (ordered from Eastman Kodak Co.).

(3) Half-filter, 3x3 in., consisting of 3x1½ in., K2 filter and 3x1½ in., 35D filter cemented side by side in glass (ordered from Eastman Kodak Co.).

(4) 50 Dummy Filter 3x3 in., cemented in glass (was available at the Observatory).

The last filter was used for focussing, because it has the same optical path without the disadvantage of the low transparency of the colored filters. The focussing was done by means of Foucault's knife edge test.

[11] W. H. Wright, *L. O. B.* **9**, 52, 1917.

A preliminary investigation was made of the type of plate which would be very sensitive both to violet and yellow light.

From the plates available at the time of the tests, the Cramer Isochromatic Instantaneous Plates were selected as most sensitive in the violet and yellow light. A second Cramer plate, the Isochromatic Presto Plate, came to our attention later and showed a slightly (about 10%) higher sensitivity than the Cramer Isochromatic Instantaneous Plate. It was not included in the preliminary tests.

The plates were developed for 5 minutes at 70° F in Eastman D-11 developer which gives great contrast and has good keeping qualities.

An acid fixing bath (Eastman Formula F1) with added hardening solution was used, and care was taken to wash the plates thoroughly.

### (c) *Observations*

The *Scutum* cloud is a fairly large cloud, covering an area of approximately six degrees by eight degrees. Six sampling areas, numbered from A to F, were selected within the cloud, well distributed, and in places where the star density is fairly uniform, and not affected by dark nebulae or star clusters. The centers of the areas are:

| Area | Central Star | α(1900.0) | δ(1900.0) | Mag |
|---|---|---|---|---|
| A | A118 AG6313, BD−6°4869 | 18ʰ39ᵐ18ˢ.24 | −6°38′15″.0 | 7.5 |
| B | B110 AG6388, BD−8.4733 | 18 45 47.67 | −8 7 37.3 | 7.4 |
| C | C 77 AG6412, BD−9.4876 | 18 47 32.09 | −9 41 50.2 | 6.5 |
| D | D143 | 18 51 7.00 | −6 17 1.0 | 13.7 |
| E | E 90 AG6513, BD−6.5005 | 18 56 5.70 | −6 20 2.1 | 7.0 |
| F | F114 AG6301, BD−8.4686 | 18 38 4.47 | −8 22 26.5 | 5.0 |

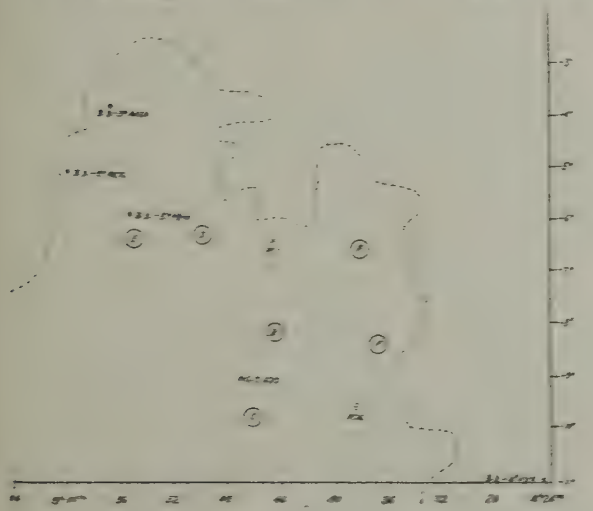


Fig. 1. Scutum cloud.

Figure 1 shows the relative positions of the six areas.[12]

[12] From E. E. Barnard, *A Photographic Atlas of Selected Regions of the Milky Way*, Carnegie Inst. of Washington, 1927, Chart 37.

On Crossley plates good definition is obtained within a radius of about 15′ to 20′ from the plate center, but in order to permit neglecting the effect of the distance from center on the magnitude, a circular area of 15′ diameter was chosen. As centers for the smaller areas on the 2-hour exposures stars A103 and A114 of the 10 min. exposure numbering system were selected which are identical with stars A2135 and A2485, respectively, of the 2-hour exposure numbering system, and all stars within the two circular areas, the one of 4¾′, and the other of 4′ diameter, were measured.

The observations fall into three classes according to the purposes for which they were obtained:

(1) Slitless spectrograms for the determination of the spectral types of the brighter stars.

(2) Comparison of the field with the stars of the North Polar Sequence for the determination of the photographic magnitudes of the standard stars.

(3) Exposures of the field with color filters for obtaining the photographic magnitudes of all stars and the color indices.

One spectrogram was taken of each area with an exposure time of 4 hours, and in addition a 15-minute spectrogram to permit classification of the brightest stars, which were overexposed on the 4-hour exposures. The comparison of the field with the North Polar Sequence consisted as a rule in consecutively exposing 10 minutes and 1 minute on the field, swinging the telescope around to the North Pole, and then exposing 1 minute and 10 minutes on the North Polar Sequence, or in exposing in reversed order. Between successive exposures the plate holder carriage was moved in declination about 2 mm, so that two separate images of the brighter stars were obtained. Of the fainter stars, only one image, i.e. the 10-minute exposure, is visible. Cramer Isochromatic Instantaneous Plates and a Wratten 35D violet filter were used. The time elapsing between the exposures of the field and the North Polar Sequence was about 20 to 30 minutes, this being the time required to turn the dome, bring the telescope into position and locate the star field. For a determination of the photographic magnitudes of the faint stars on the 2-hour color index plates of Area A, 2-hour comparisons with the North Polar Sequence would be necessary if the Cramer Plate and the 35D violet filter were used. It was considered advisable to make instead 30-minute comparisons with Eastman 40 Plates and no filter. This method will reach practically the same magnitudes, but has the disadvantage of a different color equation, which, however, can be determined and allowed for. The advantage lies in the reduction of the time elapsing between the exposures of the area and of the North Polar

Sequence. These exposures were supplemented by 2-minute comparisons with the North Polar Sequence, using Eastman 40 Plates and no filter. In order to eliminate the effect of atmospheric extinction, the exposures of the areas were taken at the altitude of the pole. A 30-minute comparison of the whole *Scutum* cloud with the North Polar Sequence was taken with the Crocker-Ross telescope, Eastman 40 Plate and no filter.

With full-filter color index determinations the arbitrary assumption is introduced that the exposures of the violet and yellow images are obtained under the same conditions of atmospheric transparency, focussing and guiding. Differences in these conditions will change the shape of the gradation curve, particularly at the faint end, and the use of the photographic gradation curve for the reduction of the yellow images will introduce a factor of uncertainty. This can, however, be eliminated by the use of a half-filter, as suggested by Dr. R. J. Trumpler[13]. The violet image on one half of the plate, and the yellow image on the other half of the plate, are obtained simultaneously under exactly the same atmospheric, focussing and guiding conditions. The gradation curve determined for the violet images can be employed for the reduction of the yellow images, and the only adjustment required is the elimination of the Purkinje-Effect, and, if desired, a zero-point shift to make the photovisual magnitude of an *Ao* star equal to its photographic magnitude. While both the full-filter and half-filter methods were applied to the short exposures up to 10 minutes, the half-filter method only can be used for the 2-hour exposures, due to the widely different observing conditions existing, either when successively exposed, or on two different nights. The full-filter 2-hour exposures were employed to reduce the systematic and accidental errors in the magnitudes and color indices obtained from the half-filter exposures. The full-filter color index plates were obtained by exposing 10 minutes and 1 minute with the K2 full-filter, changing the filter and exposing 1 minute and 10 minutes with the 35D full-filter. In the case of the half-filter color index plates, the procedure was similar, the half-filter being used in one position during the first 10-minute and 1-minute exposures, then, having been turned through 180°, used again in the second 10-minute and 1-minute exposures. Since the plate holder carriage was also moved between exposures, a group of four images is obtained, but this time the sequence of images of different wavelengths is reversed in the two halves of the plates.

Since neither the comparisons with the North Polar Sequence, which have two fields on one plate, nor the color index plates which show from 1 to 4 images of each star, are well suited for identification purposes, 10-minute exposures of the fields, with Eastman 40 Plates and no filter, were secured. The East-West line was found by unclamping the telescope and allowing the field to drift across the plate. A list of observations is attached. (Table 1.)

[13] *L.O.B.* **14**, 89, 1929.

## III. Measurement and Reduction of the Observations

### (a) *Measurement of Star Images*

For the measurement of the intensity of images there are two methods in use, one of which is based on measuring the diameter of focal images, and the other on the blackening of extra-focal images. The former will reach fainter stars, because the light is concentrated in one point, and it was adopted. A modified Hartmann microphotometer is available at the Lick Observatory which permits the measurement of focal images by comparison with a series of focal scale images. The scales were obtained by making successive exposures, with small intermediate shifts, in an exposure ratio of $\sqrt{3} : 1$, and there were, as a rule, 10 images which could be used. They were numbered in the order of decreasing diameters as 10.0, 10.5, 11.0 . . . . . to 14.5, and designated as scale readings. It was in practically all cases possible to estimate the diameter to one-fifth of the difference between successive images, or one-tenth of a scale reading. This corresponds to about one-tenth of a magnitude.

A reseau is used to define the area to be measured and consists of parallel lines and two concentric circles, the smaller one (I) having a linear diameter of 23.0 mm corresponding to 14′ 42″, and the larger one (II) having a diameter of 30.8 mm corresponding to 19′ 42″. Accordingly the area of Circle I is 0.0471 square degree, and of Circle II, 0.0847 square degree. Within Circle I, all stars visible on the 10-minute exposures were measured. Stars lying between Circles I and II and visible on the 1-minute exposures were also measured and indicated in the star lists by an * following the star number. In the case of the 2-hour exposures, two smaller circular areas, one south and the other north of the plate center, (see figure 2, plate I) were selected, and all stars visible within them measured. The diameter of the southern circle is 283″.5 and its area 0.00486 square degree, the diameter of the northern circle is 237″ and its area 0.00341 square degree. The total area in which 10-minute exposures have been completely measured is therefore 0.2826 square degree, and the combined area in which 2-hour exposures are completely measured is 0.00827 square degree.

TABLE 1

LIST OF OBSERVATIONS

| Area | No. | Date (1928) | Description | Plate | Filter | Exposure |
|---|---|---|---|---|---|---|
| A | 130 | Oct. 19 | Pg. Mg. | Cramer | 35D full | Field and N.P.S. two 1 min. exp. each |
| A | 93 | Sept. 18 | Pg. Mg. | E40 | 50 dummy | Field and N.P.S. 2 min. each |
| A | 92 | Sept. 18 | Pg. Mg. | E40 | 50 dummy | Field and N.P.S. 30 min. each |
| A | 82 | Sept. 15 | Pg. Mg. | E40 | ——— | Field and N.P.S. 30 min. each |
| A | 61 | Aug. 19 | Pg. Mg. | Cramer | 35D full | 2 hours |
| A | 64 | Sept. 5 | Pv. Mg. | Cramer | K2 full | 2 hours |
| A | 113 | Oct. 9 | Pg. and Pv. Mg. | Cramer | 35D-K2 half | 1 hour 57 min. |
| A | 111 | Oct. 4 | Pg. and Pv. Mg. | Cramer | K2-35D half | 1 hour 51 min. and 10 min. |
| A | 69a | Sept. 7 | Color index | Cramer | K2 full | 10 min and 1 min. |
| | | | | | 35D full | 10 min. and 1 min. |
| A | 119 | Oct. 16 | Color index | Cramer | K2-35D half | 10 min. and 1 min. |
| | | | | | 35D-K2 half | |
| A | 47 | Aug. 9 | Slitless spectrograms | E40 | ——— | 4 hours |
| A | 106 | Sept. 25 | Slitless spectrograms | E40 | ——— | 15 min. |
| B | 129 | Oct. 19 | Pg. Mg. | Cramer | 35D full | Field and N.P.S. 10 min. and 1 min. each |
| B | 89 | Sept. 16 | Color index | Cramer | K2 full | 10 min. and 1 min. |
| | | | | | 35 D full | 10 min. and 1 min. |
| B | 123 | Oct. 17 | Color index | Cramer | K2-35D half | 10 min. and 1 min. |
| | | | | | 35D-K2 half | 10 min. and 1 min. |
| B | 6 | July 15 | Slitless spectrograms | E40 | ——— | 4 hours |
| C | 87 | Sept. 16 | Pg. Mg. | Cramer | 35D full | Field and N.P.S. 10 min. and 1 min. each |
| C | 96 | Sept. 19 | Pg. Mg. | Cramer | 35D full | Field and N.P.S. 10 min. and 1 min. each |
| C | 120 | Oct. 16 | Color index | Cramer | K2-35D half | 10 min. and 1 min. |
| C | 126 | Oct. 18 | Color index | Cramer | K2-35D half | 10 min. each without and with diaphragm |
| | | | | | 35D-K2 half | 10 min. each without and with diaphragm |
| C | 8 | July 16 | Slitless spectrograms | E40 | ——— | 4 hours |
| C | 108 | Sept. 25 | Slitless spectrograms | E40 | ——— | 15 min. |
| D | 118 | Oct. 15 | Pg. Mg. | Cramer | 35D full | Field and N.P.S. 10 min. and 1 min. each |
| D | 98 | Sept. 20 | Color index | Cramer | K2 full | 10 min. and 1 min. each |
| | | | | | 35D full | 10 min. and 1 min. each |
| D | 127 | Oct. 18 | Color index | Cramer | K2-35D half | 10 min. each without and with diaphragm |
| | | | | | 35D-K2 half | 10 min. each without and with diaphragm |
| D | 125 | Oct. 17 | Color index | Cramer | K2 full | 10 min. and 1 min. |
| | | | | | 35D full | 10 min. and 1 min. |
| D | 12 | July 19 | Slitless spectrograms | E40 | ——— | 4 hours |
| D | 10 | July 17 | Slitless spectrograms | E40 | ——— | 1 hour 10 min. |
| E | 97 | Sept. 19 | Pg. Mg. | Cramer | 35D full | Field and N.P.S. 10 min. and 1 min. each |
| E | 94 | Sept. 18 | Color index | Cramer | K2 full | 10 min. and 1 min. |
| | | | | | 35D full | 10 min. and 1 min. |
| E | 124 | Oct. 17 | Color index | Cramer | 35D-K2 half | 10 min. and 1 min. |
| | | | | | K2-35D half | 10 min. and 1 min. |
| E | 122 | Oct. 16 | Color index | Cramer | 35D-K2 half | 10 min. and 1 min. |
| | | | | | K2-35D half | 10 min. and 1 min. |
| E | 51 | Aug. 10 | Slitless spectrograms | E40 | ——— | 4 hours |
| E | 107 | Sept. 25 | Slitless spectrograms | E40 | ——— | 15 min. |
| F | 99 | Sept. 20 | Pg. Mg. | Cramer | 35D full | Field and N.P.S. 10 min. and 1 min. each |
| F | 91 | Sept. 18 | Color index | Cramer | K2 full | 10 min. and 1 min. |
| | | | | | 35D full | 10 min. and 1 min. |
| F | 121 | Oct. 16 | Color index | Cramer | 35D-K2 half | 10 min. and 1 min. |
| | | | | | K2-35D half | 10 min. and 1 min. |
| F | 53 | Aug. 12 | Slitless spectrograms | E40 | ——— | 4 hours |
| F | 105 | Sept. 25 | Slitless spectrograms | E40 | ——— | 15 min. |
| All areas | RT1728† | Oct. 15 | Pg. Mg. | E40 | ——— | Field 30 min., N.P.S. twice 30 min. |

† This plate was very kindly taken for me by Dr. R. J. Trumpler with the Crocker-Ross telescope.



17607

(b) *Photographic Magnitudes of Standard Stars*

On each plate used for a determination of the photographic magnitudes two different fields are impressed:

(1) The area to be investigated
(2) The North Polar Sequence stars

In order to distinguish the two fields with certainty, an enlargement of a plate, showing the area only, was made. About 50 field stars, well distributed over the area and in the magnitude scale, were selected as standard stars, and about 20 stars of the North Polar Sequence near the plate center were picked out and their diameters expressed in readings of a measuring scale which was selected from several available so that the general character of the scale images as to diffuseness was the same as that of the stars to be measured.

The photographic magnitudes of the North Polar Sequence[14] are on a color system, from which ours, due to the use of a violet filter and isochromatic plate, will necessarily differ. The color equation, i.e. the color index of a K0 star with our instrumental arrangement, has to be determined next. If we plot the I. A. U. photographic magnitudes against the scale readings on 35D Cramer plates, we find that red stars fall below the line marked out by A0 stars. By plotting the vertical distance of a red star from the curve, against the known color index, the color equation can be determined. Since the measured scale readings are affected by accidental errors, the vertical distances of a number of North Polar Sequence stars are measured on six different plates, and the averages taken for each star. The relation is well represented by a linear equation of the form:

$$m_D = m_I + 0.32C_I,$$

where $m_D$ is the photographic magnitude with 35D Filter and Cramer Isochromatic Plates, $m_I$ is the photographic magnitude on I. A. U. System, and $C_I$ is the color index on I. A. U. System. New photographic magnitudes for the North Polar Sequence stars on the 35D-Cramer color system can be computed. All photographic magnitudes contained in the star catalogues in Chapter IV are on this color system. Next, the effect of differential photographic extinction, since the field exposures were taken at altitudes differing slightly from the altitude of the pole, has to be eliminated. Table 2 shows the differential extinction corrections of the comparison plates. The photographic extinction at the altitude of the pole on Mt. Hamilton was interpolated from Müller's *Extinction Table*[15] and the photographic extinction assumed as twice the visual extinction.

[14] *Trans., I. A. U.*, I, 69, 1922.
[15] Scheiner-Graff, *Astrophysik*, p. 104, 1922.

TABLE 2

TABLE OF DIFFERENTIAL PHOTOGRAPHIC EXTINCTION CORRECTIONS

| Area | Plate | Exposure in minutes | | Correction in magnitudes |
|---|---|---|---|---|
| A | 82 | 30m | Crossley | 0m00 |
| A | 92 | 30 | Crossley | +.01 |
| A | 93 | 2 | Crossley | −.03 |
| A | 130 | 1 | Crossley | −.02 |
| A | RT1728 | 30 | Crocker | −.01 |
| B | 129 | 1 | Crossley | .00 |
| B | RT1728 | 30 | Crocker | −.01 |
| C | 96 | 10 | Crossley | .00 |
| C | 96 | 1 | Crossley | +.01 |
| C | RT1728 | 30 | Crocker | −.03 |
| D | 118 | 10 | Crossley | −.10 |
| D | 118 | 1 | Crossley | −.10 |
| D | RT1728 | 30 | Crocker | +.01 |
| E | 97 | 10 | Crossley | +.01 |
| E | 97 | 1 | Crossley | .00 |
| E | RT1728 | 30 | Crocker | +.02 |
| F | 99 | 10 | Crossley | .00 |
| F | 99 | 1 | Crossley | .00 |
| F | RT1728 | 30 | Crocker | +.03 |

If we now plot the photographic magnitudes of the North Polar Sequence on a comparison plate against the scale reading, the photographic magnitudes of the standard stars of the field can be read off (see figure 3).

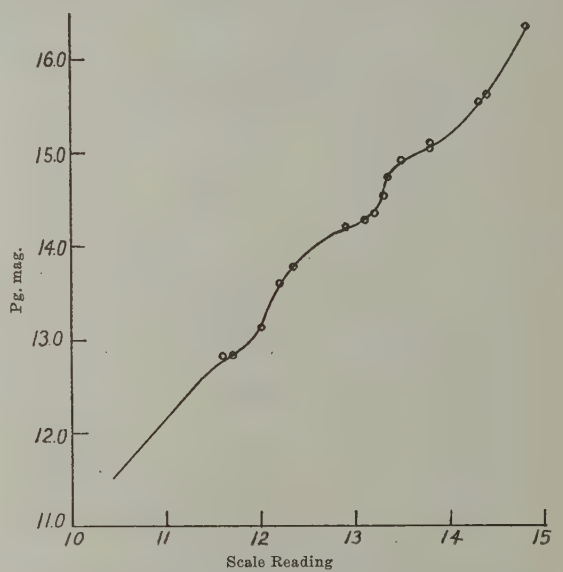



Fig. 3. Comparison of Area C with North Polar sequence.

This process is carried out for each of the six areas, and a check is afforded by the comparison plate of the whole *Scutum* cloud with the North Polar Sequence made with the Crocker-Ross telescope. The color equation with this instrument and Eastman 40 Plates was known to be $m_K = m_I - 0.10C_I$ and the photographic magnitudes of a number of Crossley standard stars to about 14th magnitude can be determined.

The transition from the Crocker (E40) to the Crossley (35D, Cramer) color system is given by the relation $m_D = m_K + 0.42C_I$ by comparing the two relations given above. However, the color index of the standard stars must already be known approximately, because $C_I$ enters into the equation, and therefore a preliminary solution for the color index (I. A. U. system) of the standard stars must be made first.

The magnitudes corrected for color equation and differential extinction should now be comparable, except for accidental errors and zero point corrections. The following mean differences were determined:

(1) Crossley 10 min. exposure minus Crocker 30 min. exposure

(2) Crossley 1 min. exposure minus Crocker 30 min. exposure

(3) Crossley 10 min. exposure minus Crossley 1 min. exposure

| Area | (1) | (2) | (3) | Mean of (1) and (2) |
|---|---|---|---|---|
| A | ..... | $+^m.17$ | ..... | $+^m.17 = L_a$ |
| B | ..... | $+.30$ | ..... | $+.30 = L_b$ |
| C | $+^m.34$ | $+.26$ | $+^m.08$ | $+.30 = L_c$ |
| D | $+.28$ | $+.24$ | $+.12$ | $+.26 = L_d$ |
| E | $+.03$ | $(-.55)$ | $(+.56)$ | $+.03 = L_e$ |
| F | $+.31$ | $+.09$ | $+.21$ | $+.20 = L_f$ |

In order to find the most probable corrections to be applied to the magnitudes determined in different ways, this method was used:

(1) Each individual comparison with the North Polar Sequence has a certain probability of giving the correct magnitudes. Weight 1 was given to a single (1 minute) Crossley comparison, weight 1½ to the mean of a 10-minute and 1-minute Crossley comparison, and weight 2 to the 30-minute Crocker comparison.

(2) The differences between the mean of the 10- and 1-minute Crossley comparisons, and the 30-minute Crocker comparisons, i.e. $L_a$, $L_b$, $L_c$, $L_d$, $L_e$, $L_f$, are used in solving for the residuals of the mean of the 10-minute and 1-minute Crossley comparisons, $a$, $b$, $c$, $d$, $e$, $f$, and for the residual $k$ of the 30-minute Crocker comparison.

The 1-minute Crossley comparison of Area $E$ with a large negative residual was not included in the solution, since this is probably due to wrong exposure time or other systematic error. To each of the differences $(a\text{-}k)$, $(b\text{-}k)$, $(c\text{-}k)$ . . . . . $(f\text{-}k)$ weight 2 was given and the 13 equations of condition become:

| a | b | c | d | e | f | k | L | Weight |
|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | 0 | 1 |
| | 1 | | | | | | 0 | 1 |
| | | 1 | | | | | 0 | 1½ |
| | | | 1 | | | | 0 | 1½ |
| | | | | 1 | | | 0 | 1 |
| | | | | | 1 | | 0 | 1½ |
| | | | | | | 1 | 0 | 2 |
| 1 | | | | | | −1 | $L_a$ | 2 |
| | 1 | | | | | −1 | $L_b$ | 2 |
| | | 1 | | | | −1 | $L_c$ | 2 |
| | | | 1 | | | −1 | $L_d$ | 2 |
| | | | | 1 | | −1 | $L_e$ | 2 |
| | | | | | 1 | −1 | $L_f$ | 2 |

A solution for the seven unknowns gives:

$$a = +^m.01 \quad b = +^m.10 \quad c = +^m.08 \quad d = +^m.06$$
$$e = -^m.08 \quad f = +^m.02 \quad k = -^m.15$$

By allowing for the calculated differences between 10-minute and 1-minute exposures, and transforming residuals into corrections through reversing the signs, the zero corrections become:

| Area | 10 min. Exp. | 1 min. Exp. |
|---|---|---|
| A | ..... | $-^m.01$ |
| B | ..... | $-.10$ |
| C | $-^m.12$ | $-.04$ |
| D | $-.14$ | $-.02$ |
| E | $+.08$ | $+.65$ |
| F | $-.13$ | $+.09$ |
| Crocker (30 min.) | $+.15$ | |

The extinction and zero corrections to the uncorrected photographic magnitudes are applied, and the means between 10-minute and 1-minute Crossley and 30-minute Crocker exposure magnitudes are taken. These values constitute the adopted photographic magnitudes of the standard stars.

The magnitudes of the fainter standard stars up to 18th magnitude in Area $A$ are determined in the same general way. Two 30-minute and one 2-minute Crossley comparison with the North Polar Sequence on Eastman 40 Plates without filter were available, and it was found that the color equation was not different from that of the I. A. U. The magnitudes were expressed in the 35D-Cramer Color System, corrected for extinction, and the means taken. There is a range from about 14th to 16th magnitude in which a considerable number of stars of Area $A$ is determined both on the long and the short exposures. A small difference was found in the magnitudes of these stars when determined from the long, and from the short exposures. This was partly due to the shape of the reduction curve determined from the 1-minute comparison. The magnitude determinations for the long exposure of Area $A$ have greater weight, and it was

decided to apply the differences as corrections to the previously adopted photographic magnitudes of the short exposure. After this adjustment the short and long exposures of Area *A* were consistent.

The final step is the determination of the photographic magnitudes of the remaining field stars by setting up the magnitude-scale reading curve for each violet filter exposure, and by reading off the magnitude values corresponding to the measured scale readings.

(c) *Determination of the Color Indices*

It would be possible to derive the photovisual magnitudes in the same manner as the photographic magnitudes, by comparing the field with the photovisual magnitudes of the North Polar Sequence stars, using a yellow filter. This would, however, increase the number of north polar comparisons, and their reductions. Also the long exposure time of two hours would make the value of the magnitudes very doubtful, due to the variation in the observing conditions in a 4-hour period. Further the difference between the two independently determined magnitudes, i.e. the color index, would be greatly affected by systematic errors. For these reasons the differential method of determining the color index was chosen. On the full-filter color index plates the intensity of both the violet and yellow images was measured in terms of the same measuring scale. The gradation curve for the photographic magnitudes was derived from the standard field stars and also used for the reduction of the intensity of the yellow images. "Quasi-photovisual" magnitudes are thus obtained, which, when deducted from the photographic magnitudes, furnish color indices giving an indication of the spectral class, if properly calibrated.

The procedure in the case of half-filter color index plates is similar, but two gradation curves are here used. The first is derived from the photographic magnitudes in the eastern half of the plate, and the second one from the photographic magnitudes in the western half of the plate. The first curve is used for deriving the quasi-photovisual magnitudes in the western half of the plate, and the second for deriving those in the eastern half of the plate. It will be necessary to investigate the existence of, and if present to correct for, a photographic Purkinje Effect. It is well known that the sensitivity of the human eye for colors is different depending on the intensity of illumination of the object. Blue will appear brighter to the eye than red in semi-darkness, while the same red will appear brighter in daylight. A similar effect was observed with the isochromatic plate. While the color index derived from the 10-minute violet and 10-minute yellow images should be the same as that derived from the 1-minute violet and 1-minute yellow images on the same plate, there was found a difference. Let the 1-minute color index be $c_1 = m_{pg1} - m_{pv1}$, and let the 10-minute color index be $c_{10} = m_{pg10} - m_{pv10}$. For the same star the two should be equal:

$$c_1 = c_{10} = C$$

But since they are different we apply small corrections $n$ to the color indices $c_1$ and $c_{10}$, which should bring about the equality:

$$c_1 + n_1 = c_{10} + n_{10} = C$$

The corrections are due to the difference between the gradation curves for the violet and yellow images, for whose reduction we have used the same, i.e. the photographic, gradation curve. The photographic magnitudes determined from 10-minute and 1-minute exposures are, except for accidental errors, equal. Consequently the correction to the photovisual magnitude equals the negative correction to the color index:

$$c + n = (m_{pg} - m_{pv}) + n = m_{pg} - (m_{pv} - n)$$

It will be assumed that $n$ is only a function of the intensity of the yellow image, as measured by the scale reading $S$ and this function $n = F(S)$ is now to be determined. In order to find the character of the dependency, the equations are transformed thus:

$$c_{10} - c_1 = n_1 - n_{10}$$

By dividing with the difference between the 1-minute and 10-minute photovisual scale readings, the ratio of these two differences expresses the change of the color index per scale reading, or the change of the correction $n$ per unit scale reading:

$$\frac{c_{10} - c_1}{S_1 - S_{10}} = \frac{n_1 - n_{10}}{S_1 - S_{10}}$$

If these differences were infinitely small, we would have the derivative of $n$ with respect to scale reading:

$$\frac{dn}{dS} = f(S), \quad n = \int f(S)\, dS + a$$

and by expressing it in terms of $S$ and integrating with respect to it, $n = F(S)$ would be obtained. However, in our problem the differences are finite, e.g. $(S_1 - S_{10})$ equals about two scale readings, and the solution has to be made by treating the differences as such. For a number of stars which have photographic and photovisual images on both the 10-minute and 1-minute exposures, the difference ratios

$$\frac{c_{10} - c_1}{S_1 - S_{10}}$$

are formed, and plotted against the average of the photovisual scale readings $\frac{1}{2}(S_1 + S_{10})$. Since the ordinate $\frac{c_{10} - c_1}{S_1 - S_{10}}$ equals $\frac{n_1 - n_{10}}{S_1 - S_{10}}$ the differences between the

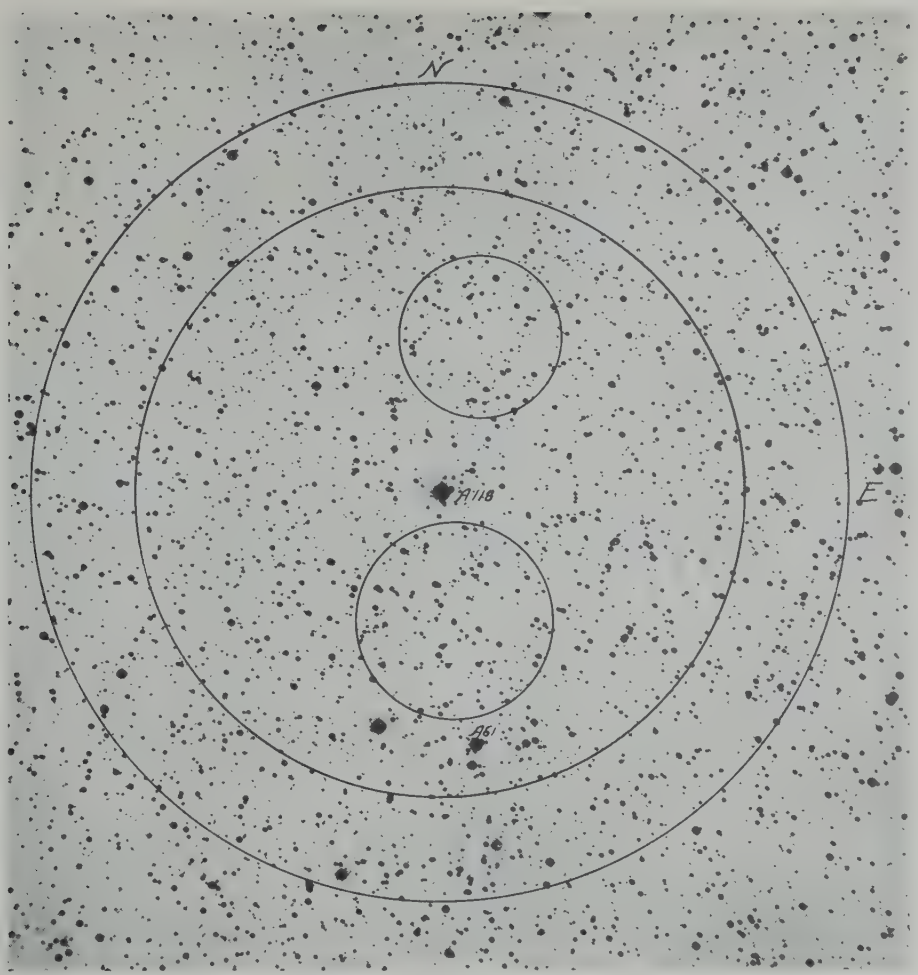



Fig. 2. Enlargement of two-hour exposure of Area A.

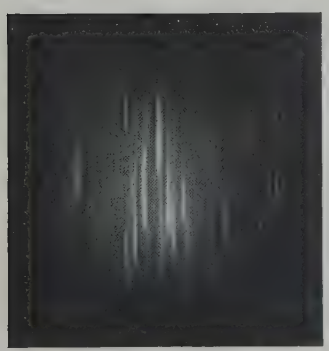


Fig. 5. Slitless Spectrogram of Area A.
Four-hour exposure.

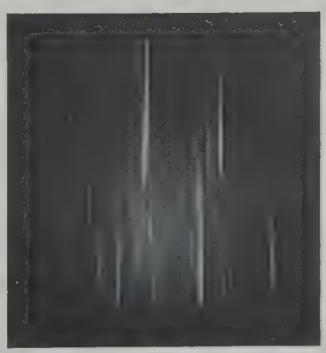


Fig. 6. Slitless Spectrogram of Area E.
Four-hour exposure.

$n$ at different scale readings are known, and $n$ can be found as a function of $S$. In most cases $\frac{n_1-n_{10}}{S_1-S_{10}}$ was a constant, which means that $n=F(S)$ is a straight line, with an angle of inclination $\alpha$ to the $S$-axis whose tangent equals that constant.

The following table shows the values of $\frac{n_1-n_{10}}{S_1-S_{10}}$ as determined from the various color index plates:

| Area | Exp. | Plate | Filter | $\frac{n_1-n_{10}}{S_1-S_{10}}$ |
|---|---|---|---|---|
| A | $10^m$ | 69a | Whole | $+0^m08$ |
| A | 10 | 119 | Half, east | +0.09 |
| A | 10 | 119 | Half, west | +0.11 |
| A | $2^h$ | 111 | Half, east and west | [ 0 ] |
| B | $10^m$ | 89 | Whole | −0.01 |
| B | 10 | 123 | Half, east | −0.06 |
| B | 10 | 123 | Half, west | +0.12 |
| C | 10 | 120 | Half, east | +0.20 |
| C | 10 | 120 | Half, west | +0.03 |
| C | 10 | 126 | Half, east | +0.05 } Diaphragm |
| C | 10 | 126 | Half, west | +0.21 } |
| D | 10 | 98 | Whole | +0.13 $(s-10.0)-0.08$ |
| D | 10 | 127 | Half, east | −0.11 } Diaphragm |
| D | 10 | 127 | Half, west | +0.09 } |
| E | 10 | 94 | Whole | +0.07 |
| E | 10 | 122 | Half, east | −0.08 $(s-10.0)+0.14$ |
| E | 10 | 122 | Half, west | −0.04 $(s-10.0)+0.10$ |
| F | 10 | 91 | Whole | +0.18 |
| F | 10 | 121 | Half, east | +0.04 |
| F | 10 | 121 | Half, west | +0.02 |

The zero point corresponding to the constant of integration, was selected so that $n$ was zero at that scale reading which corresponds to $14^m.0$. Tan $\alpha$ was in most cases positive, and in the neighborhood of 0.1. Therefore $n$, the correction to $c$, was generally positive for the images fainter than $14^m.0$, and negative for the brighter images. Or, in other words, the correction to be applied to the quasi-photovisual magnitudes read from the photographic reduction curve is negative for faint images, and positive for bright images. For a given magnitude interval the yellow images differ more than the blue images, that is, $\gamma$ is greater for the longer wavelengths (figure 4). In the 2-hour exposure of Area A $n$ assumed the opposite sign. This result was derived from only one plate, which was moderately fogged by sky-light in the violet half, and on which, due to fuzziness of the images, the probable error of a single scale reading was larger than on the shorter exposures. It did not appear sufficiently reliable, and as a compromise with the experience on the shorter exposures, $n$ was made zero, and no corrections to the photovisual magnitudes were applied. Further, only the non-fogged half-filter exposure was utilized in the determination of the photovisual magnitudes, and for the other half the full-filter yellow exposure was used. To the photovisual magnitudes of the short exposures of all areas, obtained with the K2 full-filter a systematic correction of about $+0^m.25$ had to be applied in order to make the values agree with those obtained with the K2 half-filters. The K2 full-filter, which had been the property of the Observatory for a considerable time, was more transparent than the K2 half-filter, recently ordered from the factory. The effect was independent of the wave length.

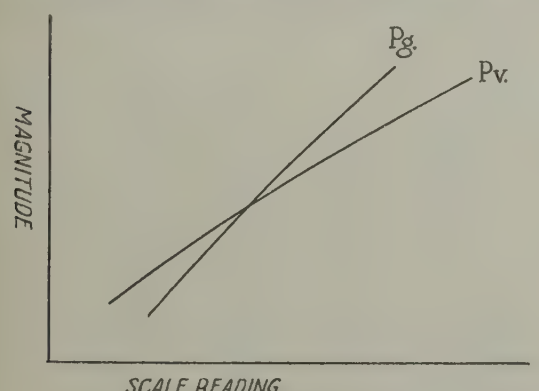


Fig. 4

The full-filter exposures give a means to correct for systematic differences between the two halves of the field. The values finally adopted are the means between the half-filter and full-filter exposures. Where available, 1-minute exposures were included.

### (d) *Determination of Spectral Types*

The 4-hour spectrogram plates obtained with the slitless quartz spectrograph contain within a circle of 10′ radius, i.e. in the area in which the brighter stars have been determined, about 10–15 easily classified spectra. (See figures 5 and 6 on plate III.) While there are in addition a number of faint spectra, they were, on account of the rapidly increasing uncertainty in the classification, not included in the calibration of color indices from spectral types. In the six areas a total of about 65 good spectra were classified according to the Henry Draper classification, by means of 52 comparison stars, whose spectra had been observed and classified by A. D. Maxwell.[16] Table 3 shows the spectral type estimates obtained.

The difference between the final photographic and photovisual magnitude is the color index peculiar to the instruments and material used, i.e. Crossley reflector, Cramer Isochromatic Instantaneous Plates, 35D and K2 Filter, D-11 Developer a.s.o. By comparing the color indices with the spectra (types of table 3)

[16] *L. O. B.* **390**, p. 70, 1927.

TABLE 3

SLITLESS SPECTROGRAMS

| Area A | | | Area B | | | Area C | | | Area D | | | Area E | | | Area F | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No. | Sp. | C | No. | Sp. | C | No. | Sp. | C | No. | Sp. | C | No. | Sp. | C | No. | Sp. | C |
| 46 | B8 | − .51 | 1 | A0 | + .04 | 1 | A0 | − .09 | 13 | F2 | + .63 | 2 | gG5 | +1.06 | 35 | A2 | + .10 |
| 59 | B5 | − .09 | 2 | dC0 | + .49 | 26 | A4 | − .06 | 59 | F0 | + .38 | 6 | A0 | + .25 | 95 | gG0 | −1.20 |
| 60 | A0 | − .24 | 58 | B5 | −1.03 | 78 | A0 | + .03 | 86 | A2 | + .27 | 14 | dG0 | + .81 | 115 | A3 | + .36 |
| 62 | A0 | − .11 | 60 | A0 | − .30 | 89 | A0 | + .23 | 88 | A0 | + .20 | 73 | dG0 | + .60 | 145 | A0 | + .21 |
| 132 | A5 | + .46 | 108 | gG0 | + .70 | 90 | A0 | − .06 | 89 | A2 | + .44 | 160 | A8 | + .42 | 146 | A0 | + .16 |
| 155 | A6 | + .46 | 109 | A0 | + .11 | 100 | F2 | + .59 | 112 | A0 | + .28 | -- | | | 147 | dG5 | +1.06 |
| 156 | A3 | + .31 | 158 | gK | +1.95 | 115 | F0 | + .38 | 147 | gG5 | +1.67 | | | | 159a | A0 | + .44 |
| 157 | K5 | +2.52 | 195 | A3 | − .19 | | | | 193 | dG0 | + .89 | | | | 282 | A0 | − .25 |
| 158 | dG5 | + .41 | 219 | gG5 | +1.24 | | | | 257 | A1 | + .50 | | | | | | |
| 180 | F5 | + .38 | | | | | | | 271 | A2 | + .30 | | | | | | |
| 236 | A0 | + .30 | | | | | | | | | | | | | | | |
| 237 | A0 | − .07 | | | | | | | | | | | | | | | |
| 238 | A0 | − .28 | | | | | | | | | | | | | | | |

and adding a constant +.90 to all color indices, in order to make the color index of an *A*o star zero, the following relation was found:

| Sp. | C. | Sp | C. | Sp | C. |
|---|---|---|---|---|---|
| B5 | − .53 | gG0 | − .98 | gK7 | −2.68 |
| A0 | 0 | gG5 | −1.40 | gM2 | +2.92 |
| A5 | +.35 | gK0 | −1.91 | dG0 | − .75 |
| F0 | + .55 | gK2 | −2.12 | dG5 | −1.02 |
| F5 | + .72 | gK5 | −2.42 | dK0 | −1.38 |

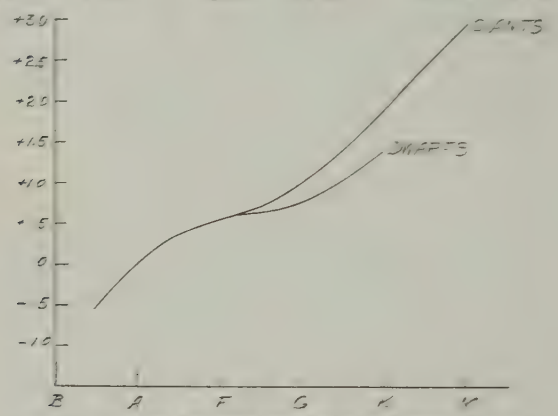



Fig. 7. Relation between color index and spectral class.

Figure 7 shows graphically the relation between color index and spectral type.

### (e) *Positions*

When measuring the color index plates for the image intensities, the plates were, by means of the trail plate, oriented as accurately as feasible so that the east-west direction was parallel to the motion in the $x$-coordinate. Both coordinates of the yellow and violet 10-minute exposures were measured to 0.01 of a micrometer revolution, and are probably certain within 0.05 to 0.10. 1 revolution = 0.04 in. = 1.015 mm. The scale on Crossley plates is approximately 1 mm = 38″.9, and therefore 1 revolution = 39″.4. The positions in the attached catalogue in $x$ and $y$ are given in micrometer revolutions. The measured coordinates $x$ and $y$ differ from the ideal coordinates $X$ and $Y$ by the effects of scale, orientation and zero point:

$$X = ax - by + c$$
$$Y = ay - bx + d$$

Where $a$ is the scale factor, $b$ the orientation factor, and $c$ and $d$ the zero point constants. Let $A$ and $D$

TABLE 4

IDENTIFICATION OF STARS

| Area No. | A. G. | B. D. | α(1900.0) | δ(1900.0) | x | y | Mag. |
|---|---|---|---|---|---|---|---|
| A118 | 6313 | −6°4869 | 18h 39m 18ˢ.24 | −6° 38′ 15″.0 | 18.87 | 34.58 | 7.5 |
| A 61 | 6315 | −6.4871 | 18 39 21.98 | −6 44 19.4 | 17.36 | 43.93 | 8.1 |
| Ay | 6316 | −6.4872 | 18 39 27.59 | −6 26 31.5 | 15.34 | 16.70 | 8.8 |
| B110 | 6388 | −8.4733 | 18 45 47.67 | −8 7 37.3 | 18.83 | 34.67 | 7.4 |
| B108 | 6387 | −8.4732 | 18 45 47.15 | −8 3 44.0 | 18.99 | 28.74 | 8.1 |
| Bx | 6381 | −7.4739 | 18 45 15.41 | −7 57 13.3 | 20.98 | 18.96 | 8.4 |
| By | 6385 | −7.4740 | 18 45 44.35 | −7 54 56.6 | 20.01 | 15.22 | 8.6 |
| C 77 | 6412 | −9.4876 | 18 47 32.09 | −9 41 50.2 | 18.88 | 34.45 | 6.5 |
| Du | 6460 | −6.4971 | 18 51 7.84 | −6 4 35.0 | 18.30 | 15.77 | 8.7 |
| Dx | 6453 | −6.4966 | 18 50 34.61 | −6 28 29.1 | 31.23 | 52.15 | 8.8 |
| E2 | 6518 | −6.5009 | 18 56 34.15 | −6 19 26.2 | 8.06 | 33.67 | 7.8 |
| E93 | 6513 | −6.5005 | 18 56 5.70 | −6 20 2.1 | 18.91 | 34.51 | 7.0 |
| Ex | 6517 | −6.5007 | 18 56 27.17 | −6 31 17.1 | 10.74 | 51.74 | 8.0 |
| F114 | 6301 | −8.4686 | 18 38 4.47 | −8 22 26.5 | 18.81 | 34.00 | 5.0 |
| F 95 | 6302 | −8.4687 | 18 38 7.35 | −8 28 0.6 | 17.83 | 42.30 | 7.4 |

be the right ascension and declination of the plate center (in degrees) and let $Y = \tan q$, then $\alpha$ and $\delta$ of any star may be found:

$$\tan(\alpha - A) = X \sec(D+q)$$
$$\tan\delta = \tan(D+q)$$

The four constants, $a$, $b$, $c$, $d$, together with $A$ and $D$ are given for each area at the head of tables 5 to 11. Table 4 is a list of reference stars.

## IV. Star Catalogue

The measured and reduced stars are arranged in the attached star catalogue (tables 5–11) according to areas. The short exposures of Area A are followed by the long exposures of Area A, and by the short exposures of Area B to Area F. Each star number consists of a prefix signifying the area and a running number, beginning with 1 for all short exposures, and with 2001 for the 2-hour exposure of Area A. An * following the number indicates that the star is in the ring-shaped supplementary field between Circles I and II. The coordinates $x$ and $y$ are measured coordinates, which can be expressed in $\alpha$ and $\delta$, as indicated in IIIe. The observed photographic magnitudes $m_D$ and color indices $C$ can be reduced to the I. A. U. color system by the formulae:

$$m_I = m_D - 0.23C$$
$$C_I = 0.73C$$

The following symbols were employed to describe partially observed stars:

$B$ = too bright
$D$ = near half-filter division line
$F$ = too faint
$I$ = intercepted
$L$ = covered by reseau line
$V$ = variability suspected

The algebraic sign was in some of these cases denoted, but no numerical value given.

TABLE 5

Area A
(Short Exposures)

A = 279° 49′ 34″ ) A118 b = +0.000. 003, 33
D = −6° 38′ 15″ ) c = +0.003, 467
a = −0.000. 189, 9 d = +0.006, 622

| No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A 1* | 5.61 | 35.55 | 11.75 | − .25 | A36 | 11.96 | 39.85 | 14.36 | + .49 | A71 | 14.70 | 31.45 | 14.95 | + .55 |
| A 2* | 4.96 | 31.81 | 13.25 | + .18 | A37 | 12.89 | 38.28 | 14.99 | + .68 | A72 | 14.84 | 27.73 | 13.34 | + .05 |
| A 3 | 8.29 | 38.31 | 14.52 | + .59 | A38 | 12.57 | 37.25 | 14.36 | + .84 | A73 | 15.73 | 28.18 | 14.50 | + .23 |
| A 4 | 7.99 | 37.45 | 14.42 | + .30 | A39 | 12.47 | 36.61 | 14.70 | + .84 | A74 | 15.89 | 27.11 | 14.01 | +1.43 |
| A 5 | 8.50 | 36.16 | 14.17 | + .54 | A40 | 12.28 | 36.36 | 12.45 | + .04 | A75 | 15.63 | 25.68 | 14.46 | + .72 |
| A 6 | 8.43 | 35.76 | 14.40 | + .54 | A41 | 11.92 | 33.70 | 14.61 | +1.76 | A76 | 14.92 | 24.78 | 15.14 | +1.75 |
| A 7 | 8.21 | 34.81 | 14.32 | + .68 | A42 | 12.16 | 32.37 | 13.51 | + .38 | A77 | 16.38 | 23.65 | 15.14 | + .83 |
| A 8 | 8.57 | 34.05 | 14.67 | + .64 | A43 | 12.55 | 30.53 | 12.91 | − .20 | A78 | 16.25 | 42.30 | 14.15 | + .65 |
| A 9 | 8.21 | 34.22 | 15.19 | ——— | A44 | 12.03 | 28.73 | 13.41 | + .09 | A79 | 17.20 | 42.54 | 15.14 | +1.14 |
| A10 | 7.44 | 34.33 | 13.57 | + .35 | A45 | 12.19 | 27.37 | 13.37 | +1.61 | A80 | 16.94 | 41.21 | 15.47 | +1.11 |
| A11 | 8.65 | 30.15 | 14.32 | + .05 | A46* | 14.75 | 48.19 | 11.81 | − .51 | A81 | 17.33 | 40.95 | 13.81 | − .10 |
| A12 | 8.44 | 30.40 | 13.90 | + .11 | A47 | 13.91 | 43.46 | 14.11 | + .73 | A82 | 16.59 | 40.20 | 14.15 | + .33 |
| A13 | 8.20 | 30.24 | 14.09 | + .29 | A48 | 14.81 | 41.00 | 14.90 | + .64 | A83 | 17.34 | 39.67 | 14.15 | +1.15 |
| A14 | 8.53 | 32.79 | 14.30 | + .34 | A49 | 13.67 | 40.30 | 14.61 | +1.25 | A84 | 17.31 | 37.27 | 14.15 | +1.36 |
| A15 | 7.88 | 32.22 | 15.22 | +1.77 | A50 | 14.19 | 38.94 | 14.36 | + .56 | A85 | 17.12 | 36.66 | 15.47 | +1.94 |
| A16* | 8.58 | 23.72 | 12.25 | + .77 | A51 | 14.10 | 36.78 | 14.77 | + .80 | A86 | 16.70 | 37.25 | 14.94 | + .74 |
| A17 | 10.10 | 40.32 | 14.45 | + .57 | A52 | 13.94 | 35.57 | 14.30 | + .33 | A87 | 16.56 | 34.40 | 14.19 | + .59 |
| A18 | 9.36 | 40.64 | 14.69 | + .58 | A53 | 13.92 | 28.25 | 15.32 | +1.29 | A88 | 16.51 | 33.69 | 15.43 | + .96 |
| A19 | 9.10 | 37.03 | 13.10 | − .14 | A54 | 14.20 | 27.66 | 13.46 | − .10 | A89 | 16.82 | 32.18 | 14.86 | + .60 |
| A20 | 9.95 | 32.22 | 15.19 | +1.90 | A55 | 13.20 | 26.65 | 15.25 | + .91 | A90 | 16.98 | 31.88 | 13.95 | + .24 |
| A21 | 9.74 | 31.17 | 14.63 | + .54 | A56 | 13.08 | 26.44 | 10.88 | − .12 | A91 | 16.17 | 31.57 | 13.22 | + .17 |
| A22 | 9.35 | 29.28 | 13.17 | − .14 | A57* | 16.77 | 20.17 | 10.83 | − .15 | A92 | 17.12 | 32.62 | 14.65 | + .73 |
| A23 | 9.88 | 28.23 | 14.42 | + .21 | A58 | 17.00 | 26.67 | 12.79 | +1.59 | A93 | 16.38 | 31.08 | 14.54 | + .18 |
| A24 | 10.96 | 36.92 | 13.18 | + .10 | A59* | 16.56 | 47.62 | 10.93 | − .09 | A94 | 16.59 | 30.21 | 14.50 | + .71 |
| A25 | 10.86 | 39.93 | 14.15 | + .79 | A60 | 17.51 | 44.64 | 11.56 | − .24 | A95 | 16.40 | 30.14 | 15.14 | + .61 |
| A26 | 10.38 | 40.30 | 14.65 | + .49 | A61 | 17.36 | 43.93 | 10.37 | + | A96 | 16.15 | 27.70 | 14.04 | + .65 |
| A27 | 11.07 | 36.46 | 13.91 | − .08 | A62 | 15.51 | 43.81 | 12.03 | − .11 | A97 | 18.31 | 44.60 | 14.94 | +1.22 |
| A28 | 11.19 | 36.36 | 14.06 | + .41 | A63 | 15.20 | 43.70 | 12.90 | + .15 | A98 | 18.58 | 43.70 | 14.74 | + .43 |
| A29 | 10.97 | 32.18 | 15.53 | +1.47 | A64 | 15.24 | 43.04 | 12.74 | +1.86 | A99 | 18.50 | 42.48 | 14.14 | +1.15 |
| A30 | 10.88 | 29.24 | 14.09 | + .14 | A65 | 15.23 | 40.94 | 14.19 | + .55 | A100 | 18.40 | 40.99 | 15.25 | + .79 |
| A31 | 10.41 | 29.05 | 15.09 | +1.95 | A66 | 15.16 | 40.65 | 14.70 | + .98 | A101 | 18.88 | 40.61 | 13.97 | + .32 |
| A32 | 10.57 | 27.26 | F | ——— | A67 | 15.20 | 39.30 | 14.70 | + .60 | A102 | 18.69 | 40.27 | 14.40 | + .38 |
| A33* | 11.56 | 20.84 | 12.26 | − .62 | A68 | 15.59 | 37.75 | 14.49 | − .39 | A103 | 18.25 | 39.89 | 14.47 | +1.19 |
| A34 | 12.95 | 40.64 | 14.16 | − .26 | A69 | 15.80 | 44.27 | 14.61 | + .46 | A104 | 18.85 | 38.84 | 14.82 | + .35 |
| A35 | 12.61 | 39.95 | 13.50 | + .17 | A70 | 15.41 | 30.25 | 14.01 | + .21 | A105 | 18.01 | 37.61 | 14.49 | + .23 |

TABLE 5—*Continued*

| No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A106 | 18.02 | 35.91 | 14.70 | +1.36 | A151 | 20.51 | 30.40 | 14.80 | + .46 | A195 | 23.28 | 26.69 | 15.18 | + .61 |
| A107 | 17.73 | 33.81 | 13.48 | +1.07 | A152 | 20.24 | 25.00 | 14.33 | +1.34 | A196 | 25.99 | 41.63 | F | + |
| A108 | 18.03 | 34.63 | F | + | A153 | 21.08 | 24.50 | 15.41 | +1.57 | A197 | 25.57 | 40.24 | 15.18 | + .84 |
| A109 | 18.36 | 32.90 | 14.61 | +1.64 | A154 | 21.18 | 24.35 | 13.79 | + .35 | A198 | 25.23 | 40.12 | 14.19 | + .51 |
| A110 | 17.71 | 31.28 | 14.50 | + .68 | A155 | 22.96 | 37.27 | 12.38 | + .46 | A199 | 24.83 | 38.85 | 14.37 | + .69 |
| A111 | 17.28 | 30.82 | 13.86 | + .16 | A156 | 22.16 | 38.21 | 13.11 | + .31 | A200 | 24.88 | 38.10 | 14.71 | +1.18 |
| A112 | 18.27 | 28.38 | 14.70 | + .44 | A157 | 22.88 | 40.73 | 13.44 | +2.52 | A201 | 25.11 | 36.79 | 14.42 | +1.22 |
| A113 | 17.98 | 27.47 | 14.24 | + .36 | A158* | 22.29 | 48.76 | 10.41 | + .41 | A202 | 24.63 | 35.14 | 13.87 | + .47 |
| A114 | 17.47 | 28.90 | 14.87 | + .51 | A159 | 22.79 | 43.66 | 13.48 | − .07 | A203 | 25.28 | 35.28 | 14.71 | + .32 |
| A115 | 17.42 | 27.28 | 15.30 | +1.62 | A160 | 23.20 | 43.05 | 15.19 | +2.30 | A204 | 25.18 | 34.79 | 14.84 | + .91 |
| A116 | 17.29 | 26.13 | F | + | A161 | 22.95 | 39.33 | 14.46 | +1.17 | A205 | 25.17 | 28.89 | 14.78 | +1.46 |
| A117 | 18.56 | 25.94 | 15.28 | + .60 | A162 | 22.11 | 38.48 | 14.42 | + .56 | A206 | 25.30 | 26.10 | 13.17 | + .13 |
| A118 | 18.87 | 34.58 | B | + | A163 | 22.38 | 37.63 | 15.30 | + .94 | A207* | 26.82 | 22.35 | 11.03 | + .52 |
| A119 | 19.47 | 42.69 | 14.40 | + .24 | A164 | 22.05 | 33.86 | 14.50 | + .59 | A208 | 26.80 | 42.29 | 15.13 | +1.79 |
| A120 | 19.06 | 38.06 | 12.87 | + .07 | A165 | 22.06 | 33.54 | 14.13 | + .37 | A209 | 26.75 | 41.38 | 14.88 | + .60 |
| A121 | 19.97 | 38.49 | F | + | A166 | 22.15 | 32.72 | 13.24 | + .42 | A210 | 26.52 | 39.58 | 12.80 | − .06 |
| A122 | 19.93 | 38.36 | F | + | A167 | 21.59 | 33.59 | 14.77 | +1.38 | A211 | 26.73 | 39.10 | 14.13 | + .19 |
| A123 | 20.20 | 36.81 | 12.84 | − .27 | A168 | 21.60 | 34.45 | 15.08 | +1.25 | A212 | 26.79 | 37.36 | 14.92 | +1.09 |
| A124 | 19.33 | 36.57 | 15.28 | +2.56 | A169 | 22.41 | 30.76 | 14.06 | +2.29 | A213 | 26.72 | 37.05 | 15.30 | +1.09 |
| A125 | 19.80 | 36.26 | 14.85 | + .61 | A170 | 22.92 | 30.17 | 14.37 | + .60 | A214 | 26.50 | 34.13 | 14.93 | |
| A126 | 19.84 | 34.24 | 14.78 | + .45 | A171 | 21.91 | 31.82 | 14.85 | + .35 | A215 | 26.56 | 33.55 | 15.47 | +1.53 |
| A127 | 19.33 | 32.50 | 14.59 | + .43 | A172 | 21.79 | 29.91 | 15.34 | +1.24 | A216 | 26.08 | 33.23 | 15.07 | +1.19 |
| A128 | 20.06 | 29.30 | 14.24 | + .58 | A173 | 22.10 | 27.60 | 14.55 | + .22 | A217 | 25.87 | 33.92 | 15.13 | +1.25 |
| A129 | 19.53 | 27.48 | 15.28 | +1.66 | A174 | 21.62 | 25.82 | 15.34 | + .96 | A218 | 26.85 | 35.07 | 14.85 | |
| A130 | 19.15 | 26.90 | 14.60 | + .60 | A175 | 21.43 | 25.64 | 14.57 | + .50 | A219 | 27.10 | 31.76 | 14.33 | + .47 |
| A131 | 19.97 | 24.20 | 14.68 | + .22 | A176 | 22.16 | 24.48 | 14.67 | + .68 | A220 | 26.97 | 31.56 | 15.41 | +1.53 |
| A132 | 21.06 | 43.30 | 10.18 | + .46 | A177 | 21.67 | 23.95 | 14.84 | + .35 | A221 | 25.98 | 27.66 | 14.33 | + .68 |
| A133 | 21.89 | 44.02 | 15.17 | +1.83 | A178 | 22.10 | 23.56 | 15.25 | +1.75 | A222* | 26.80 | 26.38 | 13.54 | + .51 |
| A134 | 20.61 | 45.38 | 14.50 | + .44 | A179* | 23.87 | 24.06 | 12.40 | − .08 | A223* | 28.33 | 26.08 | V | V |
| A135 | 21.26 | 41.57 | 13.56 | + .22 | A180 | 23.51 | 30.77 | 11.28 | + .38 | A224 | 27.89 | 33.37 | 13.54 | + .16 |
| A136 | 20.97 | 41.09 | 14.46 | + .40 | A181 | 24.20 | 41.86 | 13.22 | − .06 | A225 | 28.35 | 39.19 | 14.20 | + .32 |
| A137 | 20.90 | 39.53 | 14.84 | + .56 | A182 | 24.11 | 40.84 | 14.75 | + .31 | A226 | 27.93 | 32.47 | 14.53 | + .15 |
| A138 | 21.21 | 38.95 | 14.24 | +1.48 | A183 | 24.51 | 40.63 | 14.40 | + .49 | A227 | 27.39 | 31.11 | 14.97 | +2.00 |
| A139 | 21.08 | 38.29 | 14.61 | +1.09 | A184 | 23.73 | 39.17 | 14.02 | + .69 | A228 | 27.97 | 30.61 | 14.37 | + .76 |
| A140 | 21.16 | 38.20 | 13.41 | +1.55 | A185 | 24.02 | 38.07 | 14.42 | + .71 | A229 | 27.35 | 27.49 | 15.02 | +1.54 |
| A141 | 20.56 | 38.21 | 14.97 | + .54 | A186 | 24.14 | 37.28 | 14.30 | + .80 | A230* | 28.64 | 27.29 | 12.46 | − .21 |
| A142 | 20.91 | 36.27 | 14.71 | + .65 | A187 | 23.56 | 34.42 | 14.53 | +1.46 | A231 | 29.27 | 30.25 | 12.89 | +1.50 |
| A143 | 20.90 | 34.58 | 15.02 | + .72 | A188 | 23.44 | 32.35 | 13.83 | + .54 | A232 | 29.40 | 35.55 | 14.40 | + .16 |
| A144 | 20.75 | 34.61 | 15.52 | +1.46 | A189 | 24.24 | 32.22 | 14.23 | + .76 | A233 | 29.41 | 37.19 | 14.61 | +1.70 |
| A145 | 20.82 | 34.33 | 15.13 | +1.94 | A190 | 23.34 | 30.14 | 15.29 | +1.80 | A234 | 29.01 | 30.91 | 15.41 | +1.32 |
| A146 | 20.62 | 33.06 | 13.40 | + .25 | A191 | 24.08 | 29.42 | 14.13 | + .27 | A235* | 31.10 | 27.36 | 12.62 | + .08 |
| A147 | 20.63 | 31.89 | 14.20 | + .70 | A192 | 24.08 | 28.95 | 15.18 | +1.46 | A236* | 31.20 | 42.81 | 11.75 | − .30 |
| A148 | 20.55 | 31.24 | 14.33 | +1.24 | A193 | 23.91 | 26.43 | 14.45 | + .60 | A237* | 30.57 | 41.49 | 10.98 | − .07 |
| A149 | 20.84 | 31.23 | 14.46 | + .46 | A194 | 23.95 | 25.80 | 14.42 | + .50 | A238* | 32.13 | 35.35 | 11.48 | − .28 |
| A150 | 20.66 | 31.62 | 14.49 | + .04 | | | | | | | | | | |

TABLE 6

AREA A

(Long Exposures)

A = 279° 49′ 34″
D = −6° 38′ 15″
} A118
a = −0.000, 190, 2

b = +0.000, 004, 22
c = +0.003, 422
d = +0.006, 380

| No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A2001 | 21.65 | 38.71 | 16.23 | + .77 | A2008 | 21.75 | 38.24 | 17.87 | +1.73 | A2014 | 21.25 | 39.71 | 17.97 | +1.19 |
| A2002 | 21.59 | 38.65 | 17.35 | +1.35 | A2009 | 21.38 | 37.89 | 17.21 | − .40 | A2015 | 20.74 | 39.75 | 17.73 | +1.39 |
| A2003 | 21.46 | 38.75 | 18.40 | + .45 | A2009a | 21.48 | 37.89 | F | + | A2016a | 20.84 | 39.57 | 14.88 | + .75 |
| A2004 | 21.55 | 38.49 | 16.50 | + .84 | A2010 | 21.43 | 37.51 | 17.83 | +2.18 | A2016b | 20.84 | 39.48 | 16.94 | +1.04 |
| A2005 | 21.46 | 38.33 | 16.24 | + .94 | A2010a | 21.53 | 37.41 | F | + | A2016c | 20.96 | 39.50 | 17.21 | +1.00 |
| A2006 | 21.31 | 38.22 | 17.34 | + .91 | A2011 | 21.39 | 36.77 | 18.40 | +2.86 | A2017 | 20.94 | 39.30 | 17.38 | + .66 |
| A2007 | 21.37 | 38.16 | 17.16 | + .96 | A2012 | 21.20 | 36.10 | 17.87 | +2.11 | A2017a | 21.04 | 39.30 | F | + |
| A2007a | 21.31 | 38.06 | 18.34 | +2.61 | A2013 | 21.12 | 39.87 | 18.38 | +3.04 | A2018 | 21.09 | 38.94 | 16.92 | +1.04 |

TABLE 6—*Continued*

| No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A2019 | 21.21 | 38.61 | 17.87 | +2.53 | A2077 | 19.42 | 37.54 | 17.28 | + .76 | A2136 | 18.71 | 37.64 | 17.22 | +1.22 |
| A2020 | 20.91 | 38.89 | 18.26 | +2.14 | A2078 | 19.79 | 37.00 | 15.99 | +1.64 | A2137 | 18.65 | 37.32 | 14.76 | + .55 |
| A2021 | 20.95 | 38.76 | 18.07 | + .91 | A2079 | 19.74 | 36.85 | 15.80 | +1.53 | A2138 | 18.50 | 37.22 | 18.00 | +1.25 |
| A2022 | 20.69 | 38.75 | 17.87 | + .31 | A2080 | 19.34 | 37.03 | 17.82 | + .94 | A2139 | 18.27 | 37.20 | 17.92 | +1.10 |
| A2023 | 20.63 | 38.49 | 17.21 | +1.65 | A2081a | 19.95 | 36.29 | 17.92 | +1.07 | A2140 | 18.45 | 37.05 | F | + |
| A2024 | 21.14 | 38.30 | 16.88 | +1.16 | A2081b | 19.87 | 36.71 | 18.16 | +2.39 | A2141 | 18.48 | 36.90 | 16.98 | +1.44 |
| A2025 | 21.16 | 38.19 | 18.07 | + .91 | A2082 | 19.94 | 35.44 | 17.50 | + .90 | A2142 | 18.64 | 36.79 | 17.73 | + .63 |
| A2026 | 21.07 | 38.15 | 15.92 | + .92 | A2083 | 19.94 | 35.35 | 17.78 | + .96 | A2143 | 18.77 | 36.58 | 15.74 | + .76 |
| A2027 | 20.87 | 38.14 | 17.43 | +1.15 | A2084 | 19.65 | 35.14 | 17.70 | + .55 | A2144 | 18.33 | 36.48 | 17.64 | + .69 |
| A2028 | 20.76 | 38.04 | 15.47 | +1.14 | A2085 | 19.48 | 35.21 | 17.35 | + .73 | A2145 | 18.25 | 36.58 | 17.04 | + .56 |
| A2029 | 21.08 | 37.66 | 16.60 | +2.20 | A2086 | 19.60 | 34.80 | 14.94 | + .74 | A2146 | 18.19 | 36.41 | 16.39 | +1.81 |
| A2030 | 20.92 | 37.66 | 17.43 | + .38 | A2087 | 19.32 | 41.20 | 14.14 | + .64 | A2147 | 18.48 | 35.40 | 18.06 | +2.00 |
| A2031 | 20.85 | 37.54 | 16.94 | + .58 | A2088 | 19.18 | 40.96 | 17.97 | + .62 | A2147a | 18.33 | 35.42 | F | + |
| A2032 | 21.11 | 37.43 | 14.07 | + | A2089 | 19.03 | 41.04 | 17.78 | + .88 | A2148 | 18.21 | 35.50 | 18.38 | + .63 |
| A2033 | 21.05 | 37.59 | 18.50 | | A2090 | 18.92 | 41.04 | 16.88 | + .86 | A2148a | 18.06 | 35.20 | F | + |
| A2034 | 21.14 | 37.60 | 18.20 | +2.42 | A2091 | 19.19 | 40.54 | 17.87 | +1.02 | A2149 | 18.28 | 34.90 | 17.92 | + .46 |
| A2035a | 20.93 | 36.82 | 15.10 | + | A2092 | 18.98 | 40.69 | 17.27 | + .91 | A2150 | 18.24 | 34.71 | 16.88 | + .73 |
| A2035b | 21.16 | 36.70 | 13.70 | + | A2093 | 19.01 | 40.21 | 16.24 | + .64 | A2151a | 18.59 | 34.38 | 17.38 | + .17 |
| A2036 | 20.96 | 36.19 | 18.16 | +1.01 | A2093a | 19.40 | 40.51 | F | + | A2151b | 18.13 | 41.36 | 17.48 | + .58 |
| A2037 | 20.94 | 35.95 | 18.14 | +1.86 | A2094 | 19.32 | 39.62 | 16.72 | + .96 | A2152 | 17.63 | 41.46 | 17.70 | +2.31 |
| A2038 | 20.72 | 36.07 | 17.48 | +1.12 | A2095 | 19.34 | 39.48 | 16.67 | +1.43 | A2153 | 17.77 | 41.36 | 18.38 | |
| A2039 | 20.85 | 35.79 | 16.80 | + .98 | A2096 | 19.25 | 38.50 | 16.48 | +1.28 | A2154 | 17.81 | 41.01 | 17.98 | +2.27 |
| A2040 | 20.54 | 35.63 | 17.15 | +1.15 | A2097 | 19.34 | 38.35 | 16.07 | + .84 | A2155 | 17.99 | 40.58 | 16.56 | + .98 |
| A2041 | 20.33 | 40.65 | 17.60 | +1.32 | A2098 | 18.88 | 38.20 | 17.82 | +2.00 | A2156 | 17.92 | 40.58 | 16.56 | + .98 |
| A2042 | 20.26 | 40.55 | 17.33 | +1.19 | A2099 | 19.02 | 38.11 | 18.24 | + .29 | A2157 | 17.65 | 40.35 | 17.87 | + .71 |
| A2043 | 20.25 | 40.25 | 18.38 | + .87 | A2100 | 19.13 | 37.91 | 17.62 | +1.97 | A2158 | 18.14 | 40.09 | 17.82 | + .67 |
| A2044 | 20.53 | 39.73 | 17.87 | + .66 | A2101 | 18.85 | 37.95 | 17.62 | + .83 | A2159 | 18.11 | 39.89 | 17.60 | +2.58 |
| A2045 | 20.24 | 39.73 | 18.16 | + .51 | A2102 | 18.90 | 37.83 | 16.18 | + .60 | A2160 | 17.57 | 39.98 | 17.47 | +1.82 |
| A2046 | 20.24 | 39.56 | 18.12 | + .47 | A2103 | 18.92 | 37.28 | 18.28 | + .93 | A2161 | 17.73 | 39.42 | 18.16 | + .70 |
| A2047 | 20.14 | 39.45 | 16.18 | + .94 | A2104 | 18.96 | 37.17 | 18.16 | +2.38 | A2162 | 18.16 | 39.06 | 18.26 | +1.01 |
| A2047a | 20.04 | 39.45 | 18.74 | +2.65 | A2105 | 18.90 | 37.17 | 14.85 | + .31 | A2163 | 18.16 | 38.85 | 17.78 | +1.20 |
| A2048 | 20.15 | 38.97 | 18.16 | +1.40 | A2105a | 19.15 | 36.87 | F | + | A2164 | 18.02 | 38.57 | 16.08 | +1.81 |
| A2049 | 20.17 | 38.78 | 16.98 | + .24 | A2106 | 18.94 | 36.56 | 13.41 | + | A2165 | 17.75 | 38.53 | 17.87 | +1.09 |
| A2050 | 20.16 | 38.63 | 18.34 | +1.42 | A2107 | 19.17 | 36.30 | 17.57 | +1.43 | A2166 | 17.68 | 38.44 | 16.24 | + .59 |
| A2050a | 20.36 | 38.70 | 17.88 | +1.23 | A2108 | 19.30 | 35.67 | 17.15 | +1.09 | A2167 | 17.94 | 38.25 | 16.40 | +2.33 |
| A2051 | 20.36 | 38.26 | 17.73 | + .95 | A2109 | 19.26 | 35.36 | 15.68 | +1.78 | A2168 | 17.67 | 37.59 | 17.43 | +1.07 |
| A2051a | 20.42 | 38.11 | F | + | A2110 | 18.82 | 35.79 | 18.24 | + .99 | A2169 | 17.57 | 37.62 | 18.07 | + .66 |
| A2052 | 20.06 | 38.34 | 18.26 | +1.58 | A2111 | 19.31 | 35.24 | 17.51 | +1.81 | A2170 | 18.09 | 37.26 | 18.28 | + .73 |
| A2053 | 20.38 | 37.75 | 18.08 | + .97 | A2112 | 19.23 | 35.12 | 14.82 | + | A2170a | 18.12 | 37.56 | F | + |
| A2054 | 20.38 | 37.44 | 18.34 | +1.80 | A2113 | 19.29 | 34.61 | 16.80 | +1.09 | A2171 | 17.82 | 37.32 | 17.78 | + .88 |
| A2055 | 20.27 | 37.29 | 17.78 | +1.06 | A2113a | 19.25 | 34.71 | F | + | A2172 | 17.65 | 37.03 | 16.28 | + .84 |
| A2056 | 20.05 | 37.44 | 16.50 | + .85 | A2114 | 19.12 | 34.68 | 18.07 | +1.11 | A2173 | 18.08 | 36.56 | 18.20 | +1.05 |
| A2057 | 20.27 | 36.94 | 16.98 | − .28 | A2115 | 18.92 | 34.95 | 18.42 | +1.57 | A2174 | 17.06 | 36.44 | 17.70 | + .59 |
| A2058 | 20.31 | 36.87 | 17.63 | +1.56 | A2115a | 18.89 | 34.98 | F | + | A2175 | 17.88 | 36.12 | 14.80 | + .46 |
| A2059 | 20.40 | 36.82 | 17.34 | +1.28 | A2116 | 18.79 | 34.68 | 16.28 | + .89 | A2175a | 17.73 | 36.39 | F | + |
| A2060 | 20.48 | 36.72 | 15.16 | + .50 | A2117 | 18.59 | 41.32 | 15.04 | + .57 | A2176 | 17.99 | 35.95 | 15.76 | − .01 |
| A2061 | 20.10 | 36.18 | 16.29 | + .89 | A2118 | 18.70 | 41.07 | 16.80 | + .65 | A2177 | 17.99 | 35.52 | 16.28 | − .34 |
| A2062 | 20.40 | 36.12 | 18.20 | + .95 | A2119 | 18.27 | 41.34 | 18.02 | +1.74 | A2178 | 17.89 | 35.60 | 17.35 | + .40 |
| A2063 | 20.02 | 35.80 | 16.98 | +1.21 | A2120 | 18.37 | 40.99 | 14.07 | + | A2179 | 17.85 | 34.98 | 18.17 | + .92 |
| A2064 | 20.10 | 35.33 | 13.55 | + | A2121 | 18.37 | 40.78 | 17.87 | +1.77 | A2180 | 17.85 | 34.79 | 16.80 | +1.03 |
| A2065 | 19.94 | 40.85 | 18.07 | +1.33 | A2122 | 18.22 | 40.88 | 16.50 | +1.06 | A2181 | 17.90 | 34.43 | 15.04 | + |
| A2066 | 19.81 | 41.08 | 17.96 | + .70 | A2123 | 18.77 | 39.58 | 17.15 | +1.15 | A2182 | 17.77 | 34.48 | 17.20 | + .50 |
| A2067 | 19.76 | 40.97 | 16.98 | + .90 | A2124 | 18.28 | 39.77 | 17.97 | + .51 | A2183 | 17.27 | 41.28 | 18.20 | +1.32 |
| A2068 | 19.58 | 41.23 | 17.60 | +1.53 | A2125 | 18.77 | 39.50 | 18.10 | +2.65 | A2184 | 17.10 | 41.06 | 14.81 | + .47 |
| A2069 | 19.94 | 40.24 | 18.48 | +1.12 | A2126 | 18.60 | 39.40 | 16.80 | +1.03 | A2185 | 17.22 | 40.82 | 16.14 | + .86 |
| A2070 | 20.00 | 39.93 | 18.32 | +3.14 | A2127 | 18.35 | 39.55 | 14.60 | + .87 | A2186 | 17.20 | 40.46 | 16.12 | + .68 |
| A2071 | 19.52 | 39.64 | 15.47 | + .57 | A2128 | 18.23 | 39.48 | 17.74 | +1.22 | A2187 | 17.05 | 40.50 | 18.28 | +1.78 |
| A2072 | 19.97 | 39.10 | 17.62 | +1.06 | A2129 | 18.75 | 39.16 | 13.55 | + | A2188 | 17.25 | 40.61 | 18.44 | +2.44 |
| A2073 | 19.95 | 38.64 | 17.62 | + .93 | A2130 | 18.43 | 39.13 | 18.10 | +2.33 | A2189 | 17.09 | 39.92 | 17.44 | +1.00 |
| A2074 | 19.58 | 38.34 | 16.92 | + .59 | A2131 | 18.57 | 38.81 | 14.14 | + | A2190 | 17.36 | 39.90 | 17.53 | + .42 |
| A2074a | 19.73 | 38.30 | F | + | A2132 | 18.41 | 38.76 | 18.17 | +2.25 | A2191 | 17.47 | 39.80 | 17.34 | +1.00 |
| A2074b | 19.60 | 38.54 | F | + | A2133 | 18.55 | 38.43 | 18.28 | + .83 | A2192 | 17.52 | 39.54 | 18.34 | +1.52 |
| A2075 | 19.60 | 37.79 | 16.08 | + .94 | A2134 | 18.39 | 38.17 | 18.20 | + .55 | A2193 | 17.21 | 39.47 | 14.38 | + .88 |
| A2076 | 19.97 | 37.50 | 15.74 | + .55 | A2135 | 18.16 | 37.93 | 14.35 | + | A2194 | 17.13 | 39.40 | 17.04 | +1.04 |

TABLE 6—*Continued*

| No. | x | y | Pg.Mag. | C |
|---|---|---|---|---|
| A2195 | 17.29 | 39.24 | 17.92 | + .86 |
| A2196 | 17.38 | 38.70 | 16.02 | + .88 |
| A2197 | 17.27 | 38.58 | 18.54 | +1.76 |
| A2198 | 17.18 | 38.40 | 18.00 | +2.15 |
| A2199 | 17.23 | 38.18 | 14.30 | + .65 |
| A2200 | 17.16 | 38.29 | 17.52 | + .90 |
| A2201 | 17.38 | 37.38 | 17.60 | +1.84 |
| A2201a | 17.34 | 37.61 | F | + |
| A2202 | 17.48 | 37.28 | 18.12 | + .67 |
| A2203 | 17.47 | 37.08 | 17.21 | +1.25 |
| A2204 | 17.42 | 36.99 | 17.78 | + .68 |
| A2205 | 17.39 | 36.89 | 17.97 | +2.58 |
| A2206 | 17.19 | 36.66 | 17.78 | + .90 |
| A2207 | 17.01 | 36.49 | 15.26 | + .67 |
| A2207a | 16.98 | 36.61 | F | + |
| A2208 | 17.43 | 36.44 | 18.03 | +1.05 |
| A2209 | 17.26 | 36.36 | 17.20 | +1.14 |
| A2210 | 17.32 | 36.23 | 15.86 | +2.06 |
| A2211 | 17.46 | 36.17 | 16.50 | +2.49 |
| A2212 | 17.51 | 35.96 | 15.92 | +1.70 |
| A2213 | 17.20 | 35.80 | 14.07 | + |
| A2213a | 17.02 | 36.12 | F | + |
| A2214 | 17.36 | 35.62 | 17.28 | + .92 |
| A2215 | 17.52 | 35.48 | 16.82 | +1.11 |
| A2216 | 17.55 | 35.36 | 17.54 | +1.18 |
| A2217 | 17.20 | 35.42 | 17.35 | +1.59 |
| A2218 | 17.12 | 35.62 | 17.78 | +1.34 |
| A2219 | 17.05 | 35.18 | 15.86 | +2.06 |
| A2220 | 17.25 | 35.09 | 17.90 | +2.18 |
| A2221 | 17.05 | 34.68 | 16.65 | + .35 |
| A2222 | 17.14 | 34.50 | 17.60 | + .09 |
| A2223 | 17.39 | 34.50 | 16.02 | + .68 |
| A2224 | 17.45 | 34.38 | 17.60 | + .68 |
| A2225 | 16.49 | 41.11 | 17.78 | + .83 |
| A2226 | 16.58 | 40.95 | 18.20 | + .54 |
| A2226a | 18.61 | 40.70 | 18.64 | +1.99 |
| A2227 | 16.42 | 40.63 | 18.48 | + .83 |
| A2228 | 16.34 | 40.52 | 15.78 | + .34 |
| A2229 | 16.91 | 40.26 | 16.48 | + .58 |
| A2230 | 16.46 | 40.20 | 17.46 | − .05 |
| A2231 | 16.84 | 40.06 | 17.78 | + .83 |
| A2232 | 16.38 | 39.98 | 17.27 | + .85 |
| A2233 | 16.79 | 39.72 | 15.58 | +1.44 |
| A2234 | 16.83 | 39.62 | 18.02 | + .51 |
| A2235 | 16.90 | 39.43 | 16.88 | +1.64 |
| A2236 | 16.66 | 39.37 | 16.58 | +1.04 |
| A2237 | 16.37 | 39.24 | 18.24 | + .83 |
| A2238 | 16.53 | 38.88 | 18.20 | + .65 |
| A2239 | 16.45 | 38.73 | 14.22 | + .64 |
| A2240 | 16.97 | 38.56 | 16.58 | +1.86 |
| A2241 | 16.93 | 37.88 | 18.20 | +1.52 |
| A2242 | 16.80 | 37.72 | 18.48 | + .83 |
| A2243 | 16.97 | 37.59 | 18.36 | +1.10 |
| A2244 | 16.53 | 37.52 | 18.07 | + .66 |
| A2244a | 16.77 | 38.31 | F | + |
| A2245 | 16.35 | 37.02 | 16.50 | +1.00 |
| A2245a | 16.61 | 37.27 | F | + |
| A2246 | 16.72 | 36.58 | 17.82 | + .57 |
| A2246a | 16.75 | 36.46 | F | + |
| A2247 | 16.53 | 35.79 | 15.31 | +1.05 |
| A2248 | 16.71 | 35.72 | 16.28 | + .94 |
| A2249 | 16.42 | 35.53 | 18.40 | +1.82 |
| A2250 | 16.49 | 35.32 | 18.28 | +1.07 |
| A2250a | 16.73 | 35.22 | F | + |
| A2251 | 16.48 | 34.83 | 18.20 | +2.26 |
| A2252 | 16.30 | 40.84 | 17.48 | +1.16 |
| A2253 | 16.15 | 40.82 | 14.07 | + |
| A2254 | 15.95 | 40.74 | 17.78 | +1.28 |
| A2255 | 15.98 | 40.64 | 18.07 | + .92 |
| A2256 | 16.02 | 40.34 | 16.64 | + .42 |
| A2257 | 15.87 | 40.42 | 17.26 | + .58 |
| A2258 | 16.06 | 40.00 | 16.88 | +2.41 |
| A2259 | 16.15 | 39.70 | 18.33 | |
| A2260 | 16.05 | 39.75 | 18.14 | + .79 |
| A2261 | 15.88 | 39.70 | 18.24 | + .79 |
| A2262 | 16.20 | 39.32 | 17.46 | +1.32 |
| A2263 | 16.11 | 39.42 | 18.20 | +1.52 |
| A2264 | 15.84 | 39.38 | 17.21 | + .75 |
| A2265 | 15.93 | 38.77 | 17.40 | + .78 |
| A2266 | 15.98 | 38.40 | 18.24 | +1.42 |
| A2267 | 15.85 | 38.71 | 18.80 | |
| A2268 | 15.87 | 38.42 | 18.62 | +1.17 |
| A2268a | 15.72 | 38.22 | 18.64 | +2.43 |
| A2268b | 15.61 | 37.81 | 18.24 | +2.03 |
| A2269 | 16.22 | 37.75 | 17.78 | +1.32 |
| A2270 | 15.86 | 37.79 | 18.07 | + .72 |
| A2271 | 16.22 | 37.25 | 18.20 | + .69 |
| A2271a | 16.20 | 37.35 | F | + |
| A2272 | 15.87 | 37.22 | 17.31 | +1.31 |
| A2273 | 15.87 | 36.73 | 15.32 | + .35 |
| A2274 | 16.02 | 36.67 | 18.24 | +1.96 |
| A2275 | 16.12 | 36.63 | F | + |
| A2275a | 16.22 | 36.48 | 18.34 | +1.19 |
| A2275b | 16.32 | 36.03 | F | + |
| A2275c | 16.17 | 35.98 | F | + |
| A2276 | 15.82 | 35.85 | 17.27 | + .69 |
| A2277 | 15.86 | 35.68 | 17.45 | + .73 |
| A2278 | 15.88 | 35.39 | 16.44 | +2.37 |
| A2279 | 16.26 | 35.16 | 18.20 | + .75 |
| A2280 | 16.30 | 34.99 | 16.55 | +1.11 |
| A2281 | 15.64 | 40.38 | 17.43 | + .53 |
| A2282 | 15.76 | 40.18 | 17.87 | + .41 |
| A2283 | 15.60 | 40.05 | 17.10 | +1.10 |
| A2284 | 15.34 | 40.22 | 17.10 | +2.00 |
| A2285 | 15.45 | 39.21 | 17.15 | +1.33 |
| A2285a | 15.35 | 39.36 | F | + |
| A2286 | 15.52 | 38.95 | 17.32 | + .96 |
| A2287 | 15.33 | 38.77 | 16.58 | + .98 |
| A2288 | 15.29 | 38.99 | 18.24 | +2.74 |
| A2289 | 15.22 | 38.02 | 15.64 | +1.08 |
| A2290 | 15.48 | 37.34 | 18.16 | +2.60 |
| A2291 | 15.45 | 36.61 | 18.16 | +1.06 |
| A2292 | 15.41 | 36.25 | 14.70 | + .88 |
| A2293 | 15.52 | 36.05 | 16.85 | +2.57 |
| A2294 | 15.61 | 35.53 | 17.78 | +1.78 |
| A2295 | 15.11 | 39.44 | 14.00 | + |
| A2296 | 14.98 | 39.13 | 14.48 | + |
| A2297 | 15.10 | 38.44 | 16.98 | + .83 |
| A2298 | 14.97 | 38.13 | 16.48 | +1.82 |
| A2298a | 15.13 | 38.77 | F | + |
| A2298b | 14.98 | 38.77 | F | + |
| A2299 | 15.10 | 37.80 | 14.48 | + .41 |
| A2300 | 14.83 | 37.97 | 16.02 | + .96 |
| A2301 | 14.71 | 38.08 | 17.10 | +1.70 |
| A2302 | 14.58 | 38.26 | 16.34 | + .90 |
| A2302a | 14.73 | 36.29 | 18.64 | +2.43 |
| A2303 | 15.13 | 37.03 | 18.38 | + .63 |
| A2304 | 15.15 | 37.24 | 18.16 | +2.16 |
| A2305 | 15.21 | 37.49 | 18.12 | +1.74 |
| A2306 | 15.12 | 36.83 | 16.98 | + .90 |
| A2307 | 14.88 | 36.74 | 16.24 | + .76 |
| A2308 | 15.05 | 36.44 | 16.88 | +1.93 |
| A2308a | 15.15 | 36.40 | 16.67 | +1.21 |
| A2401 | 20.01 | 28.75 | 18.02 | +2.00 |
| A2402 | 19.94 | 28.95 | 18.32 | |
| A2403 | 20.10 | 28.58 | 18.39 | +3.20 |
| A2404 | 20.11 | 27.58 | 15.93 | +1.10 |
| A2405 | 19.94 | 28.06 | 16.65 | + .77 |
| A2406 | 19.89 | 27.80 | 14.22 | + .76 |
| A2407 | 20.11 | 27.11 | 16.88 | + .86 |
| A2408 | 20.06 | 26.87 | 17.77 | + .68 |
| A2409 | 20.20 | 26.54 | 18.02 | + .88 |
| A2410 | 20.02 | 26.48 | 18.08 | + .71 |
| A2411 | 20.08 | 26.41 | 17.58 | + .95 |
| A2412 | 19.63 | 28.92 | 16.76 | +2.24 |
| A2413 | 19.38 | 29.10 | 17.94 | +1.28 |
| A2414 | 19.52 | 28.43 | 17.60 | +2.83 |
| A2415 | 19.79 | 28.10 | 16.67 | + .57 |
| A2416 | 19.84 | 28.01 | 16.03 | + .63 |
| A2417 | 19.84 | 27.61 | 16.28 | +1.19 |
| A2418 | 19.54 | 27.50 | 16.33 | +1.01 |
| A2419 | 19.36 | 27.28 | 16.33 | +2.19 |
| A2420 | 19.24 | 27.10 | 16.76 | + .80 |
| A2421 | 19.47 | 26.67 | 15.80 | + .86 |
| A2422 | 19.75 | 26.57 | 18.26 | |
| A2423 | 19.31 | 26.66 | 17.88 | +2.56 |
| A2424 | 19.27 | 26.59 | 17.74 | +1.39 |
| A2425 | 19.77 | 26.12 | 16.38 | +1.38 |
| A2426 | 19.32 | 26.13 | 16.53 | +1.70 |
| A2427 | 19.36 | 26.00 | 15.13 | +1.71 |
| A2427a | 19.20 | 25.90 | F | |
| A2428 | 19.26 | 25.19 | 15.38 | +1.24 |
| A2429 | 19.11 | 29.62 | 17.46 | + .71 |
| A2430 | 19.00 | 29.34 | 18.49 | +2.57 |
| A2431 | 19.19 | 29.19 | 15.28 | + .45 |
| A2432 | 18.82 | 29.27 | 17.16 | +2.07 |
| A2433 | 18.72 | 29.08 | 18.02 | +2.27 |
| A2434 | 18.84 | 28.94 | 18.36 | |
| A2435 | 19.13 | 28.79 | 17.29 | +2.13 |
| A2435a | 18.93 | 28.79 | F | + |
| A2436 | 19.13 | 28.65 | 17.75 | + .64 |
| A2437 | 19.23 | 28.42 | 16.58 | + .86 |
| A2438 | 19.16 | 28.33 | 17.45 | +1.05 |
| A2439 | 18.73 | 28.30 | 16.06 | +1.93 |
| A2440 | 19.14 | 28.07 | 18.29 | + .39 |
| A2441 | 19.05 | 27.53 | 17.73 | +1.07 |
| A2442 | 18.96 | 27.44 | 18.22 | +2.28 |
| A2443 | 18.83 | 26.80 | 17.82 | +1.96 |
| A2443a | 18.83 | 27.00 | F | + |
| A2443b | 18.80 | 26.74 | F | + |
| A2444 | 19.10 | 26.24 | 15.97 | +1.21 |
| A2445 | 18.91 | 26.34 | 16.07 | +1.07 |
| A2446 | 18.59 | 26.41 | 15.91 | +1.24 |
| A2447 | 18.91 | 25.71 | 15.22 | + .64 |
| A2448 | 18.98 | 25.39 | 14.84 | + .78 |
| A2449 | 18.81 | 25.46 | 17.45 | + .76 |
| A2450 | 19.05 | 25.13 | 16.92 | +1.83 |
| A2451 | 19.02 | 24.92 | 17.34 | + .78 |
| A2452 | 18.50 | 29.57 | 18.25 | |
| A2453 | 18.14 | 30.02 | 16.60 | + .80 |
| A2454 | 18.21 | 29.31 | 18.29 | + .82 |
| A2455 | 18.27 | 28.96 | 17.74 | + .65 |

TABLE 6—*Concluded*

| No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A2456 | 18.45 | 28.99 | 17.40 | + .84 | A2504 | 16.91 | 29.29 | 17.45 | + .79 | A2552 | 16.07 | 30.10 | 13.55 | + |
| A2457 | 18.36 | 28.79 | 16.38 | + .90 | A2505 | 16.90 | 29.03 | 17.33 | + .49 | A2553 | 15.59 | 29.73 | 14.29 | + .47 |
| A2458 | 18.28 | 28.68 | 17.82 | + .74 | A2505a | 16.90 | 29.00 | F | + | A2554 | 15.93 | 29.20 | 16.70 | +1.15 |
| A2459 | 18.19 | 28.83 | 18.42 | | A2506 | 16.97 | 28.59 | 17.33 | +1.45 | A2555 | 15.69 | 28.87 | 16.48 | + .70 |
| A2460 | 18.47 | 27.82 | 18.32 | +1.06 | A2507 | 17.08 | 28.68 | 18.11 | − | A2556 | 15.88 | 28.74 | 18.16 | |
| A2461 | 18.40 | 27.47 | 16.60 | + .82 | A2508 | 17.09 | 28.42 | 16.48 | +1.33 | A2557 | 15.58 | 29.15 | 17.38 | +1.36 |
| A2461a | 18.12 | 27.88 | F | + | A2508a | 17.15 | 28.45 | F | + | A2558 | 15.57 | 28.37 | 18.24 | +2.06 |
| A2462 | 18.50 | 26.92 | 15.42 | + .50 | A2509 | 16.93 | 28.23 | 18.02 | + .77 | A2559 | 15.94 | 27.91 | 17.16 | +1.21 |
| A2463 | 18.10 | 26.84 | 14.90 | + .80 | A2510 | 16.97 | 28.12 | 17.45 | +1.35 | A2560 | 15.92 | 27.73 | 16.82 | + .96 |
| A2464 | 18.16 | 26.72 | 17.50 | +1.48 | A2511 | 16.79 | 28.28 | 17.94 | +1.19 | A2561 | 15.79 | 27.33 | 16.78 | + .74 |
| A2465 | 18.49 | 26.36 | 18.11 | +1.03 | A2512 | 17.27 | 28.02 | 17.60 | +1.44 | A2562 | 15.99 | 27.24 | 16.98 | +1.98 |
| A2466 | 18.27 | 26.00 | 18.26 | | A2513 | 17.09 | 27.85 | 18.46 | | A2563 | 16.13 | 27.05 | 17.98 | +1.59 |
| A2467 | 18.25 | 25.73 | 17.92 | +2.12 | A2514 | 16.94 | 27.31 | 18.16 | | A2564 | 16.15 | 26.90 | 18.16 | +1.11 |
| A2468 | 18.31 | 25.63 | 17.92 | + .15 | A2515 | 17.01 | 26.98 | 18.11 | | A2565 | 16.02 | 26.24 | 14.04 | + .77 |
| A2469 | 18.04 | 25.92 | 17.60 | +1.04 | A2516 | 17.25 | 26.88 | 17.10 | +1.24 | A2565a | 15.92 | 26.59 | F | + |
| A2470 | 17.95 | 25.53 | 15.75 | +1.14 | A2517 | 17.02 | 26.42 | 16.06 | +1.14 | A2566 | 15.62 | 26.92 | 15.60 | +1.72 |
| A2471 | 18.21 | 24.84 | 15.98 | + .86 | A2518 | 17.10 | 26.37 | 18.35 | +1.73 | A2567 | 15.52 | 26.68 | 14.78 | + .90 |
| A2472a | 18.46 | 24.74 | 18.65 | | A2519 | 17.15 | 26.15 | 18.22 | | A2568 | 15.77 | 25.91 | 17.87 | +1.13 |
| A2472b | 17.60 | 30.32 | 16.42 | + .91 | A2520 | 17.32 | 25.97 | 18.39 | | A2569 | 15.99 | 25.50 | 17.78 | +1.22 |
| A2473 | 17.66 | 29.98 | 17.55 | +1.37 | A2521 | 17.30 | 25.80 | 15.44 | +1.62 | A2570 | 15.65 | 25.60 | 14.04 | + |
| A2474 | 17.83 | 29.86 | 17.22 | + .66 | A2522 | 17.22 | 25.73 | 18.16 | +1.66 | A2571 | 15.61 | 25.12 | 17.10 | + .77 |
| A2475 | 17.84 | 29.74 | 16.60 | + .74 | A2523 | 16.89 | 25.81 | 17.04 | +1.24 | A2572 | 15.49 | 25.26 | 17.23 | + .57 |
| A2476 | 17.65 | 29.73 | 15.86 | +1.54 | A2524 | 17.18 | 25.29 | 18.08 | +1.87 | A2573 | 15.98 | 24.75 | 18.16 | + .69 |
| A2477 | 17.48 | 29.76 | 14.90 | +1.08 | A2525 | 16.84 | 25.22 | B | | A2574 | 15.32 | 29.60 | 17.65 | + .83 |
| A2478 | 17.94 | 29.22 | 16.92 | +2.42 | A2526 | 17.31 | 24.80 | 17.68 | +1.02 | A2575 | 15.53 | 29.40 | 18.08 | + .46 |
| A2479 | 17.75 | 29.05 | 16.48 | + .68 | A2527 | 17.14 | 24.63 | 15.86 | +2.62 | A2576 | 15.40 | 29.33 | 18.34 | |
| A2480 | 17.70 | 28.75 | 15.76 | +1.00 | A2528 | 16.61 | 30.08 | 17.60 | +1.34 | A2577 | 15.46 | 29.16 | 17.92 | +1.90 |
| A2481 | 17.46 | 28.94 | 17.44 | + .80 | A2529 | 16.68 | 29.63 | 15.90 | +1.05 | A2578 | 15.38 | 29.21 | 17.98 | +1.34 |
| A2482 | 17.35 | 29.10 | 16.65 | + .93 | A2530 | 16.27 | 29.58 | 14.47 | + .41 | A2579 | 15.31 | 28.97 | 17.65 | + .99 |
| A2483 | 17.97 | 28.46 | 17.65 | + .57 | A2531 | 16.50 | 28.95 | 17.78 | + .16 | A2580 | 15.26 | 28.75 | 14.04 | + .54 |
| A2484 | 17.77 | 27.46 | 18.02 | + .25 | A2532 | 16.41 | 28.96 | 17.78 | +1.60 | A2581 | 15.31 | 28.27 | 17.88 | + .96 |
| A2485 | 17.32 | 27.41 | 14.89 | + .07 | A2533 | 16.42 | 28.73 | 14.51 | + .63 | A2582 | 15.22 | 27.94 | 17.46 | + .80 |
| A2486 | 17.73 | 27.08 | 17.45 | + .53 | A2534 | 16.56 | 28.61 | 17.74 | +1.08 | A2583 | 15.12 | 28.08 | 17.62 | + .88 |
| A2487 | 17.45 | 26.67 | 16.27 | + .87 | A2535 | 16.49 | 28.44 | 18.08 | +1.10 | A2583a | 15.22 | 28.23 | F | + |
| A2487a | 17.45 | 26.77 | F | + | A2536 | 16.26 | 28.60 | 14.67 | + .65 | A2584 | 15.45 | 27.27 | 17.65 | +1.39 |
| A2488 | 17.49 | 26.56 | 16.22 | +1.16 | A2537 | 16.37 | 28.25 | 17.04 | +2.04 | A2584a | 15.45 | 27.37 | 18.15 | + .68 |
| A2489 | 17.62 | 26.19 | 17.10 | + .84 | A2538 | 16.25 | 28.33 | 17.16 | + .41 | A2585 | 15.31 | 26.70 | 15.76 | + .67 |
| A2490 | 17.82 | 25.94 | 14.67 | +1.16 | A2539 | 16.53 | 27.95 | 17.04 | +1.12 | A2586 | 15.18 | 26.38 | 16.64 | + .54 |
| A2491 | 17.63 | 25.84 | 16.12 | +1.93 | A2540 | 16.18 | 27.87 | 16.17 | + .99 | A2587 | 15.34 | 25.96 | 18.19 | +2.09 |
| A2492 | 17.85 | 25.42 | 16.99 | + .76 | A2541 | 16.37 | 27.47 | 18.16 | + .90 | A2588 | 15.25 | 25.34 | 17.84 | + .85 |
| A2493 | 17.50 | 25.46 | 16.70 | + .35 | A2542 | 16.37 | 27.02 | 18.49 | | A2589 | 15.34 | 25.24 | 18.12 | + .77 |
| A2494 | 17.70 | 25.22 | 15.34 | +1.08 | A2542a | 16.17 | 27.05 | F | | A2590 | 15.02 | 26.31 | 18.34 | |
| A2494a | 17.70 | 25.42 | F | + | A2543 | 16.67 | 26.32 | 16.58 | + .94 | A2591 | 14.84 | 29.04 | 17.23 | + .67 |
| A2495 | 17.90 | 24.89 | 17.91 | +1.29 | A2544 | 16.19 | 26.75 | 16.60 | + .80 | A2592 | 14.76 | 28.40 | 16.21 | +1.46 |
| A2496 | 17.81 | 24.66 | 15.50 | +1.02 | A2545 | 16.23 | 26.53 | 18.07 | | A2593 | 14.56 | 28.49 | 16.48 | +1.72 |
| A2497 | 17.15 | 30.15 | 16.98 | − .29 | A2545a | 16.26 | 26.43 | F | + | A2594 | 14.66 | 28.58 | 17.74 | +1.16 |
| A2498 | 17.17 | 30.15 | 17.52 | +1.42 | A2546 | 16.20 | 25.89 | 17.68 | +1.66 | A2595 | 14.45 | 27.72 | 17.90 | +1.16 |
| A2499 | 16.85 | 30.45 | 13.94 | + .51 | A2547 | 16.15 | 25.50 | 17.40 | + .42 | A2596 | 14.41 | 27.36 | 17.44 | +1.35 |
| A2500 | 17.14 | 29.82 | 16.01 | +1.25 | A2548 | 16.14 | 25.14 | 18.02 | + .65 | A2597 | 14.40 | 27.02 | 17.38 | +1.14 |
| A2501 | 17.05 | 29.87 | 17.52 | +1.04 | A2549 | 16.44 | 25.00 | 17.45 | +1.35 | A2598 | 14.83 | 26.66 | 17.65 | +1.85 |
| A2502 | 17.15 | 29.32 | 14.22 | + .72 | A2550 | 16.61 | 25.00 | 17.88 | +1.18 | A2599 | 14.48 | 26.43 | 15.96 | + .87 |
| A2503 | 17.03 | 29.38 | 17.50 | +1.08 | A2551 | 16.55 | 24.75 | 18.11 | | A2600 | 14.60 | 26.23 | 13.70 | + |

TABLE 7

AREA B

A = 281° 26′ 55″ } B110
D = −8° 7′ 37″ }
a = −0.000, 190

b = +0.000, 001, 07
c = +0.003, 540
d = +0.006, 603

| No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| B 1* | 4.95 | 29.10 | 10.84 | + .04 | B60 | 17.39 | 44.74 | 12.38 | − .30 | B119 | 19.92 | 35.51 | 15.98 | +1.47 |
| B 2* | 6.05 | 29.88 | 11.91 | + .49 | B61 | 17.12 | 43.53 | 15.54 | + .31 | B120 | 19.10 | 34.67 | 15.74 | + .79 |
| B 3 | 7.94 | 37.38 | 15.06 | + .62 | B62 | 16.42 | 41.78 | 15.26 | + .85 | B121 | 18.96 | 31.87 | 14.72 | + .38 |
| B 4 | 7.99 | 37.27 | 15.00 | +1.55 | B63 | 16.13 | 40.19 | 13.67 | + .02 | B122 | 19.57 | 30.40 | F | + |
| B 5 | 8.63 | 31.84 | 13.72 | | B64 | 16.13 | 39.68 | 15.47 | +1.26 | B123 | 19.98 | 28.12 | F | + |
| B 6 | 8.20 | 31.14 | 15.50 | + .22 | B65 | 17.14 | 37.21 | 15.12 | +1.54 | B124 | 19.95 | 24.05 | 14.93 | + .90 |
| B 7 | 9.56 | 37.55 | 13.79 | + .98 | B66 | 16.60 | 37.88 | 15.20 | + .41 | B125 | 19.85 | 23.42 | 15.64 | + .52 |
| B 8 | 9.95 | 41.03 | 13.59 | − .69 | B67 | 16.91 | 37.90 | F | + | B126 | 20.51 | 45.57 | 15.74 | + .54 |
| B 9 | 9.28 | 40.02 | 15.71 | +1.14 | B68 | 16.64 | 38.20 | F | + | B127 | 20.75 | 43.93 | 15.11 | + .51 |
| B10 | 9.02 | 32.15 | 15.00 | + .66 | B69 | 16.03 | 35.34 | F | + | B128 | 21.23 | 38.97 | 13.93 | + .64 |
| B11 | 11.04 | 38.73 | 15.20 | + .48 | B70 | 16.15 | 34.92 | F | + | B129 | 20.42 | 34.20 | 14.00 | + .10 |
| B12 | 11.33 | 37.65 | 15.62 | +1.64 | B71 | 16.36 | 34.83 | F | + | B130 | 21.39 | 32.37 | 14.70 | + .66 |
| B13 | 10.34 | 37.09 | 14.20 | +1.26 | B72 | 16.51 | 34.57 | F | + | B131 | 21.02 | 31.46 | 16.02 | +1.25 |
| B14 | 10.80 | 36.24 | 14.67 | +1.64 | B73 | 16.59 | 34.22 | F | + | B132 | 21.10 | 31.07 | 16.38 | +1.72 |
| B15 | 11.28 | 35.58 | 13.60 | + .33 | B74 | 16.26 | 30.58 | 15.74 | +1.97 | B133 | 21.00 | 31.02 | 15.32 | +1.24 |
| B16 | 10.23 | 35.45 | 14.83 | + .26 | B75 | 16.84 | 28.58 | 15.47 | +1.31 | B134 | 20.90 | 28.27 | 15.78 | +1.30 |
| B17 | 10.55 | 30.88 | 16.32 | +1.40 | B76 | 17.15 | 27.80 | 15.86 | + .87 | B135 | 21.18 | 28.05 | 15.88 | +1.85 |
| B18 | 11.30 | 30.92 | 15.12 | + .33 | B77 | 17.79 | 24.98 | 15.54 | + .54 | B136 | 20.55 | 27.13 | 15.56 | + .31 |
| B19 | 11.50 | 29.27 | 14.52 | + .23 | B78* | 18.00 | 21.13 | 13.64 | +1.10 | B137 | 22.12 | 24.47 | 12.91 | + .26 |
| B20 | 11.43 | 29.00 | 14.30 | − .14 | B79 | 18.43 | 45.66 | 14.04 | + .47 | B138* | 22.99 | 46.08 | 12.52 | + .60 |
| B21 | 13.80 | 44.54 | 15.00 | + .43 | B80 | 18.38 | 45.26 | 15.14 | + .50 | B139 | 22.74 | 45.23 | 14.90 | +1.42 |
| B22 | 14.25 | 43.44 | 14.25 | +1.10 | B81 | 18.22 | 44.13 | 15.74 | +1.40 | B140 | 22.72 | 44.96 | 16.25 | +1.59 |
| B23 | 14.09 | 40.62 | 16.15 | +1.33 | B82 | 18.33 | 42.35 | 15.99 | +1.09 | B141 | 22.69 | 44.79 | 14.93 | + .26 |
| B24 | 14.32 | 40.24 | 14.72 | + .15 | B83 | 18.29 | 41.50 | 14.30 | − .16 | B142 | 22.24 | 43.90 | 15.61 | + .43 |
| B25 | 14.68 | 39.34 | 13.00 | + .44 | B84 | 18.50 | 39.02 | F | + | B143 | 21.77 | 43.95 | 14.78 | + .10 |
| B26 | 13.48 | 40.10 | 15.26 | + .48 | B85 | 17.93 | 36.31 | 14.25 | +1.35 | B144 | 21.72 | 42.23 | 14.61 | + .52 |
| B27 | 13.52 | 39.76 | 15.97 | + .55 | B86 | 17.40 | 35.15 | 15.40 | +1.83 | B145 | 22.07 | 42.33 | 15.75 | + .60 |
| B28 | 14.04 | 38.38 | 14.78 | +1.19 | B87 | 18.12 | 31.72 | 15.33 | + .48 | B146 | 21.85 | 36.68 | 15.78 | +1.30 |
| B29 | 13.91 | 37.55 | 15.00 | + .28 | B88 | 17.92 | 31.64 | 15.74 | +1.40 | B147 | 22.28 | 36.96 | 15.88 | +1.43 |
| B30 | 14.11 | 36.02 | 15.66 | +1.67 | B89 | 17.98 | 30.27 | 16.02 | +1.38 | B148 | 23.01 | 36.55 | 15.11 | + .50 |
| B31 | 13.36 | 35.87 | 13.29 | − .08 | B90 | 18.54 | 30.09 | 15.60 | + .51 | B149 | 22.89 | 34.96 | 16.04 | +1.26 |
| B32 | 14.01 | 34.72 | 15.40 | + .68 | B91 | 17.86 | 28.86 | 15.69 | +1.34 | B150 | 22.67 | 34.79 | 14.70 | + .94 |
| B33 | 13.11 | 33.04 | 15.56 | +1.46 | B92 | 18.19 | 27.27 | 16.17 | +1.68 | B151 | 22.19 | 32.62 | 15.11 | + .44 |
| B34 | 13.87 | 31.73 | 15.27 | + .84 | B93 | 18.31 | 25.13 | 15.62 | +1.12 | B152 | 21.84 | 30.30 | F | + |
| B35 | 13.57 | 31.43 | 15.92 | +1.08 | B94 | 17.86 | 24.91 | 14.40 | +1.10 | B153 | 22.20 | 25.24 | 15.26 | + .58 |
| B36 | 13.92 | 27.18 | 15.40 | + .12 | B95 | 12.46 | 43.89 | 14.20 | + .26 | B154 | 22.13 | 25.10 | 14.64 | + .60 |
| B37 | 13.82 | 26.39 | 15.47 | + .61 | B96 | 12.70 | 42.55 | 15.06 | + .58 | B155 | 21.46 | 25.49 | 14.98 | − .08 |
| B38 | 13.69 | 26.77 | 14.39 | − .51 | B97 | 12.06 | 41.81 | 15.47 | +1.12 | B156 | 23.70 | 24.90 | 13.74 | +1.56 |
| B39 | 13.84 | 26.03 | 15.69 | + .34 | B98 | 12.12 | 41.37 | 14.14 | + .46 | B157 | 24.34 | 40.44 | 13.42 | +1.74 |
| B40 | 15.24 | 41.16 | 15.01 | +1.08 | B99 | 12.40 | 39.31 | 15.80 | +1.56 | B158 | 24.14 | 39.75 | 11.56 | +1.95 |
| B41 | 14.78 | 42.68 | F | + | B100 | 12.66 | 36.77 | 15.46 | + .41 | B159 | 23.62 | 43.08 | 13.38 | + .10 |
| B42 | 14.82 | 42.41 | 15.64 | + .74 | B101 | 12.32 | 35.91 | 15.69 | +1.26 | B160 | 23.13 | 43.27 | 15.96 | +1.15 |
| B43 | 14.77 | 39.87 | 15.60 | + .63 | B102 | 12.36 | 31.38 | 14.30 | + .42 | B161 | 24.14 | 42.67 | F | + |
| B44 | 14.84 | 38.43 | 15.84 | +1.04 | B103 | 12.47 | 28.90 | 16.79 | +2.36 | B162 | 23.23 | 35.67 | 14.46 | + .48 |
| B45 | 15.35 | 38.53 | 15.47 | +1.93 | B104 | 12.07 | 28.35 | 15.47 | + .90 | B163 | 24.30 | 37.17 | 15.18 | + .57 |
| B46 | 14.94 | 32.16 | 16.02 | +1.88 | B105 | 12.82 | 26.16 | 15.47 | + .82 | B164 | 24.12 | 34.92 | 14.66 | + .08 |
| B47 | 15.15 | 31.93 | F | + | B106* | 18.66 | 20.15 | 13.15 | + .38 | B165 | 24.06 | 34.49 | F | + |
| B48 | 15.86 | 32.16 | 14.20 | + .26 | B107* | 19.15 | 21.26 | 13.06 | +1.33 | B166 | 23.94 | 29.65 | 14.77 | − .01 |
| B49 | 15.39 | 29.57 | 14.72 | + .40 | B108 | 18.99 | 28.74 | 9.73 | + .70 | B167 | 23.58 | 29.68 | 15.41 | + .26 |
| B50 | 15.86 | 28.79 | 14.46 | − .03 | B109 | 19.44 | 29.49 | 12.56 | + .11 | B168 | 23.92 | 30.61 | F | + |
| B51 | 15.86 | 27.32 | 14.40 | + .46 | B110 | 18.83 | 34.67 | B | | B169 | 23.38 | 31.63 | 15.60 | + .51 |
| B52 | 15.05 | 27.37 | 14.14 | + .44 | B111 | 19.58 | 42.21 | 15.64 | + .40 | B170 | 23.49 | 29.07 | 15.34 | + .04 |
| B53 | 15.37 | 27.21 | 14.78 | +1.91 | B112 | 19.04 | 41.76 | 14.88 | + .24 | B171 | 24.09 | 30.16 | 15.95 | + .86 |
| B54 | 15.05 | 26.47 | 15.47 | + .91 | B113 | 19.45 | 41.55 | 15.21 | + .66 | B172 | 24.02 | 28.57 | 15.48 | + .64 |
| B55 | 15.45 | 28.19 | 15.08 | + .01 | B114 | 19.13 | 39.11 | 15.64 | +1.37 | B173 | 23.15 | 28.58 | 15.39 | + .08 |
| B56 | 15.25 | 25.98 | 15.93 | +1.29 | B115 | 19.38 | 38.13 | 15.08 | +2.27 | B174 | 22.98 | 28.15 | 15.56 | + .97 |
| B57 | 15.50 | 24.05 | 15.40 | + .68 | B116 | 20.06 | 37.18 | 15.08 | + .89 | B175 | 23.45 | 27.79 | 14.83 | + .06 |
| B58 | 16.21 | 30.92 | 10.91 | −1.03 | B117 | 19.12 | 36.05 | 15.82 | + .57 | B176 | 24.00 | 25.94 | 15.11 | + .44 |
| B59 | 16.18 | 31.58 | 13.14 | − .90 | B118 | 20.02 | 35.84 | F | + | B177 | 24.77 | 41.54 | 14.56 | + .09 |

TABLE 7—*Concluded*

| No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| B178 | 25.10 | 40.19 | F | + | B194* | 27.05 | 24.21 | 13.73 | + .64 | B209 | 28.27 | 37.94 | 15.86 | + .54 |
| B179 | 25.53 | 39.68 | 14.14 | + .38 | B195 | 26.21 | 37.22 | 12.52 | − .19 | B210 | 27.98 | 36.68 | 15.64 | + .50 |
| B180 | 25.13 | 37.14 | 15.52 | +1.44 | B196 | 26.44 | 38.49 | 13.04 | + .44 | B211 | 28.15 | 36.55 | 15.24 | + .42 |
| B181 | 24.94 | 36.19 | 15.84 | + .48 | B197 | 26.97 | 40.69 | 15.88 | +1.28 | B212 | 28.36 | 34.24 | 15.32 | +1.00 |
| B182 | 24.65 | 35.69 | 15.75 | +1.15 | B198 | 26.48 | 30.59 | 14.50 | + .30 | B213 | 27.69 | 33.90 | 15.62 | +1.04 |
| B183 | 24.37 | 35.51 | 15.31 | + .32 | B199 | 26.00 | 32.65 | 15.75 | +1.59 | B214 | 27.79 | 33.22 | 13.85 | +1.01 |
| B184 | 24.40 | 34.12 | 15.11 | + .44 | B200 | 26.06 | 32.30 | 15.82 | +1.16 | B215 | 28.38 | 32.08 | 15.11 | + .34 |
| B185 | 25.50 | 32.92 | 14.58 | + .33 | B201 | 26.21 | 31.12 | 14.98 | + .22 | B216 | 28.38 | 31.79 | 15.70 | + .48 |
| B186 | 25.14 | 30.44 | 14.11 | + .08 | B202 | 26.10 | 30.82 | 15.06 | +2.14 | B217 | 27.26 | 28.48 | 15.62 | + .64 |
| B187 | 24.40 | 28.62 | 15.11 | + .58 | B203 | 26.04 | 28.82 | 15.71 | + .30 | B218* | 28.66 | 27.97 | 13.27 | + .52 |
| B188 | 25.00 | 27.16 | 16.12 | +1.22 | B204 | 26.93 | 27.06 | 14.06 | + .59 | B219 | 29.07 | 31.68 | 10.40 | +1.24 |
| B189 | 25.00 | 27.00 | 15.71 | + .26 | B205 | 27.01 | 26.91 | 15.54 | + .06 | B220 | 29.02 | 39.65 | 14.66 | + .20 |
| B190 | 25.51 | 26.50 | 14.89 | +1.14 | B206 | 28.30 | 28.35 | 12.69 | + .12 | B221 | 29.13 | 36.58 | 15.04 | + .08 |
| B191 | 25.52 | 25.71 | 14.34 | +1.74 | B207 | 28.03 | 40.84 | 16.20 | +1.52 | B222 | 29.17 | 31.50 | F | + |
| B192 | 25.21 | 25.40 | 15.04 | + .52 | B208 | 27.85 | 37.94 | 15.98 | + .84 | B223* | 30.99 | 43.58 | 12.71 | +1.12 |
| B193 | 24.29 | 25.73 | 15.32 | + .56 | | | | | | | | | | |

TABLE 8

AREA C

A = 281° 53′ 1″
D = −9° 41′ 50″ } C77
a = −0.000, 190

b = —
c = +0.003, 585±
d = −0.006, 550±

| No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C 1* | 4.02 | 37.47 | 11.64 | − .09 | C39 | 15.41 | 36.95 | 14.92 | + .56 | C77 | 18.88 | 34.45 | B | |
| C 2* | 8.22 | 39.58 | 13.94 | +1.25 | C40 | 15.32 | 35.49 | 16.29 | +1.85 | C78 | 19.89 | 38.80 | 11.66 | + .03 |
| C 3 | 7.72 | 33.82 | 16.28 | +2.05 | C41 | 15 23 | 33.09 | 15.93 | +1.25 | C79 | 19.35 | 41.37 | D | |
| C 4 | 8.00 | 33.23 | 15.70 | +1.25 | C42 | 15.56 | 30.05 | 16.00 | +1.32 | C80 | 18.77 | 35.56 | 15.53 | |
| C 5* | 9.80 | 23.76 | 12.70 | − .11 | C43 | 15.50 | 28.34 | 15.99 | +1.24 | C81 | 19.20 | 34.03 | 15.72 | + .98 |
| C 6 | 9.31 | 38.59 | 14.47 | + .51 | C44 | 15.89 | 28.58 | 14.98 | +1.44 | C82 | 19.49 | 33.53 | 14.77 | + .37 |
| C 7 | 9.11 | 32.77 | 15.28 | +1.20 | C45 | 15.67 | 26.17 | 15.04 | + .68 | C83 | 19.40 | 33.57 | 15.93 | +1.38 |
| C 8 | 9.49 | 33.07 | 15.76 | +1.01 | C46 | 15.13 | 24.30 | 16.05 | +1.61 | C84 | 19.27 | 30.21 | 15.56 | +1.04 |
| C 9 | 9.95 | 32.19 | 14.06 | + .40 | C47 | 14.50 | 24.26 | 15.36 | + .80 | C85 | 19.12 | 29.39 | 15.57 | +1.66 |
| C10 | 10.25 | 34.22 | 14.36 | +1.70 | C48 | 17.15 | 43.58 | 13.98 | + .50 | C86 | 18.95 | 27.83 | 15.70 | +1.20 |
| C11 | 10.21 | 33.66 | 15.26 | +1.72 | C49 | 16.91 | 43.32 | 14.96 | +1.65 | C87 | 19.75 | 23.35 | 15.53 | +1.01 |
| C12 | 9.20 | 30.34 | 15.42 | +1.76 | C50 | 16.80 | 39.32 | 15.54 | + .82 | C88 | 19.36 | 23.60 | 16.31 | |
| C13 | 10.27 | 31.44 | 15.62 | + .80 | C51 | 16.26 | 38.25 | 15.38 | + .94 | C89 | 21.35 | 24.68 | 11.22 | + .23 |
| C14 | 10.91 | 33.02 | 15.89 | + .91 | C52 | 16.88 | 32.32 | 15.36 | + .94 | C90 | 20.81 | 44.11 | 11.10 | − .06 |
| C15 | 11.24 | 32.24 | 15.52 | + .66 | C53 | 16.30 | 31.95 | 15.56 | + .78 | C91 | 20.41 | 42.33 | 15.20 | + .80 |
| C16 | 11.24 | 30.14 | 15.52 | +1.50 | C54 | 16.32 | 29.24 | 13.56 | +1.47 | C92 | 20.16 | 40.01 | 15.93 | +1.21 |
| C17 | 11.01 | 28.25 | 14.72 | + .50 | C55 | 16.87 | 25.62 | 16.15 | +1.47 | C93 | 21.25 | 38.60 | 16.08 | +1.74 |
| C18 | 12.25 | 32.88 | 13.78 | + .85 | C56 | 16.91 | 23.50 | F | + | C94 | 20.90 | 34.24 | 15.93 | +1.73 |
| C19 | 13.02 | 38.15 | 15.05 | + .52 | C57 | 18.48 | 30.49 | 12.77 | + .18 | C95 | 21.17 | 33.84 | 15.66 | + .70 |
| C20 | 12.43 | 36.42 | 15.45 | + .37 | C58 | 17.76 | 44.06 | 15.36 | +1.44 | C96 | 21.14 | 28.89 | 14.98 | +1.70 |
| C21 | 11.65 | 33.86 | 15.55 | +1.24 | C59 | 17.52 | 42.57 | 15.48 | + .58 | C97 | 20.36 | 26.23 | 13.98 | + .78 |
| C22 | 12.03 | 31.88 | 15.45 | + .64 | C60 | 18.62 | 41.15 | 14.02 | − .05 | C98 | 21.03 | 29.59 | 16.18 | +1.09 |
| C23 | 11.96 | 29.67 | 15.78 | +1.06 | C61 | 18.22 | 40.27 | 15.56 | + .88 | C99 | 20.25 | 24.79 | 15.66 | +1.30 |
| C24 | 12.26 | 26.78 | 16.28 | +1.66 | C62 | 18.63 | 39.76 | D | | C100 | 22.20 | 35.36 | 12.54 | + .59 |
| C25 | 11.75 | 26.15 | 15.45 | + .64 | C63 | 18.79 | 38.84 | 15.57 | + .93 | C101 | 22.44 | 44.52 | 16.30 | +1.78 |
| C26 | 14.21 | 44.55 | 12.18 | − .06 | C64 | 17.41 | 34.63 | 15.57 | + .60 | C102 | 22.21 | 43.29 | 15.38 | + .59 |
| C27 | 13.95 | 39.94 | 15.48 | + .85 | C65 | 17.86 | 34.12 | 14.25 | + .22 | C103 | 23.06 | 41.88 | 15.99 | +1.60 |
| C28 | 13.64 | 39.75 | 15.74 | +1.16 | C66 | 18.23 | 33.73 | 14.10 | + .78 | C104 | 22.98 | 41.25 | 14.56 | +1.41 |
| C29 | 13.26 | 37.50 | 14.23 | − .58 | C67 | 18.49 | 33.62 | 15.76 | +2.11 | C105 | 22.05 | 40.78 | 15.98 | + .55 |
| C30 | 14.00 | 37.81 | 14.96 | + .36 | C68 | 18.01 | 32.67 | 15.97 | +1.67 | C106 | 22.22 | 38.52 | 15.38 | + .55 |
| C31 | 14.16 | 37.22 | 15.46 | + .74 | C69 | 18.47 | 31.41 | 15.57 | | C107 | 22.40 | 39.26 | 16.20 | + .42 |
| C32 | 13.29 | 33.91 | 15.83 | +1.74 | C70 | 17.65 | 31.41 | 16.16 | +1.48 | C108 | 22.48 | 37.50 | 15.43 | + .99 |
| C33 | 13.20 | 32.17 | 15.41 | +2.61 | C71 | 17.85 | 30.50 | 15.62 | + .80 | C109 | 22.48 | 40.11 | 16.09 | +1.53 |
| C34 | 14.08 | 31.71 | 15.74 | + .88 | C72 | 18.43 | 23.81 | 14.26 | + .41 | C110 | 22.04 | 35.49 | 15.53 | +1.09 |
| C35 | 13.38 | 28.74 | F | + | C73 | 18.28 | 24.38 | 15.79 | +1.02 | C111 | 22.59 | 33.94 | 14.38 | + .63 |
| C36 | 14.00 | 25.75 | 16.34 | +1.51 | C74 | 18.17 | 23.82 | 15.83 | + .97 | C112 | 22.30 | 32.32 | 15.87 | +1.35 |
| C37 | 13.64 | 24.71 | 16.08 | +1.53 | C75 | 17.35 | 23.83 | 15.60 | + .92 | C113 | 22.06 | 32.05 | 15.93 | + .89 |
| C38 | 15.93 | 39.02 | 15.68 | + .28 | C76 | 17.86 | 25.83 | D | | C114 | 21.60 | 27.25 | 15.38 | +1.76 |

TABLE 8—*Concluded*

| No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C115* | 24.36 | 46.26 | 12.30 | + .38 | C132 | 25.30 | 39.12 | 15.38 | +1.34 | C149 | 27.71 | 35.48 | 15.75 | +1.71 |
| C116 | 23.30 | 43.86 | 14.32 | + .18 | C133 | 24.66 | 38.58 | F | | C150 | 27.83 | 31.60 | 14.70 | +1.86 |
| C117 | 23.86 | 37.34 | 15.38 | +2.30 | C134 | 25.33 | 34.53 | 16.30 | +1.94 | C151 | 27.20 | 31.84 | 14.96 | + .50 |
| C118 | 24.19 | 37.11 | 15.50 | + .86 | C135 | 24.70 | 27.92 | 15.59 | +1.58 | C152 | 27.36 | 33.15 | 16.31 | +2.09 |
| C119 | 24.03 | 36.90 | 15.78 | + .94 | C136 | 25.56 | 27.26 | 14.92 | + .72 | C153 | 27.67 | 32.10 | 15.81 | |
| C120 | 24.07 | 36.59 | 15.93 | + .95 | C137 | 25.05 | 27.89 | 16.42 | +1.86 | C154 | 27.18 | 27.05 | 14.40 | + .10 |
| C121 | 23.35 | 33.70 | 16.31 | +1.88 | C138 | 24.94 | 26.90 | 16.02 | +1.28 | C155 | 27.36 | 27.11 | 15.87 | + .96 |
| C122 | 23.32 | 33.23 | 14.90 | + .64 | C139 | 25.92 | 39.49 | 15.59 | + .87 | C156 | 29.15 | 38.84 | 15.34 | + .86 |
| C123 | 23.62 | 33.13 | 15.56 | +1.48 | C140 | 26.28 | 38.62 | 14.62 | +1.94 | C157 | 28.91 | 37.89 | 15.78 | +2.28 |
| C124 | 23.16 | 31.51 | 16.42 | +1.86 | C141 | 26.85 | 38.72 | 15.90 | +1.12 | C158 | 29.11 | 36.54 | 14.58 | +1.65 |
| C125 | 24.12 | 30.04 | 15.90 | +1.50 | C142 | 26.01 | 29.68 | 15.44 | +2.12 | C159 | 28.95 | 35.46 | 15.28 | +2.01 |
| C126 | 23.21 | 27.33 | 16.96 | +2.06 | C143 | 25.97 | 30.71 | 16.42 | +1.96 | C160 | 29.63 | 32.94 | 16.31 | +1.85 |
| C127 | 23.36 | 25.28 | 15.84 | + .90 | C144 | 27.07 | 30.75 | 16.09 | +1.63 | C161 | 28.99 | 31.84 | 16.09 | +1.00 |
| C128 | 23.95 | 27.16 | 16.36 | +1.58 | C145 | 25.84 | 26.91 | 15.20 | +1.50 | C162 | 29.17 | 31.44 | 15.50 | + .88 |
| C129 | 24.12 | 24.94 | 16.23 | +1.14 | C146 | 26.96 | 27.87 | 16.12 | +1.54 | C163 | 28.89 | 30.67 | 15.86 | +1.20 |
| C130 | 25.16 | 40.35 | 16.31 | +1.75 | C147 | 27.39 | 39.27 | 15.84 | + .97 | C164 | 28.56 | 30.74 | 16.04 | +1.51 |
| C131 | 25.16 | 38.93 | 16.53 | +1.60 | C148 | 27.44 | 35.63 | 16.10 | +2.13 | C165 | 28.66 | 30.31 | 15.93 | +1.28 |

TABLE 9

AREA D

A = 282° 46′ 58″
D = −6° 4′ 35″ } Du
a = −0.000, 190, 6

b = +0.000, 001, 73
c = +0.003, 465
d = +0.003, 042

| No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D 1* | 6.04 | 42.18 | 13.67 | +1.62 | D39 | 13.11 | 36.92 | 15.11 | + .58 | D77 | 15.35 | 31.77 | 14.84 | +1.72 |
| D 2* | 6.38 | 26.61 | 13.57 | + .63 | D40 | 12.14 | 32.28 | 14.75 | +1.55 | D78 | 14.83 | 31.43 | 15.55 | +2.07 |
| D 3* | 7.22 | 26.41 | 14.11 | +1.15 | D41 | 13.08 | 33.56 | 14.22 | + .92 | D79 | 15.84 | 30.31 | 15.22 | +1.65 |
| D 4* | 7.98 | 39.60 | 13.95 | +1.63 | D42 | 11.95 | 29.28 | I | | D80 | 15.56 | 30.60 | 16.05 | + .96 |
| D 5 | 8.12 | 38.17 | 15.22 | + .92 | D43 | 12.87 | 28.98 | F | + | D81 | 14.67 | 29.80 | F | + |
| D 6 | 8.28 | 37.66 | 16.45 | | D44 | 12.66 | 28.95 | 16.05 | + .96 | D82 | 15.02 | 27.22 | 16.24 | +2.07 |
| D 7 | 8.10 | 34.12 | 16.24 | +1.76 | D45 | 12.18 | 29.99 | 15.88 | + .79 | D83 | 14.44 | 26.35 | 15.65 | + .86 |
| D 8 | 8.10 | 34.00 | 16.24 | +1.30 | D46 | 11.89 | 27.93 | F | + | D84 | 14.63 | 26.62 | F | + |
| D 9 | 8.41 | 33.24 | 16.45 | +2.63 | D47 | 11.58 | 27.81 | 13.80 | + .09 | D85 | 14.48 | 24.64 | 16.45 | +2.42 |
| D10 | 8.31 | 33.10 | 15.88 | + .94 | D48 | 12.06 | 26.23 | 14.80 | + .74 | D86* | 16.02 | 22.58 | 11.84 | + .17 |
| D11 | 8.56 | 30.34 | 13.68 | + .58 | D49 | 14.20 | 36.35 | 13.73 | + .60 | D87 | 16.97 | 23.95 | 13.53 | +1.42 |
| D12* | 8.33 | 28.20 | 13.73 | +1.98 | D50 | 13.96 | 41.81 | 14.88 | + .39 | D88 | 16.31 | 27.73 | 11.78 | + .20 |
| D13 | 10.18 | 35.02 | 11.82 | + .63 | D51 | 14.23 | 44.35 | 15.65 | + .56 | D89 | 16.21 | 36.00 | 12.62 | + .44 |
| D14* | 10.36 | 33.46 | 13.13 | + .51 | D52 | 13.98 | 39.85 | 15.08 | + .88 | D90* | 17.20 | 46.45 | 13.56 | + .67 |
| D15 | 9.58 | 40.94 | 15.65 | + .56 | D53 | 15.35 | 39.40 | 15.32 | +1.24 | D91 | 17.27 | 45.73 | 15.22 | + .82 |
| D16 | 9.78 | 40.45 | 15.76 | + .67 | D54 | 14.07 | 38.95 | 14.93 | + .79 | D92 | 17.16 | 45.22 | 15.11 | |
| D17 | 9.81 | 39.43 | 16.45 | +1.92 | D55 | 13.96 | 38.93 | 15.22 | + .88 | D93 | 17.29 | 43.77 | 15.11 | + .52 |
| D18 | 9.98 | 36.41 | 16.24 | +1.15 | D56 | 13.39 | 37.90 | 15.65 | +1.54 | D94 | 16.62 | 43.90 | 15.11 | + .52 |
| D19 | 10.17 | 33.05 | 15.44 | +2.14 | D57 | 14.10 | 39.43 | 16.05 | + .96 | D95 | 17.28 | 40.93 | 15.04 | +1.34 |
| D20 | 10.03 | 32.48 | 14.92 | + .13 | D58 | 13.41 | 36.37 | 14.62 | +2.38 | D96 | 17.14 | 40.25 | 15.88 | +1.57 |
| D21 | 10.12 | 32.58 | 14.75 | + .49 | D59 | 13.24 | 36.31 | 12.57 | + .48 | D97 | 17.30 | 39.21 | 14.58 | + .85 |
| D22 | 10.21 | 32.42 | 15.55 | +1.85 | D60 | 13.57 | 33.92 | 14.58 | + .44 | D98 | 17.14 | 39.50 | 16.45 | +2.28 |
| D23 | 9.86 | 29.25 | 15.22 | +1.69 | D61 | 13.74 | 30.40 | 16.45 | +1.36 | D99 | 16.95 | 39.17 | 15.11 | + .81 |
| D24 | 9.36 | 29.52 | F | + | D62 | 14.33 | 30.45 | 15.55 | +1.64 | D100 | 17.09 | 38.08 | 15.30 | +1.48 |
| D25 | 9.20 | 29.69 | 16.05 | +1.11 | D63 | 13.93 | 28.97 | 14.21 | +1.26 | D101 | 16.25 | 37.19 | 15.13 | + .94 |
| D26 | 11.44 | 31.36 | 13.21 | + .29 | D64 | 13.88 | 25.01 | 15.22 | +2.10 | D102 | 16.54 | 34.71 | 14.55 | +1.37 |
| D27 | 11.19 | 38.69 | 13.21 | + .23 | D65 | 13.12 | 24.93 | 15.88 | +1.78 | D103 | 16.64 | 32.79 | 16.05 | +1.59 |
| D28 | 10.90 | 36.42 | 16.45 | +1.66 | D66 | 15.49 | 29.95 | 14.18 | +1.40 | D104 | 16.95 | 32.26 | F | + |
| D29 | 10.74 | 35.83 | F | + | D67 | 15.07 | 43.27 | 13.63 | +1.35 | D105 | 17.08 | 29.49 | 14.92 | + .72 |
| D30 | 10.40 | 30.06 | 14.70 | + .60 | D68 | 14.66 | 40.09 | 14.70 | +1.22 | D106 | 16.96 | 29.07 | 15.32 | +1.26 |
| D31 | 10.43 | 29.09 | 16.45 | +2.28 | D69 | 14.94 | 39.82 | 13.68 | + .16 | D107 | 17.14 | 28.63 | 16.45 | +1.36 |
| D32 | 10.85 | 28.86 | 15.76 | + .67 | D70 | 14.63 | 38.04 | 14.83 | + .93 | D108 | 16.05 | 28.11 | 14.22 | + .60 |
| D33 | 11.32 | 28.29 | F | + | D71 | 15.48 | 38.03 | 16.24 | +1.65 | D109 | 16.32 | 28.12 | 14.94 | +1.36 |
| D34 | 11.86 | 43.08 | 14.65 | + .17 | D72 | 15.90 | 37.95 | 16.05 | +1.11 | D110 | 16.76 | 27.26 | 14.92 | + .82 |
| D35 | 12.63 | 40.53 | 14.32 | + .55 | D73 | 15.99 | 36.63 | 14.88 | +1.58 | D111 | 16.71 | 25.30 | 15.88 | +1.35 |
| D36 | 12.22 | 40.31 | 15.65 | + .71 | D74 | 15.43 | 37.23 | 15.76 | + .82 | D112 | 18.02 | 26.01 | 11.76 | + .28 |
| D37 | 12.19 | 36.52 | 15.76 | + .82 | D75 | 15.13 | 33.04 | 16.05 | +1.57 | D113 | 18.28 | 43.72 | 13.70 | + .48 |
| D38 | 11.59 | 36.48 | 15.65 | +1.06 | D76 | 15.27 | 32.46 | 14.84 | +1.98 | D114 | 17.91 | 42.00 | 15.76 | |

TABLE 9—*Concluded*

| No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D115 | 17.70 | 41.75 | 15.55 | + .89 | D168 | 22.37 | 44.24 | 15.09 | + .30 | D221 | 25.98 | 40.03 | 24.70 | + .66 |
| D116 | 17.87 | 40.24 | 16.05 | + .96 | D169 | 22.28 | 43.25 | 14.98 | + .81 | D222 | 24.99 | 39.29 | 15.54 | +1.44 |
| D117 | 17.64 | 37.01 | 15.76 | +1.52 | D170 | 23.21 | 44.39 | 15.09 | + .47 | D223 | 25.41 | 38.51 | 16.45 | +1.66 |
| D118 | 17.62 | 31.73 | 15.76 | +1.75 | D171 | 22.79 | 43.60 | F | + | D224 | 25.48 | 38.01 | 15.55 | + .61 |
| D119 | 17.65 | 34.79 | 16.24 | +1.65 | D172 | 22.11 | 43.81 | 16.45 | +2.34 | D225 | 25.56 | 37.61 | 15.88 | +1.29 |
| D120 | 17.06 | 36.84 | F | + | D173 | 22.10 | 40.66 | 15.55 | + .46 | D226 | 25.10 | 37.28 | 16.45 | +1.76 |
| D121 | 18.32 | 30.77 | 16.45 | | D174 | 22.10 | 40.26 | 15.76 | + .82 | D227 | 24.84 | 36.81 | F | + |
| D122 | 18.33 | 30.97 | 15.32 | + .66 | D175 | 23.01 | 39.58 | 14.57 | + .67 | D228 | 25.73 | 34.53 | 15.76 | + .67 |
| D123 | 17.90 | 32.17 | 16.24 | | D176 | 21.90 | 36.00 | 16.45 | +1.97 | D229 | 25.73 | 34.22 | 15.26 | +1.52 |
| D124 | 18.11 | 29.02 | 15.32 | + .66 | D177 | 22.42 | 35.85 | 16.24 | +1.97 | D230 | 24.65 | 31.94 | 15.42 | + .48 |
| D125 | 18.24 | 28.05 | 15.65 | + .86 | D178 | 23.15 | 35.31 | 16.45 | +2.21 | D231 | 24.82 | 31.31 | 14.28 | + .32 |
| D126 | 18.40 | 26.21 | 13.75 | + .43 | D179 | 22.46 | 35.08 | 16.24 | +1.15 | D232 | 24.90 | 31.90 | F | + |
| D127 | 17.32 | 25.94 | 16.24 | +1.71 | D180 | 22.78 | 34.64 | 15.22 | + .28 | D233 | 24.92 | 30.92 | 16.45 | +1.79 |
| D128 | 17.38 | 25.68 | 14.05 | + .64 | D181 | 22.72 | 34.35 | 16.45 | +1.79 | D234 | 24.45 | 28.60 | 14.93 | + .79 |
| D129 | 19.14 | 43.13 | 14.48 | + .41 | D182 | 22.82 | 34.13 | F | + | D234a | 24.43 | 29.72 | 15.19 | |
| D130 | 19.68 | 44.72 | 15.01 | + .35 | D183 | 22.94 | 31.82 | 15.22 | + .69 | D235 | 24.93 | 26.02 | 15.04 | + .73 |
| D131 | 20.19 | 41.25 | 14.35 | +1.65 | D184 | 22.25 | 31.59 | 16.24 | +2.81 | D236 | 26.32 | 31.00 | 13.22 | + .82 |
| D132 | 19.91 | 41.02 | 15.76 | + .67 | D185 | 21.94 | 31.17 | 15.14 | +1.35 | D237 | 26.40 | 42.37 | 14.18 | +1.18 |
| D133 | 19.85 | 40.65 | 15.44 | + .65 | D186 | 21.82 | 30.73 | 15.15 | + .21 | D238 | 26.11 | 41.58 | 15.43 | +1.31 |
| D134 | 19.45 | 40.11 | 14.92 | + .61 | D187 | 22.39 | 30.37 | 14.75 | + .58 | D239 | 26.45 | 41.52 | 16.45 | +2.14 |
| D135 | 19.65 | 41.71 | 16.05 | + .96 | D188 | 22.39 | 30.02 | 14.16 | + .24 | D240 | 26.17 | 39.60 | 15.09 | +1.30 |
| D136 | 19.34 | 42.81 | 16.05 | + .96 | D189 | 22.38 | 30.75 | 15.68 | + .59 | D241 | 25.50 | 39.62 | 14.80 | +1.92 |
| D137 | 19.17 | 41.34 | 15.88 | | D190 | 22.10 | 24.17 | 16.24 | +1.65 | D242 | 26.42 | 39.40 | 16.24 | +1.65 |
| D138 | 19.17 | 41.01 | 14.85 | +1.83 | D191* | 24.50 | 45.89 | 13.39 | + .79 | D243 | 26.75 | 39.34 | 15.42 | +1.52 |
| D139 | 19.62 | 38.48 | 15.55 | + .61 | D192* | 24.34 | 45.82 | 13.26 | + .17 | D244 | 25.87 | 36.32 | 15.78 | +1.42 |
| D140 | 19.33 | 38.39 | 15.55 | + .96 | D193 | 24.11 | 42.61 | 13.03 | + .89 | D245 | 25.95 | 35.95 | 15.21 | +1.19 |
| D141 | 19.21 | 38.12 | 16.45 | +1.86 | D194 | 24.01 | 44.46 | 14.52 | + .42 | D246 | 26.26 | 36.11 | 15.42 | +1.30 |
| D142 | 19.73 | 35.61 | 13.96 | +1.45 | D195 | 23.31 | 43.64 | 15.76 | + .82 | D247 | 26.96 | 35.19 | 16.05 | +1.39 |
| D143 | 18.87 | 34.66 | L | | D196 | 23.31 | 43.00 | 16.45 | +1.92 | D248 | 26.29 | 33.52 | F | + |
| D144 | 19.87 | 32.88 | 15.65 | + .56 | D197 | 23.38 | 41.95 | 16.45 | +2.29 | D249 | 26.49 | 32.55 | 15.14 | + .05 |
| D145 | 20.03 | 30.72 | 16.45 | +2.28 | D198 | 23.21 | 39.28 | 14.28 | + .76 | D250 | 25.97 | 30.86 | 16.45 | +2.34 |
| D146 | 18.96 | 29.55 | 15.01 | + .42 | D199 | 23.94 | 36.22 | 14.70 | +1.59 | D251 | 26.66 | 30.08 | 14.48 | + .58 |
| D147 | 20.57 | 44.97 | 11.65 | +1.67 | D200 | 24.29 | 36.23 | 15.37 | + .78 | D252 | 26.36 | 30.04 | 15.14 | + .61 |
| D148 | 20.90 | 39.08 | 13.46 | +1.10 | D201 | 23.11 | 34.53 | 16.45 | +1.97 | D253 | 26.56 | 29.81 | 15.65 | +1.12 |
| D149 | 21.02 | 44.61 | 16.45 | +2.24 | D202 | 23.16 | 33.25 | 15.21 | + .95 | D254 | 27.01 | 30.97 | 15.14 | + .35 |
| D150 | 21.61 | 40.73 | 16.45 | +1.92 | D203 | 23.51 | 32.89 | 15.54 | + .45 | D255 | 27.11 | 30.95 | 16.24 | +1.65 |
| D151 | 20.47 | 39.07 | 14.60 | + .60 | D204 | 24.16 | 32.59 | F | + | D256 | 26.27 | 28.45 | 13.66 | + .49 |
| D152 | 21.09 | 38.81 | 14.60 | +1.28 | D205 | 24.14 | 32.40 | 15.14 | + .44 | D257 | 25.89 | 27.89 | 15.20 | + .54 |
| D153 | 21.04 | 38.04 | 16.45 | +1.86 | D206 | 23.92 | 31.63 | 16.45 | + .44 | D258* | 28.55 | 41.91 | 13.68 | + .79 |
| D154 | 20.96 | 38.49 | 16.45 | +1.92 | D207 | 23.57 | 31.18 | 16.05 | + .28 | D259 | 27.61 | 41.70 | 15.42 | +1.43 |
| D155 | 20.80 | 37.78 | 15.32 | +1.24 | D208 | 23.57 | 30.31 | 15.55 | + .76 | D260 | 27.37 | 39.46 | 15.09 | + .60 |
| D156 | 20.87 | 36.94 | 14.53 | +1.51 | D209 | 24.13 | 27.84 | 15.32 | + .66 | D261 | 27.68 | 39.20 | 15.20 | +1.18 |
| D157 | 20.92 | 36.45 | 16.45 | +1.92 | D210 | 23.55 | 27.64 | 15.09 | + .50 | D262 | 27.81 | 30.08 | 14.74 | +1.78 |
| D158 | 20.91 | 36.44 | 16.24 | +2.00 | D211 | 23.91 | 27.15 | 15.48 | + .82 | D263 | 27.22 | 29.42 | 14.48 | +1.06 |
| D159 | 20.79 | 34.30 | 15.65 | + .86 | D212 | 23.77 | 26.72 | 16.24 | +1.71 | D264 | 27.27 | 30.18 | 15.37 | + .28 |
| D160 | 20.72 | 33.55 | 14.32 | +1.34 | D213 | 24.22 | 27.12 | 15.54 | | D265 | 27.95 | 28.01 | 15.26 | + .32 |
| D161 | 20.92 | 33.28 | 16.24 | +1.85 | D214 | 23.24 | 25.66 | 16.45 | +1.92 | D266 | 27.38 | 27.62 | 15.09 | + .50 |
| D162 | 20.53 | 30.27 | 14.94 | +1.34 | D215* | 25.23 | 23.72 | 14.29 | +1.36 | D267 | 29.95 | 33.24 | 13.07 | + .50 |
| D163 | 20.57 | 26.14 | 15.72 | +1.19 | D216 | 24.85 | 43.04 | 15.32 | +1.72 | D268 | 28.95 | 35.18 | 16.45 | +1.51 |
| D164 | 21.11 | 26.07 | 15.14 | + .55 | D217 | 25.01 | 41.42 | 14.11 | + .16 | D269 | 29.22 | 30.93 | 15.76 | + .67 |
| D165 | 20.09 | 25.66 | 15.48 | + .54 | D218 | 25.54 | 43.56 | 15.44 | + .78 | D270 | 29.05 | 30.49 | 16.24 | +1.58 |
| D166 | 22.52 | 44.71 | 16.05 | +1.26 | D219 | 25.87 | 41.78 | 14.93 | + .60 | D271* | 30.29 | 28.57 | 12.10 | + .30 |
| D167 | 22.13 | 44.70 | 15.11 | + .87 | D220 | 25.27 | 41.12 | 16.45 | +2.21 | D272* | 33.08 | 30.64 | 13.79 | + .77 |

TABLE 10

AREA E

A = 284° 1′ 26″ } E93
D = −6° 20′ 2″ }
a = −0.000, 189, 9

b = 0.000, 000, 0
c = +0.003, 593
d = +0.006, 554

| No. | x | y | Pg.Mag. | C |
|---|---|---|---|---|
| E 1* | 4.56 | 33.28 | 13.53 | +1.79 |
| E 2 | 8.06 | 33.67 | 10.92 | +1.06 |
| E 3 | 8.75 | 34.53 | 14.50 | +1.20 |
| E 4 | 8.81 | 32.81 | 14.16 | +1.12 |
| E 5* | 8.98 | 34.68 | 13.85 | +1.17 |
| E 6 | 9.58 | 35.75 | 12.02 | + .25 |
| E 7 | 9.12 | 39.39 | 14.54 | +1.44 |
| E 8 | 10.04 | 36.68 | 15.32 | + .80 |
| E 9 | 9.44 | 32.56 | 15.16 | + .57 |
| E10 | 9.21 | 29.95 | 15.30 | − .02 |
| E11 | 10.03 | 27.79 | 14.88 | +1.76 |
| E12 | 11.00 | 27.52 | 13.50 | + .81 |
| E13 | 11.53 | 33.34 | 13.41 | + .60 |
| E14 | 11.27 | 40.12 | 12.57 | + .81 |
| E15 | 10.99 | 40.98 | 14.16 | + .36 |
| E16 | 11.12 | 38.49 | 15.19 | +1.31 |
| E17 | 11.01 | 37.60 | 14.21 | +1.34 |
| E18 | 10.64 | 34.97 | 14.24 | + .62 |
| E19 | 11.30 | 32.33 | 14.62 | +1.08 |
| E20 | 11.25 | 29.11 | 14.74 | +1.72 |
| E21 | 10.39 | 28.74 | 15.55 | +1.04 |
| E22 | 10.39 | 28.40 | 15.28 | + .69 |
| E23 | 10.62 | 29.72 | F | + |
| E24 | 11.39 | 26.24 | 15.38 | + .77 |
| E25 | 11.56 | 22.67 | 13.74 | +1.02 |
| E26* | 11.88 | 23.91 | 13.19 | + .41 |
| E27* | 12.20 | 35.50 | 14.14 | +1.17 |
| E28* | 11.95 | 46.93 | 13.83 | +1.02 |
| E29 | 11.90 | 41.34 | 15.28 | +1.31 |
| E30 | 12.07 | 39.79 | 14.88 | + .46 |
| E31 | 13.02 | 38.71 | 15.21 | + .61 |
| E32 | 13.09 | 37.73 | 14.58 | + .59 |
| E33 | 12.83 | 29.64 | 15.11 | +1.05 |
| E34 | 12.88 | 27.86 | 14.94 | + .27 |
| E35 | 12.88 | 27.05 | 15.33 | +1.23 |
| E36 | 13.30 | 41.87 | 15.28 | +1.59 |
| E37 | 13.86 | 41.76 | 14.30 | +1.68 |
| E38 | 13.76 | 40.31 | 15.22 | + .64 |
| E39 | 14.14 | 39.28 | 15.43 | +1.51 |
| E40 | 13.42 | 38.88 | 14.82 | +1.40 |
| E41 | 14.16 | 38.25 | 15.10 | +1.58 |
| E42 | 13.32 | 34.90 | 15.30 | +1.20 |
| E43 | 14.14 | 33.28 | 14.98 | +1.22 |
| E44 | 14.10 | 33.22 | 15.14 | + .82 |
| E45 | 13.54 | 32.84 | 15.32 | +1.44 |
| E46 | 13.70 | 33.63 | 14.50 | − .21 |
| E47 | 13.51 | 31.94 | 14.94 | +1.40 |
| E48 | 13.77 | 31.47 | 15.38 | +1.10 |
| E49 | 13.33 | 30.57 | 14.68 | + .56 |
| E50 | 14.90 | 38.66 | 14.74 | +1.30 |
| E51 | 15.19 | 38.42 | 15.60 | + .96 |
| E52 | 15.77 | 38.34 | 15.60 | +1.24 |
| E53 | 15.99 | 36.40 | 14.30 | + .92 |
| E54 | 16.02 | 36.10 | 14.42 | + .94 |
| E55 | 15.88 | 35.53 | 14.47 | + .55 |
| E56 | 14.49 | 35.43 | 14.70 | +1.78 |
| E57 | 15.21 | 34.29 | 15.38 | + .77 |
| E58 | 14.51 | 32.68 | F | + |
| E59 | 13.34 | 26.81 | F | + |
| E60 | 16.56 | 43.91 | 14.78 | + .58 |
| E61 | 16.57 | 45.34 | 15.55 | + .99 |
| E62 | 16.68 | 43.14 | 15.38 | + .87 |
| E63 | 17.04 | 42.24 | 15.60 | +1.04 |
| E64 | 16.50 | 34.00 | 15.43 | + .82 |
| E65 | 16.23 | 33.37 | 15.43 | + .92 |
| E66 | 17.19 | 32.88 | 15.60 | +1.29 |
| E67 | 17.08 | 30.89 | 15.49 | +1.42 |
| E68 | 16.94 | 29.54 | 14.88 | + .46 |
| E69 | 17.10 | 28.67 | 15.38 | + .82 |
| E70 | 17.00 | 23.79 | 15.27 | + .71 |
| E71* | 17.46 | 23.12 | 13.20 | + .47 |
| E72 | 18.58 | 24.49 | 13.57 | + .28 |
| E73 | 18.14 | 44.37 | 13.21 | + .60 |
| E74 | 17.66 | 44.19 | 14.07 | +1.48 |
| E75 | 19.01 | 43.41 | 13.87 | +1.04 |
| E76 | 17.92 | 43.27 | 15.27 | + .71 |
| E77 | 17.73 | 42.50 | 15.55 | + .99 |
| E78 | 18.22 | 41.25 | 15.60 | +1.18 |
| E79 | 17.69 | 39.69 | 15.49 | +1.30 |
| E80 | 17.99 | 37.88 | 14.86 | +1.35 |
| E81 | 17.81 | 35.30 | 15.60 | +1.09 |
| E82 | 17.54 | 35.04 | F | + |
| E83 | 18.46 | 33.53 | 15.32 | + .76 |
| E84 | 18.08 | 32.86 | 15.15 | +1.37 |
| E85 | 18.10 | 31.51 | 15.21 | + .85 |
| E86 | 18.02 | 31.56 | 15.49 | +1.13 |
| E87 | 17.81 | 32.01 | 15.38 | + .77 |
| E88 | 17.74 | 31.39 | 15.43 | +1.36 |
| E89 | 18.29 | 30.39 | 15.43 | + .77 |
| E90 | 17.33 | 25.04 | 14.38 | + .64 |
| E91* | 19.26 | 21.47 | 14.07 | +1.19 |
| E92 | 20.05 | 26.33 | 13.35 | + .83 |
| E93 | 18.91 | 34.51 | B | |
| E94 | 19.30 | 45.91 | 13.96 | +1.33 |
| E95 | 19.07 | 45.16 | 14.92 | +1.46 |
| E96 | 19.93 | 44.88 | 15.34 | + .86 |
| E97 | 19.34 | 43.76 | 14.92 | + .98 |
| E98 | 19.54 | 43.41 | 14.96 | + .69 |
| E99 | 19.19 | 42.40 | 15.49 | +1.03 |
| E100 | 19.33 | 40.19 | 15.60 | +1.24 |
| E101 | 19.08 | 39.25 | F | + |
| E102 | 19.95 | 38.10 | F | + |
| E103 | 19.41 | 37.31 | 13.96 | + .70 |
| E104 | 19.25 | 35.08 | 15.16 | +1.40 |
| E105 | 19.87 | 31.86 | 15.43 | + .87 |
| E106 | 19.01 | 25.78 | 14.55 | + .94 |
| E107 | 19.06 | 24.80 | 14.17 | + .73 |
| E108 | 20.41 | 45.34 | 15.43 | + .82 |
| E109 | 21.16 | 44.43 | 15.60 | + .99 |
| E110 | 21.05 | 44.28 | 15.60 | + .99 |
| E111 | 21.16 | 42.27 | 15.27 | + .76 |
| E112 | 20.37 | 42.19 | 14.20 | + .48 |
| E113 | 20.99 | 41.33 | 15.49 | + .83 |
| E114 | 20.66 | 41.06 | 14.20 | +1.14 |
| E115 | 20.55 | 40.77 | 14.23 | + .29 |
| E116 | 20.88 | 40.42 | 14.58 | + .61 |
| E117 | 21.17 | 40.25 | 15.43 | + .77 |
| E118 | 21.21 | 38.87 | 15.28 | +1.99 |
| E119 | 21.10 | 36.46 | F | + |
| E120 | 21.02 | 35.66 | 14.66 | +2.25 |
| E121 | 21.53 | 30.92 | 14.11 | + .43 |
| E122 | 21.20 | 30.61 | 15.38 | + .77 |
| E123* | 22.42 | 23.35 | 13.79 | + .56 |
| E124 | 22.27 | 25.08 | 12.53 | + .75 |
| E125 | 21.97 | 33.44 | 14.05 | + .99 |
| E126 | 22.88 | 43.49 | 13.69 | +1.20 |
| E127* | 22.26 | 46.87 | 13.68 | +1.24 |
| E128 | 21.69 | 35.00 | 15.06 | +1.30 |
| E129 | 22.30 | 34.08 | 14.84 | +1.30 |
| E130 | 21.59 | 25.76 | F | + |
| E131 | 24.30 | 30.53 | 13.19 | +1.46 |
| E132 | 23.79 | 24.76 | 13.66 | + .78 |
| E133 | 23.62 | 25.56 | 13.32 | + .47 |
| E134 | 23.13 | 35.58 | 15.43 | +1.01 |
| E135 | 23.32 | 39.13 | 15.49 | + .93 |
| E136 | 23.74 | 38.18 | F | + |
| E137 | 22.99 | 40.16 | F | + |
| E138 | 24.07 | 35.95 | 15.38 | + .72 |
| E139 | 23.95 | 36.11 | 15.55 | + .89 |
| E140 | 23.49 | 35.60 | 15.49 | + .83 |
| E141 | 23.28 | 29.78 | 15.55 | +1.04 |
| E142 | 25.12 | 43.54 | 15.49 | + .98 |
| E143 | 25.16 | 43.43 | 15.43 | + .87 |
| E144 | 24.79 | 42.81 | F | + |
| E145 | 25.00 | 42.15 | 15.43 | +1.01 |
| E146 | 25.08 | 41.06 | 15.49 | +1.03 |
| E147 | 25.19 | 40.49 | 14.34 | + .56 |
| E148 | 24.82 | 39.74 | 15.60 | + .99 |
| E149 | 25.38 | 38.42 | 15.49 | + .98 |
| E150 | 25.42 | 36.45 | 14.48 | + .65 |
| E151 | 24.78 | 36.08 | 15.38 | + .87 |
| E152 | 24.70 | 34.95 | 14.48 | + .23 |
| E153 | 25.31 | 33.69 | 13.92 | + .16 |
| E154 | 25.18 | 33.51 | 15.46 | +1.14 |
| E155 | 25.14 | 33.38 | F | + |
| E156 | 24.69 | 33.61 | 15.38 | + .77 |
| E157 | 24.75 | 29.90 | 15.36 | + .88 |
| E158 | 26.36 | 28.26 | 13.29 | + .53 |
| E159 | 26.32 | 30.47 | 13.61 | + .73 |
| E160* | 26.93 | 43.24 | 12.81 | + .42 |
| E161 | 26.36 | 38.79 | F | + |
| E162 | 27.28 | 35.46 | 14.72 | +1.62 |
| E163 | 26.68 | 36.06 | 14.48 | +1.29 |
| E164 | 26.04 | 32.77 | 14.42 | + .44 |
| E165 | 25.96 | 32.59 | 15.32 | + .96 |
| E166 | 27.20 | 33.74 | 15.10 | + .71 |
| E167 | 26.60 | 34.35 | 15.60 | +1.04 |
| E168 | 27.07 | 30.90 | 15.55 | +1.29 |
| E169 | 26.44 | 28.52 | 15.38 | +1.06 |
| E170 | 26.55 | 26.92 | 15.49 | + .88 |
| E171 | 28.17 | 37.07 | 13.79 | + .54 |
| E172 | 27.28 | 40.25 | 15.32 | +1.25 |
| E173 | 27.22 | 38.50 | 15.55 | +1.04 |
| E174 | 28.17 | 37.28 | 15.00 | + .36 |
| E175 | 28.42 | 29.90 | 13.87 | + .49 |
| E176 | 28.12 | 28.94 | F | + |
| E177 | 29.92 | 31.70 | 12.60 | + .44 |
| E178 | 29.83 | 31.52 | 12.65 | + .57 |
| E179* | 29.71 | 44.77 | 13.23 | + .37 |
| E180 | 29.20 | 38.56 | 15.60 | +1.64 |
| E181 | 29.30 | 36.28 | F | + |
| E182 | 29.07 | 36.08 | F | + |
| E183 | 29.33 | 34.71 | 15.27 | + .66 |
| E184* | 31.08 | 29.54 | 13.89 | +1.23 |
| E185* | 32.41 | 40.27 | 14.13 | +1.15 |
| E186* | 33.15 | 37.60 | 13.70 | +1.04 |
| E187* | 33.80 | 34.69 | 13.87 | +1.34 |

TABLE 11

AREA F

A = 279° 31′ 7″
D = −8° 22′ 26″ } F114
a = −0.000, 190, 5

b = +0.000, 002, 5
c = +0.003, 499
d = +0.006, 526

| No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F 1* | 3.89 | 35.53 | 12.13 | + .04 | F61 | 14.54 | 40.30 | 14.66 | + .66 | F120 | 20.04 | 43.03 | 15.16 | +2.28 |
| F 2* | 5.16 | 34.46 | 12.03 | + .07 | F62 | 14.54 | 38.10 | 15.98 | +1.84 | F121 | 19.58 | 42.38 | 15.38 | + .08 |
| F 3* | 4.89 | 34.35 | 12.61 | +1.29 | F63 | 15.55 | 39.59 | 15.59 | + .76 | F122 | 19.15 | 41.59 | 14.42 | + .28 |
| F 4 | 8.42 | 35.62 | 14.63 | + .70 | F64 | 15.67 | 38.59 | 15.33 | +1.37 | F123 | 19.03 | 42.28 | 15.45 | |
| F 5 | 8.20 | 36.19 | 15.97 | | F65 | 14.96 | 37.61 | F | + | F124 | 18.96 | 41.10 | 15.50 | +1.86 |
| F 6 | 8.08 | 30.82 | 15.83 | +1.96 | F66 | 15.06 | 36.87 | 15.54 | + .64 | F125 | 19.77 | 40.13 | F | + |
| F 7 | 9.36 | 37.65 | F | + | F67 | 15.44 | 36.54 | 15.04 | + .32 | F126 | 20.24 | 40.31 | 15.14 | + .75 |
| F 8 | 9.31 | 34.22 | 14.93 | +1.41 | F68 | 15.77 | 34.01 | 14.63 | +1.35 | F127 | 19.22 | 38.60 | 15.30 | + .64 |
| F 9 | 9.15 | 32.86 | 15.42 | + .60 | F69 | 15.22 | 29.87 | 15.68 | +1.81 | F128 | 19.45 | 37.22 | 14.84 | + .84 |
| F10 | 9.89 | 30.38 | 13.61 | + .33 | F70 | 14.98 | 26.65 | 15.96 | | F129 | 19.47 | 36.78 | 15.64 | + .98 |
| F11 | 11.23 | 38.73 | 14.61 | +1.10 | F71 | 15.00 | 25.59 | 16.16 | +1.47 | F130 | 19.16 | 33.75 | 14.02 | +1.00 |
| F12 | 11.63 | 38.31 | 14.39 | + .77 | F72 | 15.26 | 25.58 | 15.54 | + .24 | F131 | 19.15 | 34.97 | I | |
| F13 | 10.65 | 33.11 | 15.19 | + .50 | F73 | 14.48 | 24.81 | 14.93 | + .57 | F132 | 19.13 | 31.90 | 13.78 | + .47 |
| F14 | 10.44 | 30.61 | 15.38 | + .60 | F74 | 16.76 | 23.14 | 12.34 | + .19 | F133 | 19.12 | 31.20 | 14.15 | + .73 |
| F15 | 11.08 | 30.29 | 13.88 | + .18 | F75 | 16.77 | 28.46 | 12.35 | + .15 | F134 | 20.09 | 31.77 | F | + |
| F16 | 10.51 | 27.16 | 15.55 | | F76* | 17.15 | 45.89 | 13.68 | +1.20 | F135 | 19.92 | 31.49 | 15.45 | + .79 |
| F17 | 11.38 | 26.48 | 15.68 | +1.90 | F77 | 17.31 | 44.32 | 14.39 | +1.53 | F136 | 19.08 | 29.29 | 14.70 | +1.70 |
| F18 | 12.11 | 30.26 | 15.45 | + .93 | F78 | 16.37 | 44.39 | 14.35 | + .41 | F137 | 19.41 | 28.99 | 14.66 | +1.74 |
| F19 | 11.85 | 25.23 | 15.03 | + .50 | F79 | 16.15 | 41.19 | 15.39 | + .56 | F138 | 19.12 | 27.78 | 14.98 | +1.72 |
| F20 | 13.02 | 35.35 | 14.70 | +1.64 | F80 | 16.92 | 39.85 | 15.33 | + .28 | F139 | 19.23 | 27.19 | 14.49 | +1.83 |
| F21 | 12.34 | 33.14 | 14.66 | + .08 | F81 | 16.35 | 37.54 | 15.50 | + .51 | F140 | 19.65 | 27.00 | 15.11 | +2.27 |
| F22 | 12.77 | 32.18 | 16.16 | +2.53 | F82 | 16.33 | 35.98 | 14.94 | + .25 | F141 | 19.53 | 26.40 | 16.16 | +1.54 |
| F23 | 12.13 | 31.55 | 15.08 | +1.70 | F83 | 16.98 | 34.48 | 15.11 | + .71 | F142 | 19.87 | 25.69 | 15.14 | + .61 |
| F24 | 11.86 | 29.00 | 15.49 | | F84 | 16.08 | 34.12 | 15.45 | + .73 | F143 | 19.93 | 24.32 | 15.54 | +1.48 |
| F25 | 12.31 | 30.40 | 15.60 | + .91 | F85 | 17.16 | 32.72 | 14.93 | + .24 | F144 | 21.39 | 30.49 | 12.84 | + .36 |
| F26 | 12.39 | 29.97 | 15.24 | +1.64 | F86 | 17.07 | 32.62 | F | + | F145 | 21.17 | 30.58 | 12.52 | + .21 |
| F27 | 12.85 | 29.57 | F | + | F87 | 16.99 | 31.87 | 15.45 | + .73 | F146 | 21.39 | 44.48 | 12.64 | + .16 |
| F28 | 11.74 | 30.33 | 14.86 | +1.37 | F88 | 17.12 | 31.07 | 15.77 | +1.45 | F147* | 21.26 | 47.06 | 13.04 | +1.04 |
| F29 | 12.09 | 28.71 | 15.33 | + .75 | F89 | 16.78 | 29.79 | F+ | + | F148 | 21.32 | 40.67 | F | + |
| F30 | 12.28 | 28.75 | F | + | F90 | 16.15 | 27.77 | F | + | F149 | 21.23 | 39.64 | F | + |
| F31 | 11.90 | 27.38 | 14.64 | + .34 | F91 | 16.71 | 27.27 | 16.16 | +2.39 | F150 | 21.50 | 37.92 | 14.70 | +1.14 |
| F32 | 11.90 | 26.29 | 13.21 | + .67 | F92 | 17.16 | 25.20 | 15.24 | + .74 | F151 | 21.28 | 37.89 | 14.36 | +1.21 |
| F33 | 12.34 | 27.08 | F | + | F93 | 16.31 | 24.47 | 14.52 | + .39 | F152 | 20.69 | 38.05 | 15.43 | + .87 |
| F34 | 11.90 | 25.29 | 13.50 | + .37 | F94 | 17.56 | 35.59 | 12.80 | + .83 | F153 | 20.68 | 33.71 | 15.45 | + .75 |
| F35* | 14.42 | 46.72 | 12.66 | + .10 | F95 | 17.86 | 42.49 | 9.86 | +1.20 | F154 | 21.07 | 29.72 | 14.09 | + .70 |
| F36 | 13.89 | 35.99 | 12.89 | + .44 | F96 | 17.75 | 44.23 | 13.01 | + .63 | F155 | 21.13 | 29.40 | 14.46 | + .92 |
| F37 | 13.57 | 25.78 | 13.09 | + .18 | F97 | 18.51 | 43.82 | 15.30 | + .47 | F156 | 21.32 | 28.60 | 13.94 | + .72 |
| F38* | 13.63 | 22.14 | 12.22 | + .06 | F98 | 18.21 | 42.86 | 14.99 | + .33 | F157 | 20.43 | 27.38 | 15.56 | +1.17 |
| F39 | 14.22 | 42.91 | 13.88 | + .12 | F99 | 18.15 | 42.76 | F+ | + | F158 | 20.70 | 26.41 | 15.73 | +1.16 |
| F40 | 14.25 | 42.51 | 14.82 | + .16 | F100 | 17.51 | 42.05 | 15.19 | +2.03 | F158a | 21.20 | 25.43 | 15.45 | |
| F41 | 14.29 | 41.86 | 15.33 | + .69 | F101 | 18.04 | 41.73 | 16.16 | +1.33 | F159 | 21.16 | 43.14 | 15.58 | +2.34 |
| F42 | 13.94 | 40.73 | 15.19 | + .61 | F102 | 18.05 | 40.68 | 15.30 | +1.43 | F159a* | 22.40 | 48.35 | 12.90 | + .44 |
| F43 | 14.02 | 39.85 | 14.82 | + .46 | F103 | 18.16 | 39.00 | 15.45 | +2.29 | F160 | 21.80 | 43.34 | 14.50 | +1.17 |
| F44 | 13.74 | 38.75 | F | + | F104 | 18.78 | 38.09 | 15.64 | +1.44 | F161 | 22.11 | 40.76 | 15.86 | +1.20 |
| F45 | 13.27 | 37.14 | F | + | F104a | 18.29 | 37.95 | 15.64 | + .98 | F162 | 22.23 | 39.47 | 13.61 | + .09 |
| F46 | 13.75 | 36.69 | 14.84 | + .32 | F105 | 18.31 | 36.99 | 14.28 | + .75 | F163 | 22.73 | 39.17 | 15.60 | + .46 |
| F47 | 13.18 | 34.53 | 15.93 | +1.45 | F106 | 18.40 | 36.31 | 15.64 | + .98 | F164 | 23.03 | 38.27 | 15.58 | +2.19 |
| F48 | 13.91 | 32.81 | 15.68 | +1.25 | F107 | 18.45 | 34.01 | 14.15 | +1.60 | F165 | 22.12 | 38.12 | 15.66 | +1.13 |
| F49 | 13.26 | 32.03 | 15.33 | + .51 | F108 | 17.52 | 32.96 | 15.45 | + .62 | F166 | 22.39 | 36.48 | 14.14 | + .64 |
| F50 | 13.55 | 30.95 | 15.50 | + .53 | F109 | 18.49 | 30.48 | 13.78 | + .55 | F167 | 22.55 | 35.89 | 14.30 | + .74 |
| F51 | 14.04 | 29.72 | F | + | F110 | 18.47 | 24.23 | D | | F168 | 22.00 | 36.33 | 15.32 | + .66 |
| F52 | 14.18 | 29.49 | 15.24 | + .65 | F111 | 18.09 | 24.35 | 15.90 | +1.64 | F169 | 21.94 | 36.07 | 15.39 | + .75 |
| F53 | 14.30 | 30.39 | 15.34 | +1.13 | F112 | 18.02 | 23.73 | 15.45 | + .31 | F170 | 21.70 | 35.77 | 15.51 | + .43 |
| F54 | 13.38 | 28.35 | 15.45 | +1.69 | F113 | 18.30 | 24.43 | 15.82 | + .52 | F171 | 22.31 | 35.78 | 15.48 | + .91 |
| F55 | 13.50 | 24.45 | 13.72 | + .31 | F114 | 18.73 | 34.00 | B | + | F172 | 22.13 | 28.74 | 14.46 | + .50 |
| F56 | 14.71 | 33.71 | 12.98 | + .54 | F115 | 19.67 | 39.21 | 12.63 | + .35 | F173 | 22.28 | 27.85 | 15.48 | + .64 |
| F57 | 15.26 | 26.55 | 12.85 | + .34 | F116 | 19.99 | 44.43 | 14.52 | + .32 | F174 | 22.37 | 28.13 | 15.66 | +1.13 |
| F58 | 15.81 | 44.84 | 14.52 | + .22 | F117 | 19.49 | 44.20 | 15.16 | + .44 | F175 | 22.61 | 27.73 | F | + |
| F59 | 15.53 | 44.63 | 15.11 | + .45 | F118 | 19.03 | 43.28 | 15.30 | + .77 | F176 | 22.30 | 26.66 | 15.64 | + .98 |
| F60 | 14.62 | 43.78 | 14.52 | + .52 | F119 | 19.84 | 43.03 | 15.45 | + .64 | F177 | 21.86 | 26.28 | 14.96 | + .57 |

TABLE 11—*Concluded*

| No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C | No. | x | y | Pg.Mag. | C |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F178 | 21.91 | 25.39 | 15.12 | +1.06 | F206 | 25.61 | 34.02 | 15.60 | | F238 | 27.60 | 37.83 | 15.36 | + .70 |
| F179 | 22.08 | 25.27 | 15.12 | + .42 | F207 | 25.37 | 31.13 | 15.58 | +1.41 | F239 | 27.93 | 37.76 | 15.60 | + .64 |
| F180 | 22.24 | 25.85 | 15.66 | + .83 | F208 | 25.28 | 30.86 | 15.76 | | F240 | 28.02 | 37.87 | 15.44 | + .70 |
| F181 | 23.24 | 27.26 | 14.46 | +1.96 | F209 | 25.08 | 29.71 | 14.48 | + .09 | F241 | 28.10 | 37.84 | 15.25 | + .51 |
| F182 | 23.91 | 30.05 | 13.44 | + .20 | F210 | 24.97 | 29.02 | 15.46 | +1.69 | F242 | 28.59 | 38.34 | 14.86 | + .24 |
| F183 | 23.16 | 43.93 | 14.64 | +1.36 | F211 | 24.75 | 28.97 | 15.76 | | F243 | 28.48 | 37.52 | 14.07 | + .21 |
| F184 | 23.97 | 40.03 | 14.07 | + .27 | F212 | 25.51 | 29.31 | F | + | F244 | 28.61 | 36.78 | 15.51 | |
| F185 | 24.33 | 40.95 | 15.64 | + .60 | F213 | 25.06 | 27.73 | F | + | F245 | 28.54 | 36.73 | 15.50 | + .94 |
| F186 | 23.36 | 39.59 | 15.70 | +1.83 | F214 | 25.08 | 25.66 | 14.50 | + .44 | F246 | 28.34 | 36.54 | 15.64 | +1.47 |
| F187 | 23.81 | 38.66 | 14.54 | + .51 | F215 | 24.24 | 26.04 | 15.46 | | F247 | 28.20 | 36.30 | F | + |
| F188 | 24.38 | 38.26 | 15.45 | +1.15 | F216 | 26.18 | 41.99 | 14.33 | +1.07 | F248 | 28.60 | 32.62 | 14.02 | + .40 |
| F189 | 23.08 | 36.83 | 14.46 | + .12 | F217 | 26.01 | 42.69 | 15.75 | +1.62 | F249 | 28.51 | 32.50 | 15.45 | + .31 |
| F190 | 22.99 | 36.12 | 15.60 | + .86 | F218 | 26.37 | 41.60 | 15.58 | +1.71 | F250 | 26.30 | 31.13 | F | + |
| F191 | 23.07 | 35.12 | 14.36 | + .56 | F219 | 27.09 | 36.85 | 15.45 | +1.09 | F251 | 28.30 | 30.37 | 15.14 | +1.04 |
| F192 | 23.02 | 31.86 | 15.42 | +1.03 | F220 | 27.03 | 32.66 | 15.60 | +1.75 | F252 | 27.46 | 30.69 | 16.16 | +1.86 |
| F193 | 24.05 | 28.88 | 15.25 | +1.60 | F221 | 26.63 | 34.46 | 15.44 | | F253 | 27.95 | 28.66 | 15.04 | + .88 |
| F194 | 23.45 | 29.23 | 15.17 | +1.65 | F224 | 26.11 | 34.36 | 15.51 | + .52 | F254* | 28.47 | 25.13 | 13.72 | + .94 |
| F195 | 23.60 | 28.96 | 15.72 | +1.06 | F225 | 27.17 | 32.15 | 15.66 | | F255* | 28.58 | 25.15 | 13.72 | +1.13 |
| F196 | 24.19 | 26.10 | 15.32 | + .82 | F226 | 26.12 | 30.89 | 15.32 | +1.32 | F256 | 29.74 | 30.18 | 11.95 | + .55 |
| F197 | 23.94 | 25.88 | 15.04 | + .56 | F228 | 26.85 | 30.38 | 15.30 | + .54 | F257 | 29.04 | 37.13 | 14.62 | + .46 |
| F198 | 23.88 | 25.84 | 14.07 | + .21 | F230 | 26.99 | 29.25 | 15.53 | + .41 | F258 | 29.53 | 35.92 | 14.76 | + .54 |
| F199 | 23.45 | 25.64 | 15.60 | + .77 | F231 | 26.72 | 27.79 | 15.60 | + .70 | F259 | 29.92 | 33.43 | 13.61 | + .70 |
| F200 | 25.24 | 26.41 | 13.91 | +2.15 | F232 | 26.70 | 27.54 | 15.98 | + .68 | F260 | 28.96 | 30.04 | F | + |
| F201 | 24.75 | 42.57 | 14.94 | + .44 | F233 | 27.10 | 28.05 | F | + | F261* | 30.33 | 26.33 | 11.33 | + .03 |
| F202 | 25.34 | 41.14 | 15.30 | + .31 | F234 | 26.29 | 27.68 | 15.76 | + .46 | F262* | 31.58 | 37.59 | 11.86 | + .29 |
| F203 | 25.37 | 42.02 | 14.96 | +1.20 | F235 | 25.83 | 25.68 | 15.48 | + .70 | F263* | 32.45 | 36.27 | 12.94 | + .33 |
| F204 | 25.69 | 41.36 | 15.69 | + .85 | F236 | 27.74 | 40.30 | 15.32 | + .85 | F264* | 33.82 | 32.14 | 12.53 | + .77 |
| F205 | 25.23 | 37.91 | 15.51 | + .85 | F237 | 28.73 | 38.93 | 15.98 | | F265* | 33.46 | 31.49 | 12.90 | +1.16 |

The probable error of a magnitude determination is about $\pm 0^m.10$, as illustrated in figure 8, which gives the relation between scale reading and photographic magnitude.

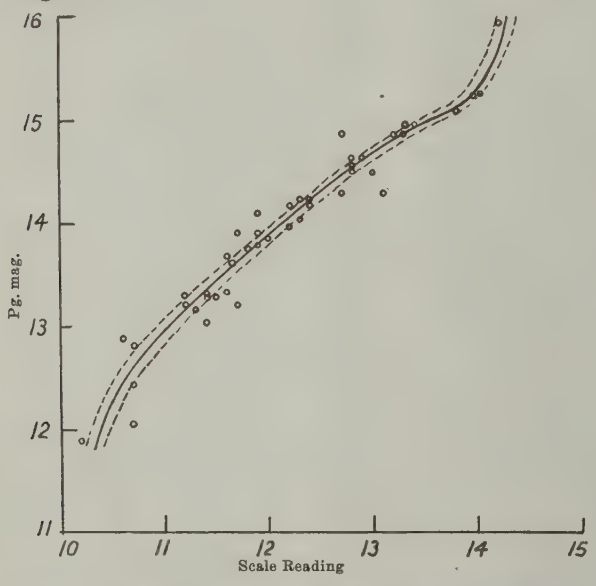



Fig. 8. Area E. Whole filter. Relation between scale reading on color index plate CK 94 and photographic magnitude determined from North Polar comparisons on plate CK 97 and RT 1728. Probable error of a magnitude determination = ± 0.10.

## V. Discussion of the Material

### (a) *Color-Magnitude Diagrams*

In order to bring the observed and reduced material, listed in the preceding catalogue, into a form suitable for statistical discussion, the stars were charted in a form similar to the Hertzsprung-Russell Diagram. As independent variables, which ordinarily are the spectral type and the absolute visual magnitude, the color index and the apparent photographic magnitude are used. The first of the two can be easily reduced to the spectral type (see IIId), while the absolute magnitudes are as yet unknown. However, if the depth of the cloud is small in comparison with its distance, absolute and apparent magnitudes will virtually differ by a constant, namely: $M-m=5+5$ log p. As dependent variable the relative star numbers are plotted. If from the characteristics of the color-magnitude diagram, the apparent magnitudes corresponding to different spectral types, can be deduced, and if, further, giants and dwarfs can be discriminated, the assignment of absolute magnitudes and the solution for the parallax $p$ become possible. The absolute magnitudes are known with great certainty from trigonometric, spectroscopic and moving cluster parallaxes.

The color-magnitude diagrams were first plotted for the central part (circle I) of each short exposure area, A to F, individually, (tables 12–17), also of the whole

## TABLE 12

AREA A, SHORT EXPOSURE

NUMBER OF STARS PER 0.0471 SQUARE DEGREE

| Pg. Mg. | C <0.60 | −0.60 to −0.35 | −0.35 to −0.10 | −0.10 to +0.15 | +0.15 to +0.40 | +0.40 to +0.65 | +0.65 to +0.90 | +0.90 to +1.15 | +1.15 to +1.40 | +1.40 to +1.65 | +1.65 to +1.90 | +1.90 to +2.15 | +2.15 to +2.40 | >+2.40 | Sum. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| <10.00 | | | | | | | | | | | | | | | |
| 10.00 to 10.50 | | | | | | 1 | | 1 | | 1 | | | | | 3 |
| 10.50 to 11.00 | | | 1 | | | | | | | | | | | | 1 |
| 11.00 to 11.50 | | | | | 1 | | | | | | | | | | 1 |
| 11.50 to 12.00 | | | 1 | | | | | | | | | | | | 1 |
| 12.00 to 12.50 | | | 1 | 1 | | 1 | | | | | | | | | 3 |
| 12.50 to 13.00 | | | 2 | 3 | | | | | | 2 | 1 | | | | 8 |
| 13.00 to 13.50 | | | 3 | 6 | 3 | 1 | | | | 2 | | | | 1 | 16 |
| 13.50 to 14.00 | | | 1 | 2 | 9 | 2 | | 1 | | | | | | | 15 |
| 14.00 to 14.50 | | | | 3 | 18 | 19 | 13 | | 9 | 2 | | | 1 | | 65 |
| 14.50 to 15.00 | | | | 1 | 10 | 24 | 9 | 4 | 5 | 3 | 2 | 1 | | | 59 |
| 15.00 to 15.50 | | | | | | 4 | 5 | 8 | 5 | 6 | 7 | 4 | 1 | 1 | 41 |
| 15.50 to 16.00 | | | | | | | 1 | 2 | 1 | 3 | 1 | | | | 8 |
| 16.00 to 16.50 | | | | | | | | | | | | | | | |
| 16.50 to 17.00 | | | | | | | | | | | | | | | |
| 17.00 to 17.50 | | | | | | | | | | | | | | | |
| Sum | | | 9 | 16 | 41 | 52 | 28 | 16 | 20 | 19 | 11 | 5 | 2 | 2 | 221 |

## TABLE 13

AREA B

NUMBER OF STARS PER 0.0471 SQUARE DEGREE

| Pg. Mg. | C <−0.60 | −0.60 to −0.35 | −0.35 to −0.10 | −0.10 to +0.15 | +0.15 to +0.40 | +0.40 to +0.65 | +0.65 to +0.90 | +0.90 to +1.15 | +1.15 to +1.40 | +1.40 to +1.65 | +1.65 to +1.90 | +1.90 to +2.15 | +2.15 to +2.40 | >+2.40 | Sum. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| <10.00 | | 1 | | | | | 1 | | | | | | | | 2 |
| 10.00 to 10.50 | | | | | | | | | 1 | | | | | | 1 |
| 10.50 to 11.00 | 1 | | | | | | | | | | | | | | 1 |
| 11.00 to 11.50 | | | | | | | | | | | | | | | |
| 11.50 to 12.00 | | | | | | | | | | | | 1 | | | 1 |
| 12.00 to 12.50 | | | 1 | | | | | | | | | | | | 1 |
| 12.50 to 13.00 | | | 1 | 2 | 1 | | | | | | | | | | 4 |
| 13.00 to 13.50 | | | | 3 | | 2 | | | | | 1 | | | | 6 |
| 13.50 to 14.00 | 1 | | | 2 | 1 | 1 | | 2 | | 1 | | | | | 8 |
| 14.00 to 14.50 | | 1 | 1 | 3 | 3 | 7 | | 2 | 2 | | 2 | 1 | | | 22 |
| 14.50 to 15.00 | | | | 7 | 10 | 3 | 2 | 2 | 1 | 2 | | 1 | | | 28 |
| 15.00 to 15.50 | | | | 5 | 5 | 21 | 8 | 5 | 3 | 2 | 1 | 2 | 1 | | 53 |
| 15.50 to 16.00 | | | | 1 | 7 | 12 | 6 | 10 | 13 | 11 | 3 | 2 | | | 65 |
| 16.00 to 16.50 | | | | | | | | 4 | 6 | 4 | 5 | 1 | 1 | | 21 |
| 16.50 to 17.00 | | | | | | | | | | | | | | 1 | 1 |
| Sum | 2 | 2 | 3 | 23 | 27 | 46 | 17 | 25 | 26 | 20 | 12 | 8 | 2 | 1 | 214 |

## TABLE 14

AREA C

NUMBER OF STARS PER 0.0471 SQUARE DEGREE

| Pg. Mg. | C <−0.60 | −0.60 to −0.35 | −0.35 to −0.10 | −0.10 to +0.15 | +0.15 to +0.40 | +0.40 to +0.65 | +0.65 to +0.90 | +0.90 to +1.15 | +1.15 to +1.40 | +1.40 to +1.65 | +1.65 to +1.90 | +1.90 to +2.15 | +2.15 to +2.40 | >+2.40 | Sum. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| <10.00 | | | | | | | | | | | | | | | |
| 10.00 to 10.50 | | | | | | | | | | | | | | | |
| 10.50 to 11.00 | | | 1 | | | | | | | | | | | | 1 |
| 11.00 to 11.50 | | | | 1 | 1 | | | | | | | | | | 2 |
| 11.50 to 12.00 | | | | 1 | | | | | | | | | | | 1 |
| 12.00 to 12.50 | | | | 1 | | | | | | | | | | | 1 |
| 12.50 to 13.00 | | | | | 1 | 1 | | | | | | | | | 2 |
| 13.00 to 13.50 | | | | | | | | | | | | | | | 0 |
| 13.50 to 14.00 | | | | | | 1 | 2 | | | 1 | | | | | 4 |
| 14.00 to 14.50 | | 1 | | 2 | 3 | 3 | 1 | | | | 1 | | | | 11 |
| 14.50 to 15.00 | | | | | 2 | 4 | 1 | | | 2 | 4 | 1 | | | 14 |
| 15.00 to 15.50 | | | | | 1 | 7 | 6 | 3 | 2 | 2 | 3 | 2 | 1 | 1 | 28 |
| 15.50 to 16.00 | | | | | 1 | 2 | 15 | 17 | 13 | 5 | 5 | 1 | 1 | | 60 |
| 16.00 to 16.50 | | | | | | 1 | 2 | 4 | 2 | 10 | 10 | 5 | | | 34 |
| 16.50 to 17.00 | | | | | | | 1 | | | 1 | | 1 | | | 3 |
| Sum | | 1 | 1 | 5 | 9 | 19 | 28 | 24 | 17 | 21 | 23 | 10 | 2 | 1 | 161 |

TABLE 15

AREA D

NUMBER OF STARS PER 0.0471 SQUARE DEGREE

| Pg. Mg. | C | <−0.60 | −0.60 to −0.35 | −0.35 to −0.10 | −0.10 to +0.15 | +0.15 to +0.40 | +0.40 to +0.65 | +0.65 to +0.90 | +0.90 to +1.15 | +1.15 to +1.40 | +1.40 to +1.65 | +1.65 to +1.90 | +1.90 to +2.15 | +2.15 to +2.40 | >+2.40 | Sum. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| <10.00 | | | | | | | | | | | | | | | | |
| 10.00 to 10.50 | | | | | | | | | | | | | | | | |
| 10.50 to 11.00 | | | | | | | | | | | | | | | | |
| 11.00 to 11.50 | | | | | | | | | | | | | | | | |
| 11.50 to 12.00 | | | | | | 2 | 1 | | | | | 1 | | | | 4 |
| 12.00 to 12.50 | | | | | | | | | | | | | | | | 0 |
| 12.50 to 13.00 | | | | | | | 2 | | | | | | | | | 2 |
| 13.00 to 13.50 | | | | | | 2 | 1 | 2 | 1 | | | | | | | 6 |
| 13.50 to 14.00 | | | | | 1 | 1 | 5 | | | 1 | 2 | | | | | 10 |
| 14.00 to 14.50 | | | | | | 3 | 6 | 2 | 2 | 3 | 1 | 1 | | | | 18 |
| 14.50 to 15.00 | | | | | 1 | 2 | 9 | 9 | 1 | 5 | 4 | 3 | 2 | 1 | | 37 |
| 15.00 to 15.50 | | | | | 1 | 7 | 15 | 14 | 3 | 11 | 4 | 4 | 2 | 1 | | 62 |
| 15.50 to 16.00 | | | | | | | 8 | 17 | 7 | 4 | 7 | 4 | 1 | | | 48 |
| 16.00 to 16.50 | | | | | | | 1 | 3 | 11 | 10 | 6 | 16 | 13 | 9 | 3 | 72 |
| Sum | | | | | 3 | 17 | 48 | 47 | 25 | 34 | 24 | 29 | 18 | 11 | 3 | 259 |

TABLE 16

AREA E

NUMBER OF STARS PER 0.0471 SQUARE DEGREE

| Pg. Mg. | C | <−0.60 | −0.60 to −0.35 | −0.35 to −0.10 | −0.10 to +0.15 | +0.15 to +0.40 | +0.40 to +0.65 | +0.65 to +0.90 | +0.90 to +1.15 | +1.15 to +1.40 | +1.40 to +1.65 | +1.65 to +1.90 | +1.90 to +2.15 | +2.15 to +2.40 | >+2.40 | Sum. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| <10.00 | | | | | | | | | | | | | | | | |
| 10.00 to 10.50 | | | | | 1 | | | | | | | | | | | 1 |
| 10.50 to 11.00 | | | | | | | | | 1 | | | | | | | 1 |
| 11.00 to 11.50 | | | | | | | | | | | | | | | | |
| 11.50 to 12.00 | | | | | | | | | | | | | | | | |
| 12.00 to 12.50 | | | | | | 1 | | | | | | | | | | 1 |
| 12.50 to 13.00 | | | | | | | 2 | 2 | | | | | | | | 4 |
| 13.00 to 13.50 | | | | | | | 4 | 1 | | | 1 | | | | | 6 |
| 13.50 to 14.00 | | | | | | 2 | 2 | 4 | 2 | 3 | | | | | | 13 |
| 14.00 to 14.50 | | | | | | 3 | 7 | 1 | 5 | 2 | 1 | 1 | | | | 20 |
| 14.50 to 15.00 | | | | 1 | | 1 | 6 | 1 | 3 | 5 | 5 | 3 | | 1 | | 26 |
| 15.00 to 15.50 | | | | | 1 | 1 | 5 | 29 | 14 | 8 | 6 | | 1 | | | 65 |
| 15.50 to 16.00 | | | | | | | | 1 | 15 | 6 | 2 | 1 | | | | 25 |
| 16.00 to 16.50 | | | | | | | | | 4 | 2 | 1 | 1 | 1 | 1 | | 10 |
| Sum | | | | 1 | 2 | 8 | 26 | 39 | 44 | 26 | 16 | 6 | 2 | 2 | | 172 |

TABLE 17

AREA F

NUMBER OF STARS PER 0.0471 SQUARE DEGREE

| Pg. Mg. | C | <−0.60 | −0.60 to −0.35 | −0.35 to −0.10 | −0.10 to +0.15 | +0.15 to +0.40 | +0.40 to +0.65 | +0.65 to +0.90 | +0.90 to +1.15 | +1.15 to +1.40 | +1.40 to +1.65 | +1.65 to +1.90 | +1.90 to +2.15 | +2.15 to +2.40 | >+2.40 | Sum. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| <10.00 | | | | | | | | | | 1 | | | 1 | | | 2 |
| 10.00 to 10.50 | | | | | | | | | | | | | | | | |
| 10.50 to 11.00 | | | | | | | | | | | | | | | | |
| 11.00 to 11.50 | | | | | | | | | | | | | | | | |
| 11.50 to 12.00 | | | | | | | | | | | | | | | | |
| 12.00 to 12.50 | | | | | 1 | 1 | | | | | | | | | | 2 |
| 12.50 to 13.00 | | | | | | 5 | 2 | 1 | | | | | | | | 8 |
| 13.00 to 13.50 | | 1 | | | | 2 | 1 | | | | | | | | | 4 |
| 13.50 to 14.00 | | | | | 2 | 4 | 2 | 2 | | | | | | 1 | | 11 |
| 14.00 to 14.50 | | | | | 2 | 5 | 4 | 5 | 3 | 1 | 2 | 1 | 1 | | | 24 |
| 14.50 to 15.00 | | | | | 1 | 9 | 10 | 3 | 2 | 5 | 3 | 3 | | | | 36 |
| 15.00 to 15.50 | | | | | 1 | 5 | 21 | 20 | 7 | 3 | 3 | 4 | 1 | 3 | | 68 |
| 15.50 to 16.00 | | | | | | 1 | 10 | 11 | 13 | 6 | 10 | 9 | 1 | 3 | | 64 |
| 16.00 to 16.50 | | | | | | | 2 | 8 | 5 | 4 | 3 | 3 | 1 | 2 | 1 | 29 |
| Sum | | 1 | | | 7 | 32 | 52 | 50 | 30 | 20 | 21 | 20 | 5 | 9 | 1 | 248 |

TABLE 18

AREA A, LONG EXPOSURE

NUMBER OF STARS PER 0.00827 SQUARE DEGREE

| Pg. Mg. | C <−0.60 | −0.60 to −0.35 | −0.35 to −0.10 | −0.10 to +0.15 | +0.15 to +0.40 | +0.40 to +0.65 | +0.65 to +0.90 | +0.90 to +1.15 | +1.15 to +1.40 | +1.40 to +1.65 | +1.65 to +1.90 | +1.90 to +2.15 | +2.15 to +2.40 | >+2.40 | Sum. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| <14.00 | | | | | | 1 | 1 | 4 | 2 | | | | | | 8 |
| 14.00 to 14.50 | | | | | | 7 | 4 | 5 | 4 | | | | | | 20 |
| 14.50 to 15.00 | | | | 1 | 1 | 5 | 6 | 2 | 1 | | | | | | 16 |
| 15.00 to 15.50 | | | | | 1 | 5 | 2 | 5 | 1 | 1 | 1 | | | | 16 |
| 15.50 to 16.00 | | | | 1 | 1 | 1 | 5 | 7 | 2 | 4 | 3 | 2 | | 1 | 27 |
| 16.00 to 16.50 | | | 1 | | | 5 | 15 | 10 | 6 | 1 | 4 | 2 | 3 | | 47 |
| 16.50 to 17.00 | | | 2 | | 3 | 6 | 17 | 18 | 5 | 3 | 3 | 2 | 2 | 4 | 65 |
| 17.00 to 17.50 | | 1 | | 1 | 2 | 11 | 19 | 17 | 19 | 2 | 3 | 4 | | | 79 |
| 17.50 to 18.00 | | | | 2 | 2 | 12 | 19 | 23 | 23 | 8 | 6 | 6 | 4 | 5 | 110 |
| 18.00 to 18.50 | | | | | 4 | 13 | 25 | 26 | 12 | 12 | 11 | 8 | 9 | 11 | 131 |
| 18.50 to 19.00 | | | | | 3 | 6 | 9 | 4 | 8 | 4 | 3 | 3 | 2 | 4 | 46 |
| Sum | | 1 | 3 | 5 | 17 | 72 | 122 | 121 | 83 | 35 | 34 | 27 | 20 | 25 | 565 |

TABLE 19

AREAS A (SHORT EXPOSURE) TO F COMBINED

Between Circles I and II only

NUMBER OF STARS PER 0.2256 SQUARE DEGREE

| Pg. Mg. | C <−0.60 | −0.60 to −0.35 | −0.35 to −0.10 | −0.10 to +0.15 | +0.15 to +0.40 | +0.40 to +0.65 | +0.65 to +0.90 | +0.90 to +1.15 | +1.15 to +1.40 | +1.40 to +1.65 | +1.65 to +1.90 | +1.90 to +2.15 | +2.15 to +2.40 | >+2.40 | Sum. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| <10.00 | | | | | | | | | | | | | | | |
| 10.00 to 10.50 | | | | | | 1 | | | | | | | | | 1 |
| 10.50 to 11.00 | | | 1 | 3 | | | | | | | | | | | 4 |
| 11.00 to 11.50 | | | 1 | 1 | | 1 | | | | | | | | | 3 |
| 11.50 to 12.00 | | 1 | 2 | 1 | 2 | 2 | | | | | | | | | 8 |
| 12.00 to 12.50 | 1 | | 1 | 4 | 2 | | 1 | | | | | | | | 9 |
| 12.50 to 13.00 | | | 1 | 2 | 1 | 3 | 1 | 1 | 2 | | | | | | 11 |
| 13.00 to 13.50 | | | | | 4 | 4 | 1 | 1 | 1 | | | | | | 11 |
| 13.50 to 14.00 | | | | | | 4 | 4 | 5 | 6 | 2 | 1 | 1 | | | 23 |
| 14.00 to 14.50 | | | | | | | | | 5 | | | | | | 5 |
| Sum | 1 | 1 | 6 | 11 | 9 | 15 | 7 | 7 | 14 | 2 | 1 | 1 | | | 75 |

measured area of the long exposure of Area A (table 18), and of the bright stars in the combined ring-shaped supplementary fields of all six areas, included between circles I and II (table 19).

Of a number of stars, only one magnitude-determination was available, because either (a) the image was too bright to be measured, or (b) too faint, or (c) could not be measured because it was covered by a reseau line, intercepted by another image, or near the half-filter division line. For a very small number of stars, neither image was available for reasons under (c). Where one magnitude was available, the algebraic sign of the color index could be derived in most cases from a simple deliberation regarding the missing image. The distribution of these partially observed stars in the color-magnitude diagram was made according to the relative star numbers obtained from the 2-hour exposure. The few stars for which neither image was measured, were distributed on the basis of general probability. One-half of a photographic magnitude, and 0.25 of the color index were chosen as intervals, and the numbers summed up horizontally and vertically. Figure 9 shows a plot of the stars observed in Area A, both with the short and long exposures. The short exposure diagram includes the bright stars between circles I and II.

In order to obtain a complete chart covering the range from 10th to 18th magnitude, the short exposure charts were made complete to 16th magnitude by extending the cumulative star numbers by means of the average rate of increase determined for each area. Due to the low limit of completeness of the short exposure of Area A, the star numbers of that area are significant only to 15.50 instead of 16.00. The long exposure chart was in the same way extended to 18.50 magnitude. In the chart combining all material (table 20), all bright stars up to 13th magnitude within circle II in all six areas were included: from $13^m00$ to $15^m50$, the stars within circle I in all six areas; and from $15^m50$ to $18^m50$ the stars of the long exposure of Area A. The surfaces covered are:

$10^m00$ to $13^m00$, 0.5082 square degree
$13^m00$ to $15^m50$, 0.2826 square degree
$15^m50$ to $18^m50$, 0.00827 square degree

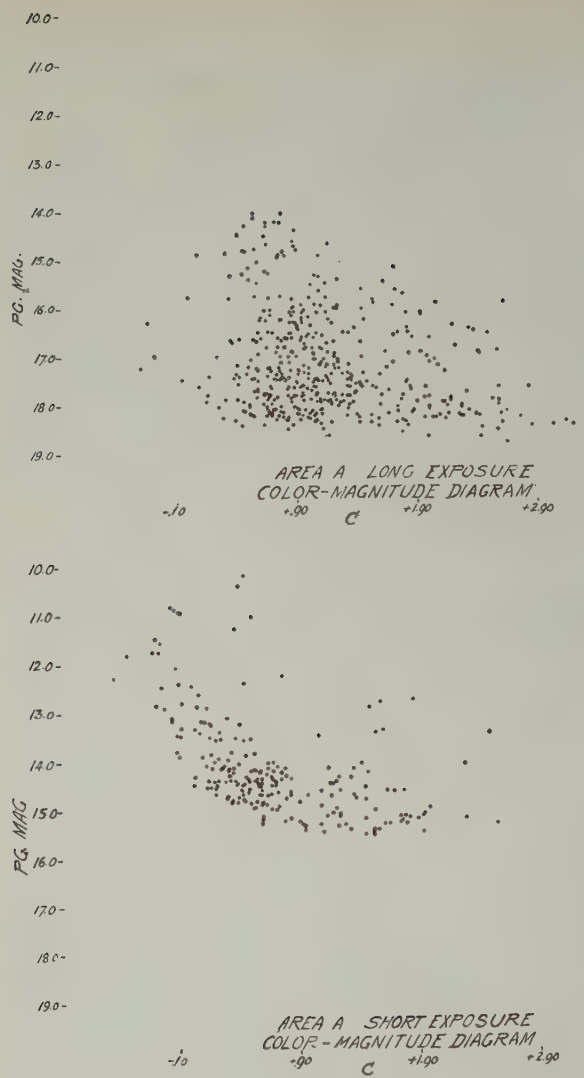



Fig. 9

The values were reduced to 0.1 square degree, and a whole magnitude was selected as magnitude interval.

From the discussion of the star density in a later paragraph it is found that Area A is by far the richest of all the six sampling areas. In combining the star numbers of the long exposure of Area A beginning at $15^m.50$, with the star numbers of the stars up to $15^m.50$, which are the means from all six areas, it is necessary to reduce the star numbers of the stars fainter than $15^m.50$ to the average density. The ratio of $N_m$ for Area A to the average $N_m$ for all six areas is 2.16 for a limiting magnitude of $15^m.00$, from the short exposure. For a limiting magnitude of $15^m.50$ from the long exposure, the ratio is 2.12. Accordingly, the star numbers of stars fainter than $15^m.50$ from Area A, are reduced to the mean density by dividing with 2.14. In order to make the half-magnitude interval $18^m.00$ to $18^m.50$ comparable with the whole magnitude intervals used throughout the table, the star numbers in that interval were multiplied by 2. The cumulative star numbers $N_m$ for increasing magnitudes are also given. (See table 20.)

The star numbers in this table, where all material has been included, are affected with the accidental errors of measurement, and the effect would be a general spreading out of the characteristic color-magnitude diagram. The effect would take place both along the color-axis and the magnitude-axis. The dispersion along the magnitude-axis, owing to the greater accuracy attainable in magnitude determinations, is less than that along the color-index-axis, and no correction was applied, but the accidental errors in the numbers were reduced by choosing whole magnitude intervals. The dispersion along the color-axis is greater, because the probable error is a considerable fraction of the color interval. Further, the probable error of a color index determination is larger than that of a magnitude determination, chiefly because the color index is the difference between two magnitude determinations. While the probable error of a single magnitude observation is about $0^m.10$, it is $0^m.10$ to $0^m.20$ for a single color index determination, but these values are reduced by taking the mean between two separate determinations.

It is possible to eliminate the effect of dispersion along the color-axis for each successive magnitude interval, by assuming that the errors of observation are distributed according to a normal frequency function:

$$A = \frac{h}{\sqrt{\pi}} e^{-h^2(x-a)^2}$$

The number observed in a single square, $N\,y(x)\,dx$, consists of a part of the actual number belonging in that square, plus a number thrown in from the two adjacent squares, plus a small number thrown in from the two following squares, etc. Accordingly, the number $N\,y(x)\,dx$, measured in the interval $x+dx$ differs from the actual number $N\,u(x)\,dx$. Provided the modulus of precision $h$ and the observed distribution defined by the function $y(x)$ is known, it is possible to find the actual distribution defined by the function $u(x)$. It can be shown[17] that

$$u(x) = y(x) - \frac{1}{4h^2} y''(x) + \frac{1}{32h^4} y^{iv}(x) + \ldots\ldots$$

The modulus of precision is $h = \frac{0.476936}{r}$, where $r$ is the probable error. The second and fourth derivatives, $y''(x)$ and $y^{iv}(x)$ can be found by interpolation, using formulae which are derived from Stirling's Formula of interpolation[18] and are applicable to tabular values:

[17] Whittaker and Robinson, *Calculus of Observations*, p. 207, 1924.

[18] H. L. Rice, *Theory and Practice of Interpolation*, p. 110, 1899.

$$y''(x)=\frac{1}{\omega^2}(b_x-\frac{1}{12}d_x)$$

$$y^{iv}(x)=\frac{1}{\omega^4}d_x$$

where $\omega$ is the interval of the argument, here 0.25, and where $b_x$ and $d_x$ are the second and fourth differences on the horizontal line through $y(x)$. A smooth curve was drawn through the observed values, and the smoothed values used for differencing. The corrections found were applied to the observed values, and again a curve was drawn through the corrected values. For magnitudes to 16$^m$.00 a probable error of ±.10, and from 16$^m$.00 to 18$^m$.00 a probable error of ±.15 in the color index determination was used. The following figure 10 shows both the observed values and those corrected for dispersion in the magnitude interval 16$^m$.00 − 17$^m$.00.

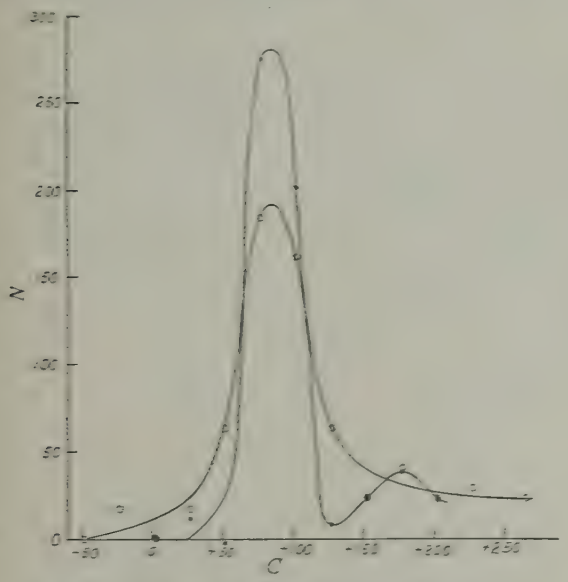



Fig. 10. Correction for dispersion magnitude interval 16$^m$.00–17$^m$.00.

○ Uncorrected for dispersion.
● Corrected for dispersion.

In table 21 are shown the star numbers of table 20 after correction for dispersion. The cumulative star numbers $N_m$ at 18$^m$.50 agree within less than ½ of 1%. The characteristic shape of the dwarf branch is easily recognized, while the giant branch is less clearly suggested. Some faint white stars are found, which may be either white dwarfs in the foreground, or ordinary white stars at a great distance behind the cloud, or possibly, since the actual number observed is only four, abnormally dispersed G-type dwarfs. Since the limit of completeness is reached at 18$^m$.00, the magnitude interval from 18$^m$.00 to 18$^m$.50 was made complete in the manner previously described, and therefore it should have lower weight than the magnitude intervals from 10$^m$.00 to 18$^m$.00. The figures are placed in parentheses in tables 20 and 21.

Owing to the size of the selected areas in which the magnitudes and color indices have been determined in this investigation only a very small number of brighter stars are found therein. In order to have available for the discussion a continuous table of magnitudes and color indices ranging from 5th to 18th magnitude, the magnitudes and spectral types of the stars brighter than 8$^m$.5 were taken from the *Henry Draper Catalogue* within an area limited by $\alpha=18^h\ 30^m$ and $19^h\ 00^m$, $\delta=-2°$ and $-12°$ (1900.0). The area equals 74.16 square degrees, and the star numbers were reduced to 0.1 square degree to conform to the system adopted previously. In order to reduce the Harvard photographic magnitudes ($H\ P_g$) used in the *Henry Draper Catalogue* to the photographic magnitudes ($m_D$) on the 35D Filter system used throughout this investigation, the following formula was employed:

$$m_D=HP_g+0.75C_H$$

TABLE 20

NUMBER OF STARS PER 0.10 SQUARE DEGREE IN THE SCUTUM CLOUD

All Material Combined. Uncorrected for Dispersion

| Pg Mg. C | <−0.40 | −0.40 to −0.35 | −0.35 to −0.30 | −0.30 to −0.15 | −0.15 to +0.40 | +0.40 to +0.65 | +0.65 to +0.90 | +0.90 to +1.15 | +1.15 to +1.40 | +1.40 to +1.65 | +1.65 to +1.90 | +1.90 to +2.15 | +2.15 to +2.40 | >+2.40 | Sum. | $N_m$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| <10.00 | | 0.2 | | | | | 0.2 | | 0.2 | | | 0.2 | | | 0.8 | 0.8 |
| 10.00–11.00 | 0.2 | | 0.6 | 0.8 | | 0.4 | | 0.6 | 0.4 | 0.3 | | | | | 3.3 | 4.1 |
| 11.00–12.00 | | 0.2 | 0.8 | 0.8 | 1.2 | 0.8 | | | | | 0.2 | 0.2 | | | 4.2 | 8.3 |
| 12.00–13.00 | 0.2 | | 1.4 | 2.7 | 2.4 | 2.2 | 1.0 | 0.2 | 0.4 | 0.4 | 0.2 | | 0.4 | 0.4 | 11.1 | 19.4 |
| 13.00–14.00 | 0.7 | | 1.4 | 5.7 | 8.5 | 7.8 | 3.9 | 2.1 | 1.4 | 2.5 | 0.4 | | 1.1 | | 35.2 | 54.6 |
| 14.00–15.00 | | 0.7 | 0.7 | 7.1 | 24.4 | 36.1 | 16.6 | 8.5 | 13.4 | 8.8 | 7.4 | 2.5 | | | 127.3 | 181.9 |
| 15.00–16.00 | | | | 8.6 | 12.5 | 35.2 | 62.0 | 61.5 | 27.1 | 36.4 | 30.0 | 19.3 | 3.5 | 7.2 | 303.3 | 485.2 |
| 16.00–17.00 | | | 17 | | 17 | 63 | 184 | 161 | 63 | 23 | 40 | 23 | 29 | 23 | 643 | 1128.2 |
| 17.00–18.00 | | 6 | | 17 | 23 | 132 | 228 | 230 | 251 | 58 | 52 | 56 | 23 | 29 | 1107 | 2235.2 |
| 18.00–18.50* | | | | | (69) | (290) | (427) | (405) | (354) | (307) | (165) | (108) | (101) | (105) | (2301) | 3285.7† |
| Sum. | 1.1 | 7.1 | 21.9 | 42.7 | 158.9 | 507.5 | 992.7 | 878.9 | 910.9 | 886.4 | 805.2 | 241.2 | 208.0 | 254.6 | 4526.2 | |

* Star numbers reduced to a whole magnitude interval.
† $N_m$ refers to 18.50.

TABLE 21

NUMBER OF STARS PER 0.10 SQUARE DEGREE IN THE SCUTUM CLOUD

All Material Combined. Corrected for Dispersion

| Pg. Mg. | C: <−0.60 | −0.60 to −0.35 | −0.35 to −0.10 | −0.10 to +0.15 | +0.15 to +0.40 | +0.40 to +0.65 | +0.65 to +0.90 | +0.90 to +1.15 | +1.15 to +1.40 | +1.40 to +1.65 | +1.65 to +1.90 | +1.90 to +2.15 | +2.15 to +2.40 | >+2.40 | Sum. | $N_m$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| <10.00 | | 0.2 | | | | | 0.2 | | 0.2 | | | 0.2 | | | 0.8 | 0.8 |
| 10.00–11.00 | 0.2 | | 0.6 | 0.8 | | 0.4 | | 0.6 | 0.4 | 0.3 | | | | | 3.3 | 4.1 |
| 11.00–12.00 | | | 0.5 | 1.2 | 1.3 | 0.8 | | | | | 0.2 | 0.2 | | | 4.2 | 8.3 |
| 12.00–13.00 | 0.2 | | 1.4 | 3.0 | 2.8 | 1.9 | 1.0 | 0.1 | 0.4 | 0.4 | 0.2 | | | | 11.4 | 19.7 |
| 13.00–14.00 | 0.7 | | 0.9 | 6.0 | 9.2 | 8.5 | 3.3 | 1.9 | 1.4 | 2.5 | 0.3 | | 0.4 | 0.4 | 35.5 | 55.2 |
| 14.00–15.00 | | | | 4.3 | 33.3 | 41.6 | 15.7 | 7.0 | 13.3 | 8.7 | 7.3 | 2.6 | 1.1 | | 134.9 | 190.1 |
| 15.00–16.00 | | | | 2.0 | 9.7 | 28.6 | 75.1 | 74.8 | 13.5 | 37.6 | 31.1 | 21.4 | | | 293.8 | 483.9 |
| 16.00–17.00 | | | (17) | | 2.0 | 25 | 275 | 202 | 8 | 24 | 39 | 22 | 29 | 23 | 666 | 1149.9 |
| 17.00–18.00 | | (6) | | | | 24 | 303 | 323 | 260 | 32 | 54 | 56 | 38 | 30 | 1126 | 2275.9 |
| 18.00–18.50* | | | | | | (80) | (630) | (496) | (183) | (200) | (192) | (133) | (160) | (170) | (2244) | 3397.9† |
| Sum | 1.1 | 6.2 | 20.4 | 17.3 | 58.3 | 210.8 | 1303.3 | 1105.4 | 480.2 | 305.5 | 324.1 | 235.4 | 228.5 | 223.4 | 4519.9 | |

* Star numbers reduced to a whole magnitude interval.

† $N_m$ refers to 18.50.

TABLE 22

NUMBER OF STARS PER 0.10 SQUARE DEGREE IN THE SCUTUM CLOUD

Within the limits: $\alpha = 18^h\ 30^m$ to $19^h\ 00^m$, $\delta = -2°$ to $-12°$ } 1900.0

From Henry Draper Catalogue

| Pg. Mg. | C: <−0.60 | −0.60 to −0.35 | −0.35 to −0.10 | −0.10 to +0.15 | +0.15 to +0.40 | +0.40 to +0.65 | +0.65 to +0.90 | +0.90 to +1.15 | +1.15 to +1.40 | +1.40 to +1.65 | +1.65 to +1.90 | +1.90 to +2.15 | +2.15 to +2.40 | >+2.40 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5.00–6.00 | | | | 0.001 | | 0.001 | | 0.001 | | | | 0.001 | | |
| 6.00–7.00 | | 0.001 | 0.001 | 0.001 | 0.004 | 0.001 | 0.001 | | 0.001 | | | 0.001 | | |
| 7.00–8.00 | 0.005 | 0.003 | 0.003 | 0.016 | 0.003 | 0.004 | 0.001 | | 0.001 | | | 0.004 | | |
| 8.00–8.50* | (0.005) | | (0.016) | (0.040) | (0.008) | (0.005) | (0.008) | (0.003) | | (0.003) | | (0.003) | | |

* Star numbers reduced to a whole magnitude interval.

$C_H$ is the color index used in *H.D.C., H.A.* 71. The Harvard color index of a K0 star was taken as +1.0. The complete color-magnitude diagram of the brighter stars, on the same system as that of the fainter stars, is given in table 22.

(b) *Distance and Dimensions of Scutum Cloud*

From the corrected color-magnitude diagram (table 21) it is possible to determine that magnitude at which the maximum number of stars of a given spectral class is reached. Stars in parentheses are, for reasons given previously, not included. K. Lundmark[19] has determined absolute visual magnitudes for each spectral class from trigonometric, moving cluster, spectroscopic and hypothetical parallaxes. The visual absolute magnitudes are converted into photographic absolute magnitudes on the 35D Filter color system by the relation:

$$M_D = M_V + 1.36 C_I$$

where $C_I$ is the international color index ($C_I = +1.40$ for gK0).

[19] *Publ. A. S. P.* **34**, 150, 1922.

| Sp. | $M_V$ | $M_D$ |
|---|---|---|
| B5 | − .50 | − .79 |
| A0 | +1.20 | +1.20 |
| A5 | +2.20 | +2.52 |
| F0 | +2.50 | +3.03 |
| dF5 | +3.50 | +4.27 |
| dG0 | +4.50 | +5.42 |
| dG5 | +4.80 | +5.85 |
| gG0 | −1.50 | − .37 |
| gK0 | +1.05 | +2.95 |

The following table shows the values of the photographic magnitude $m$ of the frequency maximum, the absolute photographic magnitude $M$, and $(m-M)$ derived for various spectral types:

| Sp. | B8 | A0 | A4 | F0 | dG1 |
|---|---|---|---|---|---|
| C | −0.19 | 0 | +0.30 | +0.55 | +0.75 |
| m | 12.7 | 13.5 | 14.4 | 14.8 | 18.0 |
| M | + .4 | + 1.2 | + 2.3 | + 3.0 | + 5.6 |
| (m-M) | 12.3 | 12.3 | 12.1 | 11.8 | 12.4 |

The average $(m-M) = 12.2$ corresponds to a distance of 2800 parsecs, or 9100 light-years. These values are in good agreement with unpublished data by Dr. R. J. Trumpler, who finds $d = 2200$ parsecs for M11 and $d = 2750$ parsecs for M26. Both objects are star clusters in the *Scutum* cloud.

The apparent dimensions of the *Scutum* cloud are 6°x8°, and with this value of the parallax, the approximate actual dimensions become 310x390 parsecs. If the depth were of similar extent, the cloud could be considered as roughly spherical with a diameter of 300 to 400 parsecs. The effect of the distribution within the cloud on the color-magnitude diagram would be a widening-out by $\pm^m.15$ along the magnitude-axis. This effect would be very small compared with the natural dispersion of the absolute magnitudes, which is about one magnitude. However, in a later discussion the depth of the cloud can be found from the star density curves.

### (c) *Luminosity Function*

The frequency curves of the photographic magnitudes for various color intervals have been drawn on the following diagram. Since the distance of the stars within the *Scutum* cloud is in a first approximation a constant, the apparent magnitudes differ from the absolute magnitudes by a constant, and figure 11 will illustrate the frequency of the absolute magnitudes, provided the effect of the foreground stars, not members of the cloud and at varying distances, is eliminated. The increase in star numbers from white to red stars is seen to be enormous and the maximum shifts to fainter magnitudes as we proceed to the red stars, indicating that the majority of red stars must be dwarfs.

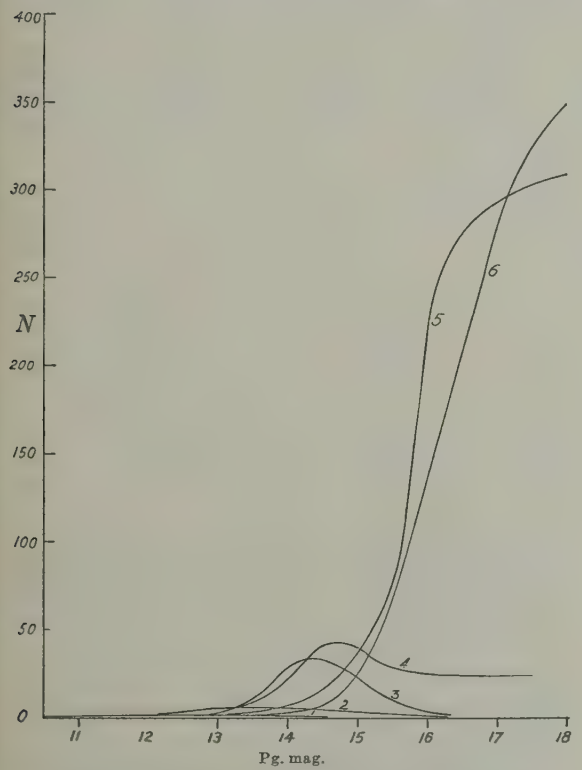



Fig. 11. Frequency curves of various color types.

| No. | C | SP. |
|---|---|---|
| 1 | <— .60 to — .10 | B5–B9 |
| 2 | — .10 " + .15 | B9–A1 |
| 3 | + .15 " + .40 | A2–A6 |
| 4 | + .40 " + .65 | A6–F3 |
| 5 | + .65 " + .90 | F3–dG3 |
| 6 | + .90 " + 1.15 | dG3–dG7 |

The form of the curves resembles a normal frequency function, with a dispersion of the absolute magnitudes, of about $\sigma = \pm 0^m.5$ to $1^m.0$. The agreement of this value with the generally adopted dispersion in the Hertzsprung-Russell diagram indicates that the depth of the cloud cannot be very great in comparison with its distance.

### (d) *Star Density*

A comparison of the relative star numbers in the six areas (see table 23) shows that Area A, (short exposure), is richest in stars, while Area C appears to be poorest, the numbers up to $15^m.0$ being in a ratio of almost 5 : 1. The other areas are intermediate. The average ratio of increase of $N_m$ derived from the whole material covering the range from 11th to 18th magnitude is 2.49. In the column "Cumulative Number" for all material combined, both the star numbers and their logarithms are given. A comparison with the mean distribution of stars according to Seares[19] is possible after the limiting magnitudes of the *Scutum* cloud, which are on the 35D-Cramer photographic scale, have been reduced to the international photographic scale by the formula:

$$m_I = m_D - 0.32 C_I$$

An approximate average value of $C_I$ for each magnitude can be derived from the color-magnitude diagram (table 21).

| Limit 35-D Pg.Mg. | Limit I.A.U. Pg.Mg. | Scutum Cloud log $N_m$ | Seares $\lambda=0$ | Log of ratio |
|---|---|---|---|---|
| 10.00 | 10.05 | 9.903 | 9.909 | 9.994 |
| 11.00 | 11.05 | 0.613 | 0.340 | 0.273 |
| 12.00 | 12.05 | 0.919 | 0.766 | 0.153 |
| 13.00 | 13.02 | 1.288 | 1.171 | 0.117 |
| 14.00 | 14.00 | 1.737 | 1.569 | 0.168 |
| 15.00 | 14.90 | 2.260 | 1.920 | 0.340 |
| 16.00 | 15.85 | 2.686 | 2.275 | 0.411 |
| 17.00 | 16.80 | 3.052 | 2.610 | 0.442 |
| 18.00 | 17.77 | 3.349 | 2.933 | 0.416 |

The values of log $N_m$ are referred to 0.10 square degree. It is seen that the star density in the *Scutum* cloud is somewhat higher than the mean up to about 13th magnitude, and increases very rapidly for the fainter stars, reaching a value of two to three times the mean at 18th magnitude. This increase is caused by the large number of dwarf stars which make their appearance beyond 13th magnitude.

[19] *Mt. W. C.* **346**, p. 44, 1928.

TABLE 23

COMPARISON OF STAR DENSITY

| Lim. Pg. Mag. | Area A (Short Exp.) $N_m$ | Area A (Short Exp.) Rich. | Area A (Long Exp.) $N_m$ | Area A (Long Exp.) Rich. | Area B $N_m$ | Area B Rich. | Area C $N_m$ | Area C Rich. | Area D $N_m$ | Area D Rich. | Area E $N_m$ | Area E Rich. | Area F $N_m$ | Area F Rich. | Areas A to F Average $N_m$ | All Material Combined* $N_m$ | All Material Combined* log $N_m$ | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10.00 | | — | | | 2 | 3.0 | | — | | — | | — | 2 | 3 | 0.7 | 0.8 | 9.903 | |
| | | | | | | | | | | | | | | | | | | 5.12 |
| 11.00 | 4 | 1.4 | | | 4 | 1.4 | 1 | .5 | | — | 2 | .9 | 2 | .9 | 2.16 | 4.1 | 0.613 | |
| | | | | | | | | | | | | | | | | | | 2.02 |
| 12.00 | 6 | 1.5 | | | 5 | 1.3 | 4 | 1.5 | 4 | 1.5 | 2 | .5 | 2 | .5 | 3.84 | 8.3 | 0.919 | |
| | | | | | | | | | | | | | | | | | | 2.34 |
| 13.00 | 17 | 1.72 | | | 10 | 1.02 | 7 | .71 | 6 | .61 | 7 | .71 | 12 | 1.22 | 9.83 | 19.4 | 1.288 | |
| | | | | | | | | | | | | | | | | | | 2.82 |
| 14.00 | 48 | 1.82 | | | 24 | .91 | 11 | .42 | 22 | .83 | 26 | .99 | 27 | 1.02 | 26.33 | 54.6 | 1.737 | |
| | | | | | | | | | | | | | | | | | | 3.34 |
| 15.00 | 172 | 2.16 | | | 74 | .92 | 36 | .45 | 77 | .96 | 72 | .90 | 87 | 1.09 | 79.67 | 181.9 | 2.260 | |
| 15.50 | | | 342 | 2.12 | 127 | | 64 | | 139 | | 137 | | 155 | | 160.67 | | | 2.66 |
| 16.00 | (557) | | | | (212) | | 124 | | (252) | | (260) | | (289) | | | 485.2 | 2.686 | |
| | | | | | | | | | | | | | | | | | | 2.32 |
| 17.00 | | | | | | | | | | | | | | | | 1128.2 | 3.052 | |
| | | | | | | | | | | | | | | | | | | 1.98 |
| 18.00 | | | | | | | | | | | | | | | | 2235.2 | 3.349 | |
| | | | | | | | | | | | | | | | | | | 2.03 |
| 18.50 | | | | | | | | | | | | | | | | (4536.2)† | (3.657) | |

$N_m$ = Cumulative number.
Rich. = Relative richness.
Ratio = Ratio of increase.

* Reduced to 0.1 square degree.
† Reduced to a whole magnitude interval.

STAR DENSITIES

| | Sp. | $B_0$-$B_9$ | B9-A5 | A5-F5 | gG | gK | dG |
|---|---|---|---|---|---|---|---|
| Scutum Cloud | N per 1.0 sq. degree | 47 | 620 | 2193 | (174) | (375) | (34360) |
| Scutum Cloud | log NS | 5.75–10 | 6.87–10 | 7.42–10 | (6.32–10) | (6.65–10) | (8.62–10) |
| Local System | log $N_L$ v. Rhijn | 6.60–10 | 6.61–10 | 7.57–10 | 5.88–10 | 6.68–10 | 7.27–10 |
| Local System | log $N_L$ Maxwell | 5.48–10 | 6.80–10 | 6.28–10 | 6.18–10 | 6.64–10 | 7.94–10 |

STAR DENSITIES

| Spectral Class | Local System From density law (Method B) Dx$10^6$ | Local System From density law (Method B) Per cent | Scutum Cloud From density law (Method B) Dx$10^6$ | Scutum Cloud From density law (Method B) Per cent | Scutum Cloud From star counts (Method A) N | Scutum Cloud From star counts (Method A) Dx$10^6$ | Scutum Cloud From star counts (Method A) Per cent |
|---|---|---|---|---|---|---|---|
| B5-B8 | 13 | .1 | 8 | .2 | 1,730 | 15.2 | .2 |
| B9-A1 | 170 | 1.8 | 30 | .7 | 8,300 | 72.8 | .9 |
| A2-A6 | 470 | 4.9 | 172 | 3.9 | 28,000 | 246 | 3.2 |
| A6-F3 | 1,400 | 14.6 | 800 | 18.1 | 101,000 | 886 | 11.3 |
| F3-dG3 | 6,000 | 62.3 | 3,250 | 73.8 | 647,000 | 5,670 | 72.6 |
| gG3-gG7 | 880 | 9.1 | (24) | 0.5 | 32,200 | (282) | 3.6 |
| gG8-gK4 | 690 | 7.2 | (122) | 2.8 | 73,200 | (642) | 8.2 |
| Sum | 9,623 | 100.0 | 4,406 | 100.0 | 891,430 | 7,814 | 100.0 |

Two methods have been employed for the derivation of the star density in the *Scutum* cloud. The first (A) assumes a depth of the cloud, and furnishes the average density within the cloud from the observed star numbers. The second (B) derives the density law, and gives the star density at various distances from the Sun.

*Method A*

The partial volume of the *Scutum* cloud contained in 1.0 square degree, for a mean distance of 2800 parsecs and a depth of 350 parsecs, becomes: $V = 840{,}000$ cubic parsecs. To find the number of stars of various spectral types per square degree, belonging to the *Scutum* cloud, the star numbers in the proper vertical columns, or fractions thereof, in table 21 are added and multiplied by 10. The foreground stars are excluded by omitting the stars which are more than two magnitudes brighter than the frequency maximum of the given color interval. In the following table the star numbers in the *Scutum* cloud per 1.0 square degree, and per cubic parsec ($N_S$) are shown. The numbers of G- and K-type stars are estimated.

For comparison purposes the star numbers per cubic parsec in the neighborhood of the Sun, as derived by v. Rhijn[20] and Maxwell[21] are also listed. Compared with V. Rhijn's figures, the *Scutum* cloud appears to be

[20] *Publ. Kapteyn Astr. Lab. Groningen*, **38**, 74, 1925.
[21] *L. O. B.* **13**, 86, 1927.

decidedly poorer in B-type stars, equally rich in A, F, and G-type giant stars, and considerably richer in G-type dwarf stars, than the local system. When compared with Maxwell's figures, the cloud seems to be equally rich in B, A, and G-type giant stars, and richer in F-type and G-type dwarf stars, than the neighborhood of the Sun. On the whole, therefore, the star density in the *Scutum* cloud will be, at least as far as the G-type stars, of the same order as that in our local system, under the assumption of a depth of 350 parsecs, and with this approximate method.

*Method B*

The number of stars per $\omega$ square degrees lying at a distance between $r$ and $r+dr$, and between magnitude $m$ and $m+dm$ is:

$$\psi(m, r)dm\ dr = \frac{\omega}{(57.3)^2} r^2 D(r)\varphi(m+5-5\log r)\ dm\ dr$$[22]

where $r$ is the distance in parsecs, $D(r)$ the density function, i.e. the total number of stars per unit volume at distance $r$, and $\varphi(m+5-5\log r)$ the luminosity function. $\varphi(m+5-5\log r)\ dm = \varphi(M)\ dM$ is the fraction of stars of absolute magnitude $M \pm \frac{dM}{2}$. Substitute for the variable $r$ a new parameter $y = 5\log r$, and for $D(r)$ the density function $\Delta(y)$, i.e. the total number of stars per unit volume at a distance corresponding to the parameter value $y$. Then we have the following relations for $\omega = 0.1$ square degree, which is the adopted unit area:

$$r = 10^{0.2y} \qquad dr = 0.46\text{x}10^{0.2y}dy = 10^{-0.34+0.2y}dy$$

$$\frac{0.1}{(57.3)^2} = 10^{-4.516+0.4y}$$

$$\psi(m, r)\ dr = \Psi(m, y)\ dy$$

With these relations: $\Psi(m, y)\ dm\ dy = 10^{-4.856+0.6y}\Delta(y)\ \varphi(m+5-y)\ dm\ dy$. Let $\int_{-\infty}^{+\infty} \Psi(m, y)\ dm = n(y)$, the number law. Then $n(y)\ dy$ is the fraction of stars of all apparent magnitudes within the distance limits $y \pm \frac{dy}{2}$ within the area $\omega$. If we consider that $\int_{-\infty}^{+\infty} \varphi(m+5-y)\ dm = 1$, and integrate $\Psi$ with respect to $m$, we obtain:

$$n(y) = \Delta(y)\ 10^{-4.86+0.6y} \text{ or } \Delta(y) = n(y)\ 10^{4.86-0.6y}$$

If the stars of a small spectral subdivision were all of the same absolute magnitude $M_o$, we would have $y = m+5-M_o$ and $n(y) = n(m+5-M_o)$, i.e. $n(y)$ could be derived for every color interval from $n(m)$ in table 21, by simply finding the argument $y$ corresponding to each $m$.

[22] Maxwell, *L. O. B.* **13**, 85.

The density law $\Delta(y)$ can then be derived from $n(y)$ by the above formula, provided the latter is given for infinitely small intervals of $y$. However, for convenience sake, the number of stars with a modulus of distance $y+\Delta y$, i.e. in a finite interval, has been determined. The absolute magnitudes are not constant but subject to a dispersion $\sigma$, which was assumed as $\pm 0.67$ for dwarfs and types A and B, and $\pm 0.80$ for giants.[24] The values $n(y)$, derived from tables 21 and 22 have to be corrected for both these effects.

(a) If $M$ varies within the limits $M \pm \Delta M$ according to an error function, and if all these stars are at a given distance, then the observed values of $n(y)$ plotted as a function of the parameter $y = m-(M \pm \Delta M)+5$ will be arranged in the form of an error curve, and it appears as if the stars which are actually at the same distance, were spread out in space between the limits $y \pm \Delta y$. The curve $n(y)$ would have been affected in the same manner if the values of $y$ had been subjected to errors of observation, and it is therefore possible[25] to derive from the observed curve one which is freed of the effect of dispersion:

$$n(y) = n_o(y) - \frac{1}{4h^2} n_o''(y) + \frac{1}{32h^4} n_o^{IV}(y) + \ldots\ldots$$

It is merely necessary to know the measure of precision $h = \frac{0.707}{r}$, where $r$ is the mean error, and the second and fourth derivative, which may be found by numerical differentiation.

(b) The effect of using finite intervals in $y$ on the median value of $y$ can also be allowed for. Let $n_{\Delta y}(y)$ be the number of stars of a spectral subclass observed in the interval $(y+\Delta y)$, then

$$n_{\Delta y}(y) = \int_{y-\frac{1}{2}\Delta y}^{y+\frac{1}{2}\Delta y} n(y)\ dy. \quad \text{Let} \int n(y)\ dy = N(y),$$

$$\text{then } n_{\Delta y}(y) = N\left(y+\frac{\Delta y}{2}\right) - N\left(y-\frac{\Delta y}{2}\right)$$

$$\text{Since } N\left(y+\frac{\Delta y}{2}\right) = N(y) + \tfrac{1}{2}\Delta y \frac{dN(y)}{dy} + \tfrac{1}{2}\frac{\Delta y^2}{4}\frac{d^2N(y)}{dy^2} + \frac{1}{6}\frac{\Delta y^3}{8}\frac{d^3N(y)}{dy^3} + \ldots\ldots$$

$$\text{and } N\left(y-\frac{\Delta y}{2}\right) = N(y) - \tfrac{1}{2}\Delta y \frac{dN(y)}{dy} + \tfrac{1}{2}\frac{\Delta y^2}{4}\frac{d^2N(y)}{dy^2} - \frac{1}{6}\frac{\Delta y^3}{8}\frac{d^3N(y)}{dy^3} + \ldots\ldots$$

$$n_{\Delta y}(y) = \Delta y\, n(y) + \frac{1}{24}\Delta y^3\, n''(y)$$

$n(y)$ is the function wanted.

[24] Maxwell, *L. O. B.* **13**, 83.

[25] *Calc. of Obs.*, Whittaker and Robinson, p. 207, 1924.

$n(y)=\frac{n_{\Delta y}(y)}{\Delta y}-\frac{1}{24}\Delta y^2\frac{d^2n(y)}{dy^2}$, neglecting higher terms. If the interval $\Delta y$ equals 1, and if we designate the observed value by $n_o(y)$, the values corrected for finite intervals become:

$$n(y)=n_o(y)-0.04\,n''(y)$$

(c) Combining the two corrections under (a) and (b), and letting $n''(y)=n_o''(y)$, we find

$$n(y)=n_o(y)-\left[\frac{1}{4h^2}+0.04\right]n_o''(y)+\frac{1}{32h^4}n_o^{IV}(y)$$

$n_o(y)$ is taken from tables 21 and 22.

The dispersion in absolute magnitudes will be increased by using finite spectral type intervals. If the range in $M$ is $\pm\Delta M$, then the probable error is $\pm\frac{\Delta M}{2}$, and the mean error $\sigma_2=\pm1.48\,\frac{\Delta M}{2}$. Let the natural dispersion in absolute magnitude be $\sigma_1$, then the effective dispersion can be set equal to $\pm\sqrt{\sigma_1^2+\sigma_2^2}$. The measure of precision $h=\frac{0.707}{\sigma}$ is computed for each spectral class investigated. No negative values of $n(y)$ were admitted, and the resulting values of $n(y)$ were dispersed with $\sigma$ according to the normal error function to make sure of satisfactory agreement with the observed values $n_o(y)$.

With the values of $n(y)$ thus obtained, the density curves for the different spectral classes are computed from the formula:

$$\Delta(y)=n(y)10^{4.86-0.6y}$$

In order to obtain a linear scale for the abscissa, the distance $r$ in parsecs was used in place of the parameter $y=5\log r$. The density function $D(r)\ 10^6$ was plotted as ordinate, i.e. the number of stars of each spectral subdivision per $10^6$ cubic parsecs, at the distance $r$.

Since the material from the *Henry Draper Catalogue* was used to a limiting magnitude of $8^m.5$, and since the observed material consists practically of stars fainter than $10^m.0$, the gap in the number law $n_o(y)$ between $8^m.5$ and $10^m.0$ was bridged over by interpolation. In view of the small number of brighter stars available, the means of several magnitude intervals were taken in the neighborhood of the Sun. In the following tables are shown the magnitude intervals, $y$, $r$, log $D(r)$, $D(r)10^6$, and the measure of precision $h$.

STAR DENSITY CURVES

B5-B8 h=.882

| m | y | r | log D(r) | D(r)10⁶ |
|---|---|---|---|---|
| 7.0–10.0 | 13.7 | 550 | 4.83–10 | 6.8 |
| 10.0–11.0 | 15.7 | 1380 | 5.22 | 16.6 |
| 11.0–12.0 | 16.7 | 2185 | 3.53 | 0.3 |
| 12.0–13.0 | 17.7 | 3470 | 4.53 | 3.4 |
| 13.0–14.0 | 18.7 | 5500 | 3.53 | .3 |

B9-A1 h=1.00

| m | y | r | log D(r) | D(r)10⁶ |
|---|---|---|---|---|
| 5.0–10.0 | 11.3 | 182 | 6.03 | 107 |
| 10.0–11.0 | 14.3 | 725 | 6.25 | 178 |
| 11.0–12.0 | 15.3 | 1150 | 5.52 | 34 |
| 12.0–13.0 | 16.3 | 1820 | 5.08 | 12 |
| 13.0–14.0 | 17.3 | 2880 | 5.47 | 30 |
| 14.0–15.0 | 18.3 | 4570 | 4.38 | 2.4 |
| 15.0–16.0 | 19.3 | 7250 | 3.44 | 0.3 |

A2-A6 h=.94

| m | y | r | log D(r) | D(r)10⁶ |
|---|---|---|---|---|
| 6.0–11.0 | 11.31 | 182 | 6.66 | 457 |
| 11.0–12.0 | 14.31 | 724 | 6.33 | 214 |
| 12.0–13.0 | 15.31 | 1150 | 5.92 | 83 |
| 13.0–14.0 | 16.31 | 1820 | 5.36 | 23 |
| 14.0–15.0 | 17.31 | 2880 | 6.22 | 170 |
| 15.0–16.0 | 18.31 | 4570 | 4.46 | 3 |
| 16.0–17.0 | 19.31 | 7240 | 3.04 | 0.1 |

A6-F3 h=.88

| m | y | r | log D(r) | D(r)10⁶ |
|---|---|---|---|---|
| 6.0–10.0 | 9.78 | 91 | 7.12 | 1320 |
| 10.0–11.0 | 12.28 | 288 | 7.08 | 1200 |
| 11.0–12.0 | 13.28 | 457 | 6.78 | 600 |
| 12.0–13.0 | 14.28 | 725 | 6.56 | 364 |
| 13.0–14.0 | 15.28 | 1150 | 6.61 | 407 |
| 14.0–15.0 | 16.28 | 1820 | 6.70 | 500 |
| 15.0–16.0 | 17.28 | 2880 | 5.94 | 87 |
| 16.0–17.0 | 18.28 | 4570 | 5.28 | 19 |
| 17.0–18.0 | 19.28 | 7250 | 4.66 | 5 |

F3-dG3 h=.73

| m | y | r | log D(r) | D(r)10⁶ |
|---|---|---|---|---|
| 6.0–10.0 | 8.24 | 45 | 7.78 | 6020 |
| 10.0–11.0 | 10.74 | 141 | 7.85 | 7080 |
| 11.0–12.0 | 11.74 | 224 | 7.76 | 5750 |
| 12.0–13.0 | 12.74 | 355 | 7.20 | 1590 |
| 13.0–14.0 | 13.74 | 562 | 7.01 | 1020 |
| 14.0–15.0 | 14.74 | 890 | 6.49 | 309 |
| 15.0–16.0 | 15.74 | 1410 | 6.89 | 775 |
| 16.0–17.0 | 16.74 | 2240 | 7.41 | 2570 |
| 17.0–18.0 | 17.74 | 3550 | 6.69 | 490 |
| 18.0–19.0 | 18.74 | 5620 | 6.45 | 282 |

gG3-gG7 h=.75

| m | y | r | log D(r) | D(r)10⁶ |
|---|---|---|---|---|
| 7.0–10.0 | 11.68 | 219 | 6.94 | 870 |
| 10.0–11.0 | 13.68 | 550 | 6.60 | 400 |
| 11.0–12.0 | 14.68 | 872 | 6.11 | 129 |
| 12.0–13.0 | 15.68 | 1380 | 5.74 | 55 |
| 13.0–14.0 | 16.68 | 2190 | 4.84 | 6.9 |
| 14.0–15.0 | 17.68 | 3470 | 5.24 | 17.4 |
| 15.0–18.0 | 19.68 | 8720 | 4.47 | 3.0 |

gG8-gK4 h=.86

| m | y | r | log D(r) | D(r)10⁶ |
|---|---|---|---|---|
| 6.0– 9.0 | 9.62 | 83 | 6.84 | 690 |
| 9.0–10.0 | 11.62 | 209 | 6.50 | 316 |
| 10.0–11.0 | 12.62 | 330 | 6.38 | 240 |
| 11.0–12.0 | 13.62 | 525 | 6.52 | 331 |
| 12.0–13.0 | 14.62 | 832 | 5.38 | 24 |
| 13.0–14.0 | 15.62 | 1320 | 4.96 | 9 |
| 14.0–15.0 | 16.62 | 2090 | 5.36 | 23 |
| 15.0–16.0 | 17.62 | 3300 | 6.02 | 105 |
| 16.0–17.0 | 18.62 | 5250 | 5.68 | 48 |

The density curves are plotted in the following diagrams, figures 11–13, the numerical values of the foregoing table being supplemented by the value of the distance of the center of the cloud of 2800 parsecs, as determined in Vb.

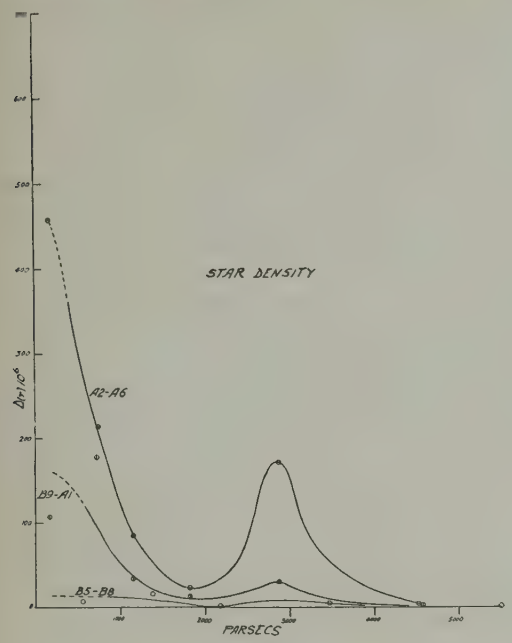



Fig. 12

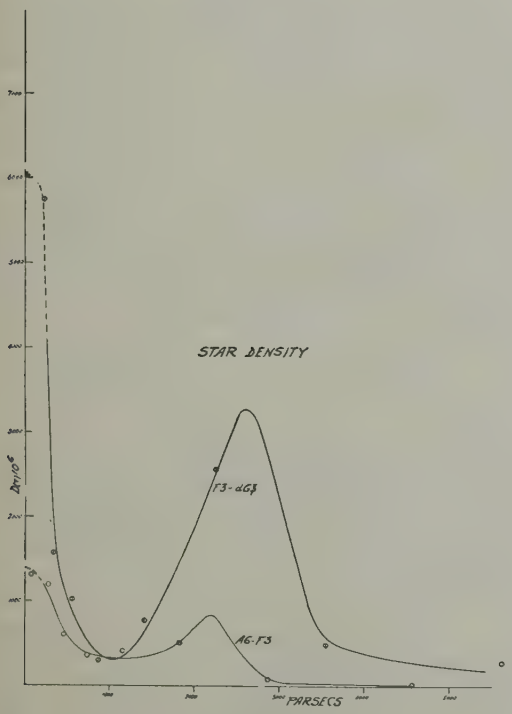



Fig. 13

Fig. 14

All spectral classes indicate a pronounced separation of the local system and the *Scutum* cloud near 1500 parsecs. The depth of the cloud appears to be from 500 to 1000 parsecs, and therefore greater than previously assumed in Vb. Accordingly it is to be expected that the star density would be less than that derived on the assumption of a depth of 350 parsecs. The following table gives the star density in the neighborhood of the Sun, and at the center of the *Scutum* cloud, both values being derived from the density law on diagram 12–14. For comparison purposes the star density derived from star counts (tables 21 and 22) is also given. The number of stars in the cloud, $N$, is obtained by multiplying the number of stars per 0.1 square degree (table 21), by 480, which gives the total number of stars per 48 square degrees, the area of the cloud. The number of stars in the interval $F3-dG3$ to magnitude $18^{m}.0$ was multiplied by a factor of 2, since only about one-half of these stars are included to a limiting magnitude of $18^{m}.0$. (See figure 11.) The total volume of the cloud is found on the basis of a distance of 2800 parsecs, and a depth of 1000 parsecs, to be $V = 114 \times 10^{6}$ cubic parsecs. The star density per $10^{6}$ cubic parsecs is accordingly $D = \frac{N}{V} \times 10^{6}$. The densities of red giant stars are somewhat less certain than those of the white giants and the dwarfs, because the former must be separated from the dwarf stars in the color-magnitude diagram by assuming a certain density for the dwarf stars in the local system.

From the density law it appears that the star density in the *Scutum* cloud is about one-half of that in the neighborhood of the Sun, while from the star counts, making the somewhat arbitrary assumption of a depth of 1000 parsecs, it is about four-fifths of that in the neighborhood of the Sun. The former ratio (one-half) deserves higher weight, since the same reduction method was employed for the determination of the density in the local system and in the *Scutum* cloud. The proportion of stars in the various spectral classes is on the whole not very different, but there is some evidence, both from the density law and from the star counts, to suggest a larger proportion of dwarfs, and therefore a smaller proportion of white and red giants in the *Scutum* cloud, as compared with the local system.

### Total Number of Stars in the Scutum Cloud

The total number of stars in each spectral class is obtained by integrating the density law between the distance limits of the cloud:

$$N = \int_{r_1}^{r_2} \frac{\omega}{(57.3)^2} r^2 D(r)\, dr$$

The distance limits $r_1$ and $r_2$ and the density law for each spectral class are available on figures 11–13, where the integration can be carried out graphically. The logarithms of $N$ are given in the following table. For comparison purposes the logarithms of the star numbers derived from the star counts per 0.1 square degree, and multiplied by 480, to include the whole cloud, are also given.

Total Number of Stars in Scutum Cloud

| Class | log $N$ From integration of density law | log $N$ From star counts |
|---|---|---|
| B5-B8 | 2.757 | 3.238 |
| B9-A1 | 3.456 | 3.919 |
| A2-A6 | 4.099 | 4.447 |
| A6-F3 | 4.646 | 5.004 |
| F3-dG3 | 5.660 | 5.811 |
| gG3-gG7 | (3.438) | (4.508) |
| gG8-gK4 | (4.136) | (4.865) |

## VI. Summary

(1) The object of the present investigation was to make a detailed study of the characteristics of the *Scutum* cloud, one of the most conspicuous formations of the Milky Way, in order to find the relation between color index and magnitude, the distance and dimensions, and the luminosity and density functions in the cloud. The study includes stars from the 5th to the 18th magnitude.

(2) Briefly, the method consists in the determination of the photographic magnitudes and spectral types of all stars in a given area. The photographic magnitudes are obtained by making successive exposures of the field and the standard stars of the North Polar Sequence on the same plate. While the spectral types of the brighter stars up to about 13th magnitude can be found directly from slitless spectrograms, those of the fainter stars are determined by means of color indices secured with a violet and yellow filter, either in quick succession with full filters, or simultaneously with a half-filter, consisting of one half of a yellow, and one half of a violet filter, side by side. The second method is superior to the first because the violet image on one half of the plate and the yellow image on the other half of the plate are obtained under exactly the same atmospheric, focussing and guiding conditions. The gradation curve determined for the violet images can be employed for the reduction of the yellow images, and the only adjustment required is the photographic Purkinje-Effect elimination. The method is differential and avoids or reduces systematic errors due to widely varying observing conditions during long exposures.

(3) The observations were made with the Crossley Reflector, equipped with a plate holder or a two-prism slitless quartz spectrograph. Cramer Isochromatic Instantaneous Plates and Wratten K2 and 35D Filters were used, giving color indices which are about 35% larger than those on the I. A. U. color system.

(4) Six well distributed circular sampling areas were selected for observation. Stars from $10^m.00$ to $13^m.50$ were measured completely in a total area of 0.51 square degree, from $13^m.50$ to $16^m.00$ within 0.28 square degree, and from $16^m.00$ to $18^m.00$ within 0.008 square degree.

(5) A color-magnitude diagram was established by plotting color indices versus photographic magnitudes after combining all material and reducing the star numbers to 0.1 square degree. It is found that the relative luminosities of the various spectral types are the same as in the neighborhood of the Sun, and that both giant and dwarf stars are present.

(6) The effect of the errors of observation on the color-magnitude diagram is analyzed and largely eliminated.

(7) From the corrected color-magnitude diagram that magnitude at which the maximum number of stars of a given spectral class is reached, is determined. From the relation $M = m + 5 + 5 \log p$, the solution for $p$ is made for five different spectral classes and a distance of 2800 parsecs is found. The dimensions are approximately 310x390 parsecs.

(8) By plotting the star numbers of various color types against the photographic magnitude, it is seen that the increase in the star numbers from white to red stars is enormous, and that the maximum shifts to fainter magnitudes as we proceed to the red stars.

The form of the curves is the normal frequency function, owing to the natural dispersion of the absolute magnitudes of about $\sigma = \pm 0.5$ to 1.0. The agreement with the generally adopted dispersion in the Hertzsprung-Russell diagram indicates that the depth of the cloud cannot be very great in comparison with its distance.

(9) A comparison of the star density in the cloud with the mean distribution of stars according to Seares shows that it is somewhat higher than the mean up to about 13th magnitude, but increases very rapidly for the fainter stars, reaching a value of two to three times the mean of 18th magnitude. The increase is caused by the large number of dwarf stars which make their appearance beyond 13th magnitude.

(10) Under the assumption of a depth of the cloud of 350 parsecs, it seems, that compared with the star density in the neighborhood of the Sun according to v. Rhijn and Maxwell, the *Scutum* cloud is on the average about equally rich in A, F, G-type and K-type giant stars, and decidedly richer in G-type dwarf stars, while there is an indication that B-type stars are less numerous. In certain parts of the cloud the star density is considerably higher than the average for the whole cloud.

(11) The density law is derived in a more rigorous manner. All spectral classes indicate a pronounced separation between the local system and the *Scutum* cloud near 1500 parsecs. The depth of the cloud appears to be from 500 to 1000 parsecs. In the combined spectral classes investigated, the average star density in the *Scutum* cloud is about one half of that in the neighborhood of the Sun. The distribution of the stars in the spectral classes is on the whole not very different, but there seems to be evidence to suggest a larger proportion of dwarfs, and therefore a smaller proportion of white and red giants in the *Scutum* cloud, as compared with the local system, in general agreement with the proportions derived by assuming a depth of 350 parsecs.

Lick Observatory,
June, 1929.
Issued October 8, 1929.

## Acknowledgments

I am under obligation to Director W. W. Campbell and Associate Director R. G. Aitken of the Lick Observatory for according me the use of the observatory instruments and material, thereby making possible the investigation, and to the staff, in particular to Dr. R. J. Trumpler who suggested the problem and has taken the greatest personal interest in it, offering numerous valuable suggestions and his friendly advice throughout the progress of the work. Further I am much indebted to my wife who assisted very ably in the laborious reductions and in the preparation of the manuscript for publication.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

# LICK OBSERVATORY BULLETIN

NUMBER 417

## ELEMENTS AND EPHEMERIS OF COMET *b* 1929 (NEUJMIN)

BY

ERNEST CLARE BOWER AND HOWARD C. WILLIS

This comet was discovered by G. Neujmin at Simeis, Russia on 1929, Aug. 2. The magnitude was given as 13.5. It was described on Aug. 15, by Krieger and Bobrovnikoff, Lick Observatory, as having a stellar nucleus surrounded by nebulosity, and by Van Biesbroeck, Yerkes Observatory, as having a nucleus of magnitude 15, surrounded by a coma of 40″.

The following observations have been received:

| | 1929 U. T. | α(1929.0) | δ(1929.0) | Observer, Place | Authority |
|---|---|---|---|---|---|
| (1) | Aug. 2.9547 | 21h 16m 04s.61 | −12° 45′ 41″ | Neujmin, Simeis | Telegram, H.C.O. Card No. 88 |
| (2) | Aug. 4.9693[2] | 21 15 19.4 | −12 55 09 | Wolf, Heidelberg | Telegram, H.C.O. Card No. 90 |
| (3) | Aug. 5.98922 | 21 14 55.91 | −12 59 58.9 | Wachmann, Bergedorf | Beob. Zirk. No. 30, 1929 |
| (4) | Aug. 6.89257 | 21 14 35.07[1] | −13 04 24 | Neujmin, Simeis | Beob. Zirk. No. 28, 1929 |
| (5) | Aug. 6.9318 | 21 14 34.4 | −13 04 32 | Wolf, Heidelberg | Telegram, H.C.O. Card No. 90 |
| (6) | Aug. 10.9287 | 21 13 02.5 | −13 24 09 | Wolf, Heidelberg | Telegram, H.C.O. Card No. 94 |
| (7) | Aug. 11.21866 | 21 12 55.71 | −13 25 35.2 | Van Biesbroeck, Williams Bay | Telegram, Letter |
| (8) | Aug. 12.35045 | 21 12 29.87 | −13 31 07.8 | Krieger and Bobrovnikoff, Mount Hamilton | Telegram, Letter |
| (9) | Aug. 13.9450 | 21 11 54.4 | −13 39 00 | Wolf, Heidelberg | Telegram, H.C.O. Card No. 94 |
| (10) | Aug. 14.23653 | 21 11 48.34 | −13 40 25.0 | Van Biesbroeck, Williams Bay | Letter |
| (11) | Aug. 15.27784 | 21 11 25.64 | −13 46 32.6 | Van Biesbroeck, Williams Bay | H.C.O. Card No. 94, Letter |
| (12) | Aug. 15.39694 | 21 11 22.68 | −13 46 05.6 | Krieger and Bobrovnikoff, Mount Hamilton | Telegram, Letter |
| (13) | Aug. 28.18044 | 21 07 50.14 | −14 44 26.1 | Van Biesbroeck, Williams Bay | Telegram, Letter |
| (14) | Aug. 31.86444 | 21 07 19.67 | −14 58 51 | Wolf, Heidelberg | Beob. Zirk. No. 32, 1929 |
| (15) | Sept. 3.19383 | 21 07 08.68 | −15 07 09.1 | Van Biesbroeck, Williams Bay | Telegram, Letter |
| (16) | Sept. 4.26387 | 21 07 06.09 | −15 10 46.2 | Krieger[3], Mount Hamilton | |
| (17) | Sept. 9.14609 | 21 07 15.55 | −15 25 16.6 | Van Biesbroeck, Williams Bay | Letter |

Determination of a satisfactory elliptic orbit for this comet has been delayed on account of erroneous designation of certain observations. Thus, the two Simeis observations (1) and (4) have been announced as apparent places but evidently correspond to astrographic places. The first two Heidelberg observations (2) and (5) originally were telegraphed as astrographic places and later appeared on *H.C.O. Card* No. 90 as apparent places.

No satisfactory solution could be obtained with 2-day intervals from (1), Aug. 2; (2), Aug. 4; and (5), Aug. 6, the first three available observations, on account

[1] These observations are announced as apparent places but comparison with the ephemeris indicates that they are probably astrographic, or mean, places.

[2] The position originally given for this date was later found to belong to Minor Planet 1929 PA (*Beob. Zirk.* 1929, Nos. 28, 29).

[3] Measures and reductions by F. L. Whipple at Students' Observatory, Berkeley.

of the erroneous designation of (1). When (7), Aug. 11, became available, an orbit was attempted on the basis of (1), Aug. 2; (5), Aug. 6; and (7), Aug. 11. This orbit was nearing completion when *H.C.O. Card* No. 90 was received with the information that (2) and (5) were apparent places. Accordingly, further work on this orbit was discontinued.

When (8), Aug. 12, became available, a new direct solution was based on (1), Aug. 2; (5), Aug. 6; and (8), Aug. 12. As the available five observations (1), (2), (5), (7), and (8) could not be satisfactorily represented either by a parabola or an ellipse, an approximate parabolic orbit and ephemeris was released, *H.C.O. Card* No. 93, with the statement that the orbit was uncertain on account of large inconsistencies in the five available observations. Inquiry addressed by airmail to Harvard College Observatory concerning the contradictory designation of observations (2) and (5), originally as astrographic by telegram and later as apparent on *H.C.O. Card* No. 90, reestablished these observations as astrographic. Thus the inconsistencies in the five available observations referred to above arose from the erroneous designation of (1), Aug. 2, as an apparent place and the use of (5), Aug. 6, as an apparent place.

Additional observations in the mean time having become available, a differential correction of the parabolic orbit was undertaken on the basis of (1), Aug. 2; (12), Aug. 15; and (13), Aug. 28 for a parabolic orbit. All but the first declination were readily represented by a single differential correction. An outstanding residual of $-214''$ in this first declination indicated ellipticity of the orbit and, in addition, possibly some remaining discrepancy in one of the basic observations.

The differential correction was then completed without hypothesis concerning the ellipticity, resulting in the following representation of all the available observations, including the new observation (15), Sept. 13:

| | $\Delta\alpha$ | $\Delta\delta$ | | $\Delta\alpha$ | $\Delta\delta$ |
|---|---|---|---|---|---|
| (1) | − 2″.6 | +1″.0 | (9) | + 4″.6 | + 0″.7 |
| (2) | +31.5 | +7.0 | (10) | + 9.5 | + 1.0 |
| (3) | +25.7 | +7.6 | (11) | + 5.4 | −61.1 |
| (6) | +18.6 | +1.9 | (12) | 0.0 | 0.0 |
| (7) | +15.8 | +0.8 | (13) | + 2.8 | + 0.2 |
| (8) | +11.5 | +1.8 | (15) | +44.5 | + 7.4 |

As the orbit was based on (1), (12), and (13), inspection of these residuals at once reveals the inconsistency of (1) which, according to published announcement, had been consistently used in all work to this point as an apparent place. The differential correction without hypothesis concerning the eccentricity was then revised after substitution of position (2), Aug. 4, for (1), Aug. 2. This differential correction left the following residuals in the subsequent observations (15), Sept. 3, and (17), Sept. 9 by Van Biesbroeck, which were kindly communicated by mail:

| | $\Delta\alpha$ | $\Delta\delta$ |
|---|---|---|
| (15) | +13″.6 | −1″.4 |
| (17) | +25.2 | +3.1 |

A final differential correction with the substitution of (17), Sept. 9 as third place gave the following orbit, *H.C.O. Card* No. 97:

ELEMENTS, 1929.0

Epoch = 1929 Aug. 15.39072 U. T.
$M$ = 4°.35780
$\omega$ = 140.51614
$\Omega$ = 158.23103
$i$ = 3.68489
$e$ = 0.584958
$a$ = 4.91640
$\mu$ = 0°.0904136
$P$ = 10.9011 years

$$x = +2.366912\ (\cos E - e) + 3.493693 \sin E$$
$$y = -4.028932\ (\cos E - e) + 1.832684 \sin E$$
$$z = -1.528526\ (\cos E - e) + 0.579319 \sin E$$

This orbit represents the observations to date as follows:

| | $\Delta\alpha$ | $\Delta\delta$ | | $\Delta\alpha$ | $\Delta\delta$ |
|---|---|---|---|---|---|
| (1) | −39″.3 | − 6″.2 | (10) | +6″.4 | − 0″.1 |
| (2) | − 0.2 | − 0.3 | (11) | +4.6 | −61.5 |
| (3) | − 1.5 | + 2.1 | (12) | −0.4 | 0.0 |
| (4) | −42.0 | −10.1 | (13) | −5.0 | + 0.3 |
| (5) | − 0.6 | + 1.2 | (14) | +0.8 | − 2.6 |
| (6) | + 6.2 | − 1.7 | (15) | −0.9 | − 2.5 |
| (7) | + 4.2 | − 2.6 | (16) | −3.0 | − 3.9 |
| (8) | + 2.8 | − 0.7 | (17) | −0.4 | − 0.2 |
| (9) | + 0.8 | − 0.6 | | | |

On the assumption that (1) and (4) are astrographic places, the residuals would become:

| | $\Delta\alpha$ | $\Delta\delta$ |
|---|---|---|
| (1) | 0″ | +1″ |
| (4) | −3 | −3 |

Position (11) was kindly checked by Van Biesbroeck but no error could be found, either in the measures or reductions. The discrepancy of approximately one minute of arc in declination remains unexplained but may be due to an erroneous star place.

EPHEMERIS

| 1929 U. T. | α(1929.0) | δ(1929.0) | ρ | Mag. | Annual aberration α | Annual aberration δ |
|---|---|---|---|---|---|---|
| June 2 | $21^h\ 00^m\ 03^s.68$ | $-12°\ 38'\ 24''.8$ | 1.3837 | 14.6 | +0$^s$.64 | +1″.4 |
| June 10 | 21 07 34.70 | −12 05 39.8 | 1.3109 | 14.5 | +0.75 | +2.0 |
| June 18 | 21 13 29.97 | −11 41 14.9 | 1.2452 | 14.4 | +0.86 | +2.7 |
| June 26 | 21 17 43.99 | −11 26 41.0 | 1.1874 | 14.3 | +0.97 | +3.5 |
| July 4 | 21 20 12.76 | −11 23 10.9 | 1.1387 | 14.2 | +1.07 | +4.2 |
| July 12 | 21 20 57.73 | −11 31 22.1 | 1.1003 | 14.1 | +1.16 | +4.8 |
| July 20 | 21 20 10.27 | −11 50 40.7 | 1.0734 | 14.1 | +1.24 | +5.4 |
| July 28 | 21 18 09.10 | −12 19 26.8 | 1.0591 | 14.1 | +1.30 | +5.8 |
| Aug. 5 | 21 15 19.85 | −12 55 05.9 | 1.0583 | 14.1 | +1.33 | +6.0 |
| Aug. 13 | 21 12 16.38 | −13 34 08.7 | 1.0718 | 14.1 | +1.34 | +6.1 |
| Aug. 21 | 21 09 34.32 | −14 12 44.4 | 1.0995 | 14.2 | +1.31 | +6.0 |
| Aug. 29 | 21 07 43.36 | −14 47 34.6 | 1.1412 | 14.3 | +1.26 | +5.8 |
| Sept. 6 | 21 07 06.84 | −15 16 04.7 | 1.1964 | 14.4 | +1.19 | +5.4 |
| Sept. 14 | 21 08 00.73 | −15 36 21.1 | 1.2640 | 14.6 | +1.09 | +5.0 |
| Sept. 22 | 21 10 30.08 | −15 47 27.6 | 1.3429 | 14.7 | +0.99 | +4.5 |
| Sept. 30 | 21 14 32.33 | −15 49 10.7 | 1.4320 | 14.9 | +0.87 | +4.0 |
| Oct. 8 | 21 20 02.06 | −15 41 34.3 | 1.5302 | 15.1 | +0.74 | +3.4 |
| Oct. 16 | 21 26 50.37 | −15 24 57.8 | 1.6364 | 15.2 | +0.61 | +2.8 |
| Oct. 24 | 21 34 45.88 | −14 59 58.2 | 1.7495 | 15.4 | +0.48 | +2.2 |
| Nov. 1 | 21 43 38.12 | −14 27 14.4 | 1.8685 | 15.6 | +0.36 | +1.5 |
| Nov. 9 | 21 53 18.01 | −13 47 21.3 | 1.9926 | 15.8 | +0.23 | +0.9 |
| Nov. 17 | 22 03 36.20 | −13 01 01.3 | 2.1206 | 15.9 | +0.10 | +0.2 |
| Nov. 25 | 22 14 24.32 | −12 08 59.3 | 2.2518 | 16.1 | −0.02 | −0.5 |
| Dec. 3 | 22 25 36.28 | −11 11 52.9 | 2.3853 | 16.3 | −0.13 | −1.2 |
| Dec. 11 | 22 37 06.63 | −10 10 19.8 | 2.5202 | 16.4 | −0.24 | −1.9 |
| Dec. 19 | 22 48 50.10 | − 9 05 02.7 | 2.6556 | 16.5 | −0.35 | −2.6 |

The magnitude was computed from $m = g + 5 \log \rho r$, where $g$ was found to be 12.4.

The ephemeris was extended back for two months before discovery in order to facilitate the location of the comet on earlier plates. The annual aberration given in the last two columns should be added to astrographic places, corrected for parallax, for comparison with positions interpolated from the ephemeris with the observed time diminished by the light time.

BERKELEY ASTRONOMICAL DEPARTMENT,
September 26, 1929.
Issued October 8, 1929.

UNIVERSITY OF CALIFORNIA PUBLICATIONS

ASTRONOMY

STEWARD OBSERVATORY TUCSON, ARIZONA

# LICK OBSERVATORY BULLETIN

NUMBER 418

## A SPECTOGRAPHIC STUDY OF β CEPHEI

BY

H. S. MENDENHALL

### INTRODUCTION

*β Cephei* ($\alpha = 21^h\ 27^m4$; $\delta = +70°\ 07'$ : 1900.0) is described in the *Henry Draper Catalogue*[1] as of visual magnitude 3.32 and spectral class $B_1$. It was found to have a variable radial velocity by Frost and Adams,[2] in 1902, from observations secured with the Bruce spectrograph of the Yerkes observatory. From eight plates obtained in May, 1906, Frost[3] found the period to be about four hours and thirty minutes. A velocity curve[3] was obtained by him from plates taken on July 6 and August 27 of 1906 which led him to conclude that the eccentricity of the orbit must be small and that the two bodies must be revolving nearly in contact with each other.

A study of *β Cephei* was made by C. C. Crump[4] at the University of Michigan in 1912, using a single prism spectrograph that gave a linear dispersion of 44 A/mm. He determined the period, by means of Frost's observations, together with his own, to be accurately $4^h\ 34^m\ 17^s4$, on the average, for the six year interval preceding 1912. The other orbital elements were determined by Crump from his own velocity curve by a careful least squares solution. He also determined definitive elements for an anomalous velocity curve of Oct. 1, 1912, and concluded that the change in amplitude of the curve at that epoch must be real. The velocity of the center of mass of the system was shown to be changing by a comparison of his value with the two previously obtained by Frost. In addition to the elements for an elliptical orbit, Crump found it necessary to postulate a secondary oscillation in the velocity in order to account satisfactorily for the observed velocity curve.

But few years have passed since 1912 during which *β Cephei* has not been made the subject of a special investigation at some one of the observatories equipped for radial velocity work. Velocity curves and, in most cases, orbital elements were determined at Poulkovo in 1914, 1916, and 1917 by Gerasimovic[5] and Belopolsky[6], at Ottawa in 1919, 1920, 1922, and 1923 by Boothroyd and Van Arnam,[7] Henderson, and Henroteau,[8, 9] and at the Lick Observatory in 1918, 1921, 1926, 1927, and 1928 by Henroteau, Miss Hobe, and the writer.

The discovery of the variation in light of *β Cephei* was made by Guthnick[10] at Berlin-Babelsberg from photo-electric measures made in August and September, 1913, by Guthnick and Prager. The observations were continued in January and February, 1914,[11] and in January and February, 1917.[12] The range of variation in light was found to be about 0.05 magnitude in a period equal to that for the velocity variation. The light curves obtained in February, 1917, indicated that the variation might not be identical for different epochs.

[1] *H. A.* Vol. **99**.
[2] *Ap. J.* **15**, 340, 1902.
[3] *Ap. J.* **24**, 260, 1906.
[4] *Pub. Detroit Obs.* **2**, 144, 1916.
[5] *Bull. de l'Obs. Central de Russie*, Poulkovo, **7** No. 80.
[6] *Bull. de l'Academie des Sciences de Russie*, 1918.
[7] *Pop. Ast.* **36**, 347, 1928.
[8] *Pub. Dom. Obs.* **5**, 77, 1922.
[9] *Pub. Dom. Obs.* **9**, 36, 1925.
[10] *A. N.* 4701, 1913–14.
[11] *Berlin-Babelsberg*, **1**, 23, 1914.
[12] *Berlin-Babelsberg*, **2**, 78, 1918.

A photometric study of $\beta$ *Cephei* was made by the photo-electric method by Miss E. E. Cummings[13] in 1921 at the Lick Observatory. She determined the mean light curve for the summer of 1921 and compared it with those previously obtained by Guthnick and Prager. She also made four comparisons of photometric with spectrographic data, which showed that the light maximum occurs (in the mean) about 0.033 day before the maximum velocity of approach, or about 0.015 day after the time of maximum contraction on the pulsation hypothesis. The mean curve, with a secondary maximum on the ascending branch, is found to be very similar in shape even when derived at intervals of several years. The disparity between individual curves observed at separate epochs, noted by Guthnick and Prager, was again pointed out by Miss Cummings.

The variation in light of $\beta$ *Cephei* was studied by Miss M. Gussow[14] in 1925 at Berlin-Babelsberg. The mean curve found by her shows a secondary maximum on the descending branch instead of the ascending branch as found by previous observers, proving that the mean light curve is not constant in form.

$\beta$ *Cephei* has been taken by Ludendorff[15] as the typical star of a small group of variables of early spectral type with small light changes, to form a subdivision of the $\delta$ *Cephei* type variables because of the similar relation of the phase of the light curve to that of the velocity curve. The underlying cause of this variability and phase relation is probably very similar for these stars.

Approximately a thousand spectrograms of $\beta$ *Cephei* have been obtained since 1900 but as yet its behavior has not been completely determined nor satisfactorily interpreted and explained. Previous investigators have drawn the following conclusions: (1) The radial velocity varies with a constant period of 0.1904795 day. (2) The amplitude of the velocity variation ranges from 20 to 40 km./sec. with (3) a period of the order of a month. (4) A secondary oscillation exists in the radial velocity curve relative to simple elliptical motion having one-third of the period and one-tenth of the amplitude of the principal variation. (5) The velocity of the system, or of the center of mass, varies from one revolution to the next and from year to year. (6) A periodic variation in luminosity exists, ranging from 0.04 to 0.07 magnitude, with a phase lag for luminosity relative to positive radial velocity ranging from approximately one-fourth to one-half of the entire period.

The present investigation comprises an examination of existing data and conclusions concerning $\beta$ *Cephei* and a collection and discussion of new material. Three general topics will be considered, namely, the variation of the orbital elements as derived on the binary hypothesis, the effect of stellar atmospheric level to be expected on the pulsation hypothesis, and a test of a proposed spectrophotometric method for determining the light variation at any point in the spectrum, including the ultraviolet region.

## Radial Velocity Observations

*Lick Observatory observations.*—The majority of the observations of $\beta$ *Cephei* obtained by the writer, and all earlier observations of this star secured at the Lick Observatory, were made with the new Mills spectrograph attached to the 36-inch refractor. This spectrograph contains three dense flint prisms set for minimum deviation at $\lambda$4500 A. With the camera of 21 inches focal length that was used, the linear dispersion at $\lambda$4500 A is 10.9 A/mm. The instrument is furnished with a curved slit and a set of small lenses and reflecting prisms for introducing the comparison spectra. A full description of the new Mills spectrograph is given in *Publ. Lick Obs.*, Vol. 16, page ix. A titanium spark was used for the comparison spectra, a part of the spectrum being introduced at the beginning of the exposure, a part at the middle, and a part at the end, in order to eliminate the effects of any possible instrumental changes occurring during the exposure. The slit was set at a width of 0.0010 inch, which necessitated an exposure of from 25 to 40 minutes on Eastman 33 plates for this star.

The data for the 119 Lick observations obtained with the new Mills spectrograph in the years 1903 to 1928 are collected in Table 1. Columns one and two contain respectively the Greenwich Civil Time, and the corresponding Julian date of the observation. The fourth column gives the radial velocity of the star relative to the Sun. The number of lines upon which the determination of the velocity depends is quoted in column five, and the initials given in column six refer to the one who measured the spectrogram. They have the following significance: Cs, H. D. Curtis; Ht, F. Henroteau; H, Miss A. M. Hobe; and Md, H. S. Mendenhall.

[13] *L. O. B.* **11**, 115, 1923–24.

[14] *A. N.* 5484, 201, 1925.

[15] *Handbuch der Astrophysik*, **7**, 222.

## TABLE I

LICK OBSERVATIONS OF $\beta$ *Cephei* WITH THE NEW MILLS III-PRISM SPECTROGRAPH

| Date G. C. T. 1903 | Jul. Day 241+ | Phase | V | No. of lines | Meas. by | Date G. C. T. 1926 | Jul. Day 242+ | Phase | V | No. of lines | Meas. by |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Sept. 16 | 6374.875 | | − 6.1 | 7 | Cs | July 1 | .949 | .155 | + 6.3 | 14 | Md† |
| | | | − 5.3 | 13 | H | | .989 | .005 | − 4.6 | 18 | Md† |
| | | | − 8.7 | 15 | Md* | | 4699.011 | .027 | − 7.8 | 14 | Md† |
| 20 | 6378.962 | | − 4.4 | 6 | Cs | 6 | 4704.006 | .070 | −19.0 | 14 | Md† |
| | | | − 3.2 | 13 | H | 12 | 4709.785 | .134 | + 6.0 | 14 | Md‡ |
| | | | − 3.7 | 14 | Md* | | .808 | .157 | + 1.8 | 12 | Md‡ |
| 1918 | 242+ | | | | | | .833 | .182 | + 0.1 | 17 | Md‡ |
| May 15 | 1729.851 | | +11.7 | ? | Ht | | .856 | .015 | −13.8 | 16 | Md‡ |
| | | | +12.2 | 11 | H | | .880 | .039 | −17.1 | 16 | Md‡ |
| | | | + 8.6 | 14 | Md | | .903 | .062 | −23.7 | 14 | Md‡ |
| | .884 | | + 6.2 | ? | Ht | | .926 | .085 | −14.4 | 16 | Md‡ |
| | | | + 8.0 | 12 | H | | .956 | .115 | − 2.9 | 16 | Md‡ |
| | | | + 3.3 | 17 | Md | | .982 | .141 | +10.5 | 11 | Md‡ |
| | .908 | | − 3.5 | ? | Ht | | 4710.008 | .167 | + 9.9 | 14 | Md‡ |
| | | | − 2.7 | 13 | H | Aug. 8 | 4736.728 | .029 | −11.5 | 20 | Md |
| | | | − 5.8 | 13 | Md | | .756 | .057 | −19.0 | 17 | Md |
| | .931 | | −14.5 | ? | Ht | | .790 | .091 | −14.6 | 11 | Md |
| | | | −13.5 | 11 | H | | .859 | .160 | + 7.3 | 20 | Md |
| | | | −17.5 | 13 | Md | | .895 | .006 | + 1.7 | 20 | Md |
| | .954 | | −21.4 | ? | Ht | | .923 | .034 | − 9.9 | 19 | Md |
| | | | −22.4 | 12 | H | | .949 | .060 | −16.9 | 19 | Md |
| | | | −26.6 | 14 | Md | Aug. 11 | 4739.744 | .187 | − 1.8 | 14 | Md |
| | .985 | | −19.1 | ? | Ht | | .775 | .028 | − 8.2 | 16 | Md |
| | | | −21.5 | 13 | H | | .804 | .057 | −20.1 | 18 | Md |
| | | | −24.7 | 18 | Md | | .834 | .087 | −17.5 | 13 | Md |
| 1921 | | | | | | | .874 | .127 | + 4.7 | 16 | Md |
| Nov. 6 | 3000.573 | | + 9.2 | 11 | H | | .901 | .154 | + 9.2 | 16 | Md |
| | .586 | | +11.6 | 11 | H* | | .927 | .180 | + 7.1 | 15 | Md |
| | .596 | | +10.2 | 11 | H* | 12 | 4740.826 | .127 | + 4.7 | 15 | Md |
| | .610 | | + 2.9 | 12 | H* | 21 | 4750.023 | .181 | + 5.3 | 13 | Md |
| | .631 | | − 6.9 | 14 | H | 24 | 4753.018 | .128 | + 4.9 | 15 | Md |
| | .648 | | −14.5 | 12 | H | Sept. 12 | 4771.890 | .142 | + 7.5 | 15 | Md |
| | .666 | | −17.9 | 12 | H* | | .922 | .174 | + 6.8 | 16 | Md |
| | .684 | | −14.3 | 12 | H* | | .955 | .017 | − 3.5 | 14 | Md |
| | .702 | | −13.1 | 13 | H* | | .989 | .051 | −16.8 | 14 | Md |
| | .720 | | − 5.6 | 12 | H | | 4772.024 | .086 | −13.8 | 16 | Md |
| | .738 | | + 2.5 | 13 | H | 14 | 4773.838 | .186 | + 5.2 | 19 | Md |
| | .760 | | + 9.9 | 13 | H* | | .858 | .016 | − 5.1 | 14 | Md |
| | .778 | | +12.3 | 13 | H | | .961 | .119 | − 0.4 | 10 | Md |
| | .794 | | + 9.1 | 11 | H | | .982 | .140 | +12.0 | 11 | Md |
| | .811 | | + 1.1 | 14 | H | Oct. 7 | 4796.723 | .023 | − 7.7 | 12 | Md |
| | .827 | | − 1.8 | 13 | H | | .743 | .043 | −17.6 | 14 | Md |
| 1926 | | | | | | | .765 | .065 | −22.6 | 15 | Md |
| June 15 | 4682.912 | .119 | + 3.2 | 17 | Md | | .791 | .091 | −18.0 | 18 | Md |
| | .932 | .139 | + 5.2 | 11 | Md | | .813 | .113 | − 6.9 | 18 | Md |
| 19 | 4686.916 | .123 | + 1.4 | 16 | Md | | .835 | .135 | + 2.7 | 15 | Md* |
| 23 | 4690.799 | .006 | − 0.4 | 11 | Md | | .855 | .155 | +12.8 | 16 | Md |
| | .817 | .024 | −10.0 | 14 | Md | | .876 | .176 | +10.9 | 14 | Md |
| | .844 | .051 | −19.4 | 15 | Md | | .898 | .008 | + 3.3 | 11 | Md |
| | .861 | .068 | −21.6 | 15 | Md | | .919 | .029 | − 7.9 | 10 | Md |
| | .878 | .085 | −17.0 | 17 | Md | Oct. 24 | 4813.602 | .139 | +11.1 | 12 | Md |
| | .894 | .101 | −12.3 | 15 | Md | | .621 | .158 | +12.7 | 11 | Md |
| | .910 | .117 | − 7.0 | 14 | Md | | .640 | .177 | + 4.4 | 10 | Md |
| | .927 | .134 | + 4.0 | 14 | Md | | .665 | .012 | − 2.5 | 17 | Md |
| | .944 | .151 | + 7.4 | 14 | Md* | | .685 | .032 | −13.7 | 17 | Md |
| | .963 | .170 | + 7.1 | 15 | Md | | .704 | .051 | −21.5 | 15 | Md |
| | .983 | .000 | − 2.0 | 15 | Md | | .723 | .070 | −20.0 | 15 | Md |
| | 4691.011 | .028 | − 9.6 | 19 | Md | | .743 | .090 | − 9.8 | 13 | Md |
| July 1 | 4698.880 | .086 | −14.0 | 11 | Md† | | .762 | .109 | − 4.0 | 19 | Md |
| | .904 | .110 | −10.4 | 18 | Md† | | .782 | .129 | + 5.1 | 16 | Md |
| | .927 | .133 | + 0.9 | 15 | Md† | | .801 | .148 | +11.0 | 11 | Md |

TABLE I—(*Concluded*)

| Date G. C. T. 1927 | Jul. Day 242+ | Phase | V | No. of lines | Meas. by |
|---|---|---|---|---|---|
| Jan. 6 | 4887.756 | .007 | + 1.3 | 14 | Md |
| | .849 | .100 | −13.5 | 18 | Md |
| 7 | 4888.589 | .077 | −14.7 | 18 | Md |
| | .614 | .102 | − 4.2 | 16 | Md |
| | .642 | .130 | + 5.8 | 19 | Md |
| | .675 | .163 | + 3.9 | 20 | Md |
| | .712 | .010 | − 4.5 | 15 | Md |
| | .751 | .049 | −17.8 | 21 | Md |
| 1928 | | | | | |
| Sept. 10 | 5500.800 | .088 | −16.8 | 14 | Md |
| | .824 | .112 | − 4.5 | 16 | Md |
| | .851 | .139 | + 9.8 | 18 | Md |
| | .878 | .166 | +11.3 | 13 | Md |
| | .904 | .001 | + 0.8 | 16 | Md |
| | .938 | .035 | −16.0 | 17 | Md |
| | .965 | .062 | −21.5 | 16 | Md |
| | .992 | .089 | −13.1 | 15 | Md |

* Measured twice. † Eastman 40 plate. ‡ Poor; ½ weight.

A three-prism spectrograph was designed in 1925 by T. S. Jacobsen to give spectra extending farther into the violet than those obtained with the Mills spectrograph. The prisms are mounted in a wooden box without temperature control, but spectra of excellent definition, with a linear dispersion of 21 A/mm. at λ4200A, have been obtained. A more complete description of this instrument has been previously given by the designer.[16]

Seventeen violet spectrograms of *β Cephei* were obtained in 1926 on Eastman 33 plates with an exposure of 20 minutes each. An iron spark spectrum was used for comparison. The data for these observations are given in Table II.

TABLE II

LICK OBSERVATIONS OF *β Cephei* WITH THE III-PRISM VIOLET SPECTROGRAPH

| Date G. C. T. 1926 | Jul. Day 242+ | Phase | V | No. of lines | Meas. by |
|---|---|---|---|---|---|
| July 19 | 4717.011 | .121 | − 0.9 | 45 | Md |
| 28 | 4725.682 | .031 | −18.5 | 48 | Md |
| | .698 | .047 | −21.3 | 49 | Md |
| | .713 | .062 | −20.6 | 37 | Md* |
| | .812 | .161 | + 7.0 | 51 | Md |
| | .829 | .178 | + 1.7 | 47 | Md |
| | .846 | .005 | − 7.8 | 47 | Md |
| | .870 | .029 | −17.1 | 49 | Md |
| Aug. 5 | 4734.010 | .168 | + 3.7 | 48 | Md |
| 14 | 4742.832 | .037 | −15.7 | 51 | Md |
| | .849 | .054 | −20.7 | 51 | Md |
| | .866 | .071 | −15.5 | 50 | Md |
| | .887 | .092 | − 9.9 | 50 | Md |
| | .952 | .157 | + 4.5 | 50 | Md |
| | .970 | .175 | + 6.2 | 48 | Md |
| | .990 | .005 | − 4.5 | 49 | Md |
| | 4743.009 | .024 | − 8.4 | 50 | Md |

* Underexposed.

[16] *L. O. B.* **12**, 138, 1926.

*Method of measurement and reduction.*—The Mills plates have all been measured on a standard measuring engine having a screw-pitch of ½ mm. One or two settings were made on each comparison line and two or three on each star line. Each plate was measured in both the direct and reverse position in order to eliminate as far as possible the effect of personal equation. From 10 to 20 star lines and from 10 to 20 comparison lines were measured on each plate.

The wave-lengths adopted for the stellar and comparison lines are given in the third column of Table III, and are based upon recent laboratory determinations by the investigators noted in the final column. Identification of the absorbing atom for each of the stellar lines is quoted in column one, the absence of such notation indicating that the wave-length refers to a titanium line of the comparison spectrum. The measures of the spectrograms were reduced in the usual manner by means of a standard dispersion table computed from the Hartmann-Cornu dispersion formula. To the value of the radial velocity thus derived a correction was applied for the observer's velocity due to the orbital and rotational motions of the Earth. A further correction has been applied to each observed velocity for the effect of flexure in the tube of the 36-inch telescope. Data for its derivation were taken from the investigation by Jacobsen.[17] This correction for *β Cephei* although small, ranging from +0.20 to −0.20 km./sec. for different positions of the instrument, has been used in obtaining the values of the radial velocities for the years 1926 to 1928, given in Table I. Measures of the individual lines on each spectrogram, after correction for the radial velocity obtained from the mean of all lines on the plate, were utilized for a determination of the wave-lengths of the lines in the spectrum of *β Cephei*. The wave-lengths thus derived are given in the second column of Table III.

TABLE III

| Absorbing atoms | λ, I. A., in *β Cephei* | Assumed λ, I. A. | Authority |
|---|---|---|---|
| | | *4386.858* | H. Crew[18] |
| He I | 4387.922 | 87.928 | P. W. Merrill[19] |
| | | *95.036* | F. L. Brown[20] |
| O II | 4395.950 | 95.950 | A. Fowler[21] |
| | | *4411.080* | H. Crew |
| O II | 4414.899 | 14.888 | J. S. Clark[22] |
| O II | 4416.968 | 16.974 | J. S. Clark |
| | | *17.718* | H. Crew |
| | | *27.098* | F. L. Brown |
| He I | 4437.543 | 37.548 | P. W. Merrill |
| | | *43.799* | F. L. Brown |

[17] *L. O. B.* **12**, 138, 1926.
[18] *Ap. J.* **56**, 53, 1922.
[19] *Bull. Bureau Standards* **14**, 162, 1917.
[20] *Ap. J.* **60**, 108, 1924.
[21] *Proc. Roy. Soc.* **110**, 476, 1926.
[22] *Ap. J.* **40**, 332, 1914.

TABLE III—(*Concluded*)

| Absorbing atoms | λ, I. A., in *β Cephei* | Assumed λ, I. A. | Authority |
|---|---|---|---|
| N II, O II | 4446.984 | 47.040 | J. S. Clark |
| O II | 4452.402 | 52.380 | A. Fowler |
| | | *57.427* | F. L. Brown |
| | | *68.493* | F. L. Brown |
| *O II(?) | 4469.642 | 69.320 | A. Fowler |
| He I | 4471.527 | 71.477 | P. W. Merrill |
| Mg II, Mg II | 4481.221 | 81.230 | A. Fowler |
| | | *81.259* | F. L. Brown |
| | | *4501.269* | F. L. Brown |
| | | *12.734* | F. L. Brown |
| | | *22.797* | F. L. Brown |
| | | *27.305* | F. L. Brown |
| N II | 4530.493 | 30.421 | J. S. Clark |
| | | *49.624* | H. Crew |
| Si III, N II | 4552.602 | 52.630 | A. Fowler |
| | | *63.757* | F. L. Brown |
| Si III | 4567.844 | 67.872 | A. Fowler |
| | | *71.970* | F. L. Brown |
| Si III | 4574.753 | 74.777 | A. Fowler |
| | | *89.961* | H. Crew |
| O II | 4590.983 | 90.983 | J. S. Clark |
| O II | 4596.148 | 96.189 | J. S. Clark |
| N II | 4601.637 | 4601.490 | J. S. Clark |
| N II | 4607.174 | 07.167 | J. S. Clark |
| O II, O II | 4609.542 | 09.610 | A. Fowler |
| N II | 4613.690 | 13.884 | J. S. Clark |
| | | *17.270* | F. L. Brown |
| | | *23.098* | F. L. Brown |

*Probably line for which Struve gives λ=4470.046 (Rowland's Scale). *Ap. J.* **62**, 198, 1925.

Two Mills plates of *β Cephei* taken in 1903 and measured by both H. D. Curtis and Miss A. M. Hobe, and six obtained in 1918 that both Miss Hobe and F. Henroteau had measured, were remeasured by the writer with the following results:

TABLE IV

COMPARISON OF MEASURES OF MILLS PLATES OF *β Cephei*

| Plate taken | Curtis and Henroteau | Miss Hobe | H. S. Mendenhall | Mean of Cs, Ht, and Hobe | Mendenhall minus Mean | Residuals, Δ+3.2 |
|---|---|---|---|---|---|---|
| 1903 | − 6.1 | − 5.3 | − 8.7 | − 5.7 | −3.0 | +0.2 |
| 1903 | − 4.4 | − 3.2 | − 3.7 | − 3.8 | +0.1 | +3.3 |
| 1918 | +11.7 | +12.2 | + 8.6 | +12.0 | −3.4 | −0.2 |
| 1918 | + 6.2 | + 8.0 | + 3.3 | + 7.1 | −3.8 | −0.6 |
| 1918 | − 3.5 | − 2.7 | − 5.8 | − 3.1 | −2.7 | +0.5 |
| 1918 | −14.5 | −13.5 | −17.5 | −14.0 | −3.5 | −0.3 |
| 1918 | −21.4 | −22.4 | −26.6 | −21.9 | −4.7 | −1.5 |
| 1918 | −19.1 | −21.5 | −24.7 | −20.3 | −4.4 | −1.2 |

The next to last column gives the difference between the velocity derived by the writer and the mean of previous results. The average value of this difference is −3.2 km./sec. Adding 3.2 to each value in the difference column furnishes the residuals quoted in the final column. This limited material indicates that the mean square error for a single measurement of a plate is about ±1.5 km./sec. The "probable error" would be about 2/3 of this or ±1 km./sec. A comparison of the wave-lengths used by Miss Hobe (Albrecht's provisional table) with those in Table III gives a mean difference corresponding to 3.6 km./sec., weights having been assigned to the lines according to the frequency of their appearance in the reductions. This difference is of the right sign and order to account for the observed systematic difference in the measures. It is of interest to note that this result is in harmony with Albrecht's prediction[23] that the large $K$ term in the $B$ type stars could be accounted for largely by the wave-lengths used in the reductions.

TABLE V

| Assumed λ I. A. | Absorbing atoms | Est. int. | Assumed λ I. A. | Absorbing atoms | Est. int. | Assumed λ I. A. | Absorbing atoms | Est. int. |
|---|---|---|---|---|---|---|---|---|
| *3927.921* | | | *4227.437* | | | *18.549* | | |
| *30.298* | | | 41.787 | N | 2 | 19.222 | O II | 3 |
| | | 3 | 53.79 | O II, S | 3 | 20.92 | He I, O II | 4 |
| 45.050 | O II | 2 | 67.16 | C II | 3 | 32.82 | O II | 2 |
| 54.368 | O II | 3 | 75.80 | O II | 1 | 43.759 | He I | 5 |
| *56.59* | | | *94.128* | | | *4325.765* | | |
| 64.727 | He I | 3 | 4303.82 | O II | 2 | 36.86 | O II | |
| | | 2 | 17.160 | O II | 2 | 40.466 | H I (γ) | 15 |
| *69.260* | | | 19.647 | O II | 2 | 45.570 | O II | 1 |
| 70.075 | H I (ε) | 20 | 25.77 | O II | 1 | 47.429 | O II | 2 |
| 73.266 | O II | 3 | 4035.05 | N | 2 | 49.434 | O II | 3 |
| 82.725 | O II | 1 | 41.34 | N | 1 | 51.275 | O II | 3 |
| 94.995 | N II | 3 | 69.79 | O II | 4 | 66.906 | O II | 3 |
| *4005.244* | | | *71.741* | | | 87.931 | He I | 8 |
| 09.33 | He I | 4 | 72.156 | O II | 3 | 4414.888 | O II | 3 |
| 26.18 | He I, N | 12 | 75.869 | O II | 3 | *15.125* | | |
| 4143.759 | He I | 5 | 85.01 | O II | 2 | 16.974 | O II | 2 |
| *43.70* | | | 89.07 | Si IV, O II | 4 | 37.548 | He I | 2 |
| 53.310 | O II | 2 | 97.31 | O II, N | 1 | 47.035 | O II, N | 1 |
| 69.23 | O II | 2 | 4101.736 | H I (δ) | 16 | 71.477 | He I | 7 |
| 85.453 | O II | 2 | 04.90 | O II | | 81.229 | Mg II | 4 |
| 89.793 | O II | 2 | 16.13 | Si IV | 2 | *4528.618* | | |

[23] *Ap. J.* **55**, 361, 1922 and 57, 57, 1923.

The seventeen violet spectrograms were measured and reduced in the manner previously described for the treatment of the Mills observations. The wavelengths of stellar and iron spark comparison lines adopted as the basis of the reduction are given in columns one and four of Table V, in which the values in italics are the lines of the iron comparison from the determinations by Babcock.[24] Columns two and five contain the identification of the stellar absorbing atom. The third and sixth columns give the intensities of the stellar lines as observed on the present spectrograms, the estimates being on an arbitrary scale in which 20 has been assigned to a line of maximum intensity. The effects of the diurnal and annual motions of the observer were removed from the velocity given for each plate, as usual. The measured position of each line was not corrected for the effect of the curvature of the lines resulting from the use of a straight slit, but a constant correction, corresponding to the curvature[25] at λ4200A, has been added to each measured velocity given in Table II.

*Velocity curves.*—The velocities derived from the 95 Mills plates of 1926, and 1927 were reduced to the epoch of October 7, 1926, J. D. 2424796.700 with a period of 0.1904795 day. The phase given in the third column of Table I refers to this epoch and period. The velocities were then combined into 19 normal places each covering 0.01 day and including from two to eight measures. The combination of the velocities was made in accordance with their weights, the measures of each plate having been assigned a weight of 1, 2, or 3, depending upon the number of lines measured and the consistency of the results. Table VI contains the normal places and the sum of the weights in each case.

TABLE VI

TABLE OF NORMAL PLACES FOR VELOCITY OF β *Cephei* FROM MILLS PLATES OF 1926, 27 AND 28.

| Phase | V | Wt. | Phase | V | Wt. |
|---|---|---|---|---|---|
| 0.0045 | + 0.33 | 15 | 0.1037 | −8.09 | 9 |
| 137 | − 4.85 | 10 | 144 | −4.60 | 12 |
| 272 | − 9.42 | 13 | 268 | +4.26 | 10 |
| 341 | −14.04 | 8 | 339 | +6.07 | 15 |
| 466 | −18.52 | 5 | 428 | +9.80 | 5 |
| 534 | −19.78 | 10 | 539 | +9.21 | 9 |
| 634 | −20.79 | 9 | 631 | +7.57 | 9 |
| 728 | −17.94 | 5 | 739 | +7.84 | 7 |
| 867 | −15.53 | 11 | 831 | +4.13 | 7 |
| 0.0908 | −15.96 | 5 | 0.1945 | +0.33 | 15 |

The mean square error of the velocity derived from a single plate of unit weight is about ±2¼ km./sec. relative to the mean velocity curve. The middle error, or "probable error" of a normal place of weight 15 is then ±0.38 km./sec. and for weight 5, ±0.67 km./sec.

[24] *Cont. Mt. Wilson*, No. 343.

[25] *L. O. B.* **12**, 138, 1926.

The velocities combined into normal places were first corrected for the effect of the change of velocity during each exposure. This correction was determined on the assumption that a sine curve of appropriate period and amplitude was a fair approximation to the true velocity curve. The error of the observed velocity would then be the difference between the ordinate at the middle of the exposure and the average ordinate for the time covered by the exposure. Or, the error, $\epsilon = \sin T - \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} \sin \phi \, d\phi$, where $T$ is the phase, $\phi$, at the time of the middle of the exposure and $t_1$ and $t_2$ are the phases at the times of beginning and ending the exposure. Putting $(t_2 - t_1) = t$, it follows that $\epsilon = \sin T \, (1\text{-}2/t \cdot \sin t/2)$, or the true velocity at the recorded time, $V = V' + \Delta \cdot V'$, where $\Delta$ is the factor $(1\text{–}2/t \cdot \sin t/2)$ which can be tabulated against the length of the exposure time or put in the form of a graph. The resulting corrections range generally from 0.0 to 0.6 km./sec., both positive and negative.

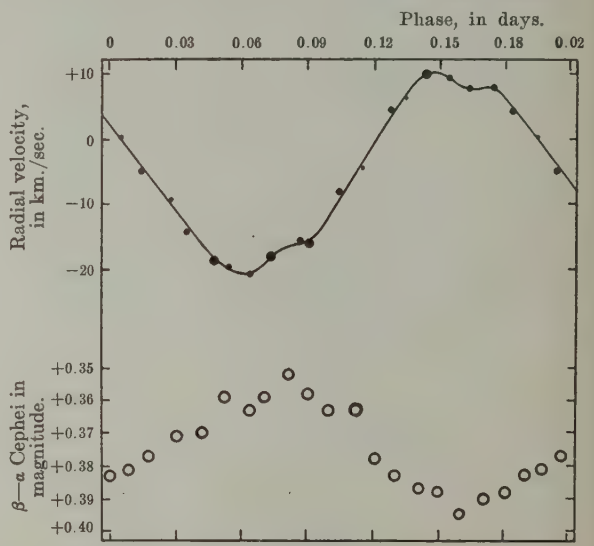



Fig. 1. Upper curve: Velocity curve determined from measures of 95 Mills III-prism spectrograms. The normal places are plotted with circles of radius equal to the "probable error" in each case.

Lower curve: Light curve determined by photo-electric measures at Berlin-Babelsberg in Sept., 1925.

The normal places are shown in Figure 1, plotted as circles with radii equal to the probable errors. A smooth curve has been drawn to represent these points as closely as possible. The reality of the indentations in the curve near the maximum and minimum points seems established by the relative smallness of the middle errors for the plotted points. The chance of two such similar indentations, correspondingly placed on the curve, being the result of a combination of accidental errors is vanishingly small. The possibility of a correlation between a small variation in the phase

or shape and in the amplitude of the curve at different epochs has been considered as an explanation of the shape of the mean curve, but no such correlation could be discerned. The light curve obtained at Berlin-Babelsberg in 1925 has been included in Figure 1. A few of the velocity curves obtained at different epochs are given in Figure 2. The lack of evident indentations in these may be due to the fact that the time covered by the average exposure is about equal to that involved in the entire indentation. Provisional orbital elements derived from the curve of September 1928 are given in Table VII of the next section.

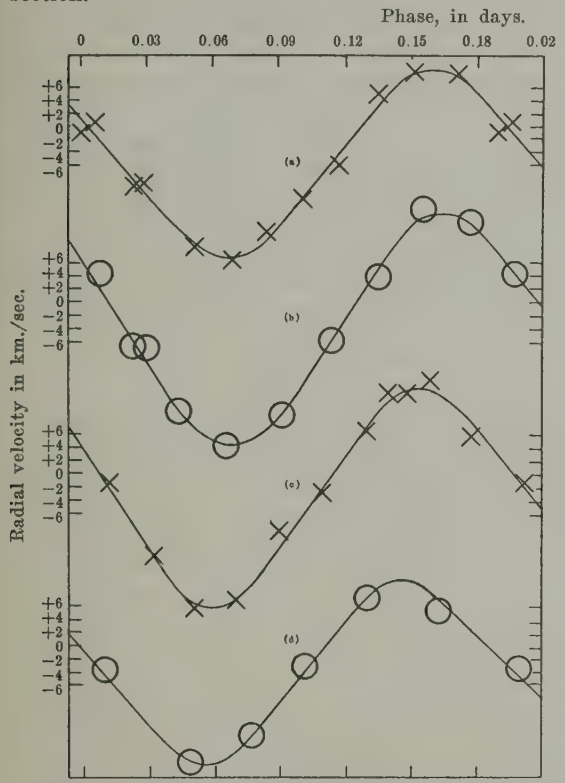



Fig. 2. Velocity curves from Mills spectrograms obtained at separate epochs. The epoch of curve (a) is June 23, 1926, of (b) is October 7, 1926, of (c) is October 24, 1926, and of (d) is January 7, 1927.

The radial velocities derived from the 17 violet spectrograms are plotted in Figure 3 after being reduced to the same phase by means of the period 0.1904795 day. The measures of each plate are plotted separately, symbols being used to designate the epoch of observation in each case. These velocities suggest a somewhat differently shaped curve than that derived from the Mills plates, but the points are too few in number, especially along the ascending branch of the former curve, to establish the reality of the observed difference in form. A real discrepancy between the two curves might be explainable in terms of a difference in effective stellar atmospheric levels at which the lines originate for the two different spectral regions. This phase of the subject will be taken up in a following section. Approximate orbital elements derived from the violet spectrographic velocities are included in Table VII of the next section.

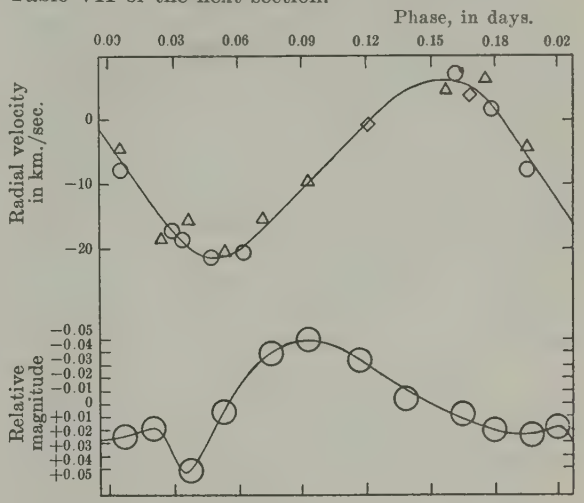



Fig. 3. Upper curve: Velocity curve from observations with III-prism violet spectrograph on July 19, July 28, August 5, and August 14, represented by a diamond, circles, a square, and triangles respectively.

Lower curve: Light curve for ultra-violet region determined from spectro-photometric observations described on pages 146 to 150 of this paper.

Positive reproductions of the spectrum of *β Cephei* as obtained with the Mills spectrograph and with the violet spectrograph are shown in Plate IV, Figures (a) and (b) respectively. A few prominent lines are marked on each.

## Variation of Elements

It is clearly recognized that the short period velocity variation in the case of *β Cephei* is probably not due to orbital motion of this star as a component of a binary system in the period indicated. Frost[26] in 1906, called attention to the fact that the major axis of the orbit is small in comparison with the probable size of the star. The mean of five determinations of the parallax is 0".006, the largest being 0".018. Assuming the effective temperature to be 21000° K we find the radius of the star to be 1½ or 3½ million km. for the two values of the parallax quoted above. The values of $a \sin i$ found for this star are less than 50000 km. or 1/30 of its probable radius. Evidently recourse must be had to the pulsation hypothesis of Shapley and Eddington,[27] or to some modification of it such as proposed by Jeans.[28] However, the orbital elements

[26] *Ap. J.* **24**, 260, 1906.
[27] *M. N.* **79**, 2, 1918.
[28] *M. N.* **85**, 797, 1925.

determined from the velocity curves serve as a convenient means of describing quite closely the shapes of these curves. They will be used in this section as a means of studying the variation in shape of the velocity curves derived at separate epochs.

All published sets of orbital elements of *β Cephei* have been collected in Table VII, together with elements estimated by the writer in cases where only the velocities were given. In the first column are given the dates approximately covered by the observations upon which the elements are based. The next six columns contain the orbital elements other than the period which was assumed to be 0.1904795 day in nearly every case. The values of $a \sin i$ are given in units of 1000 km. The next, or 8th column gives the number of plates involved, those in parentheses being also included in other groups. The Roman numerals in the 9th column indicate the number of prisms in the spectrograph used. These are followed by the linear dispersion in Angstrom units per mm. at the center of the plates. The next two columns give respectively the names of the observers responsible for the measurements and reductions of the plates and the names of the computors who derived the orbital elements from the velocities. The elements that have been estimated by the writer are provisional, being derived from the velocity curve by the method of Lehmann-Filhes without a least square solution in any case.

Eighteen complete sets of elements have been tabulated, together with incomplete data in several cases. In a few instances more than one set of elements are based upon the same observational material. A considerable variation in the elements is evident from the tabulated values. Each of the first four elements and the period will be studied separately.

*The Period, P.*—Crump's[29] value of the period, 0.1904795 day, has generally been accepted as correct. Over 40,000 "revolutions" have been made by *β Cephei* from 1906 to 1928, so that the average period over this interval may be determined with high accuracy.

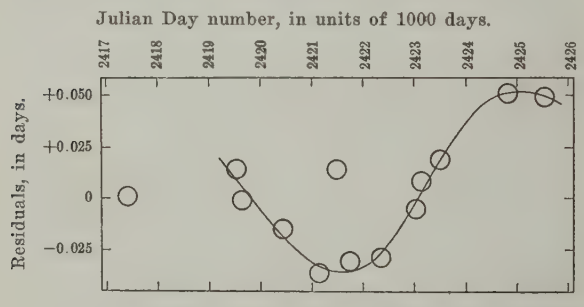



Fig. 4. Residuals, observed minus computed, for the times of "aphelion" using a period of 0.1904795 day and zero epoch in July, 1906.

[29] *Pub. Detroit Obs.* **2**, 144, 1916.

Using as a reference point the intersection of the descending branch of the velocity curve with the axis representing the velocity of the system, i.e., the aphelion point, one finds the average period to have been 0.1904806 day—an increase of 11 in the seventh decimal place over Crump's period. The aphelion time has been computed for each epoch appearing in Table VII by means of Crump's period on the assumption that true period and average period coincided for the epoch July 6, 1906. A fairly smooth curve, as shown in Figure 4, is obtained by plotting the residuals, observed minus computed, for the time of aphelion at each epoch. No reason is apparent for the conspicuous disagreement of the residuals for the epochs of 1906 and 1917. However, it seems safe to conclude that the period of the velocity variation must itself be varying about an average value close to Crump's result of 0.1904795 day. Apparently the period was shortest in 1912 and longest in 1922. The average value of the period between 1906 and 1912 was 0.1904795 day. The average value between 1906 and 1922 was again 0.1904795 day. It may be seen from the curve that the period of the velocity variation may be approximately expressed as

$$P = 0^d.1904795 + 0^d.0000130 \cos \{18^\circ (t-1922)\},$$

the coefficient of the second term being estimated from the slope of the curve at the points of inflection on the zero axis. By 1932 the period should have decreased so that the average value of the period from 1906 to 1932 will again be very nearly 0.1904795 day.

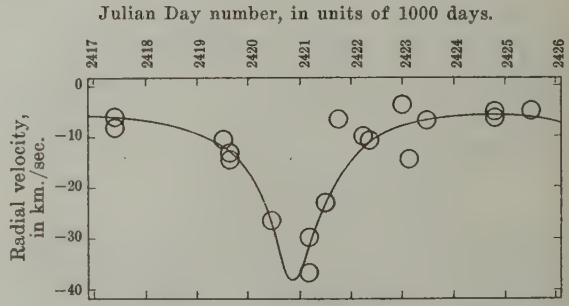



Fig. 5. Values of the systemic velocity, $\gamma$. The smooth curve was drawn to represent an assumed period of variation of $\gamma$ of 20 years.

*The Velocity of the System.*—The values of $\gamma$ given in Table VII are plotted in Figure 5. A considerable variation in $\gamma$ is apparent from the figure. Excluding the observations of 1914, 16, and 17, from Poulkovo[30], [31] however, the values remain rather close to −10 km./sec. The value of $\gamma$ is, of course, dependent upon the wave-lengths used in the reductions. A com-

[30] *Bull. de l'Obs. Central de Russie*, Poulkovo, **7** No. 80.

[31] *Bull. de l'Académi des Sciences de Russie*, 1918.

parison of the wave-lengths used by Belopolsky and by the writer indicates that the differences average close to zero, while a similar comparison for the wave-lengths used by Crump and by Henroteau gives a difference of only 1.4 km./sec. relative to those used by the writer. One concludes that the velocity of the system has probably varied as shown by the smooth curve drawn through the plotted points. Large discrepancies exist for several of the points relative to this curve. The four results for $\gamma$ that have been derived from Lick Observatory observations are all within 2 km./sec. of a constant value and deviate as much as 10 km./sec. from the curve as drawn. Moreover, the rapid change in $\gamma$ shown by the Russian observations of 1916 is in the opposite direction from that given by the curve, being a decrease instead of an increase. Therefore this curve, and conclusions drawn from it, must be accepted with caution.

TABLE VII

ORBITAL ELEMENTS DERIVED FROM VELOCITY CURVES OF $\beta$ *Cephei*

| Dates | $\gamma$ in km./sec. | $K$ in km./sec. | $e$ | $\omega$ | $T$ in Julian days 241+ | $a \sin i$ 1000 km. | No. of plates | No. prisms and A/mm. | Velocity by | Elements by |
|---|---|---|---|---|---|---|---|---|---|---|
| 1902 | | | | | | | 11 | III, 11 | Frost and Adams[32] | |
| 1903, Sept. 16 and 20 | | | | | | | 2 | III, 11 | H. D. Curtis | |
| 1906, July 6 and Aug. 27 | − 5 | 17 | .0 | | | 45 | 34 | II, 22 | Frost[33] | Frost |
| July 6 and Aug. 27 | − 5.7 | 18 | .04 | 94 | | 48 | (24) | II, 22 | Frost | Belopolsky |
| July 6 | − 4.9 | 16.9 | .03 | 165 | 7398.859 | | (12) | II, 22 | Frost | Crump |
| July 6 | − 7.8 | 16.3 | | | 7398 | | (12) | II, 22 | Frost and Struve[34] and Barrett | Mendenhall |
| 1912, Apr. 29 | −10.2 | | | | 9522 | | | | Frost[35] | Frost |
| Aug. 8–Nov. 21 | −14.1 | 15.8 | .04 | 3 | 9638.812 | 41 | 163 | I, 44 | Crump[35] | Crump |
| Oct. 1 | −12.9 | 19.2 | .05 | 17 | 9638.820 | | (18) | I, 44 | Crump | Crump |
| 1914, Oct. 9–21 | −26.1 | 16.8 | .16 | 184 | 242+ 0415.285 | 43 | 35 | III, 30 | Belopolsky[36] and Gerasimovic | Gerasimovic |
| 1916, Sept. 12–Oct. 31 | −32.4 | 19.0 | .10 | 334 | 1119.363 | 48 | 55 | III, 30 | Belopolsky[37] | Belopolsky |
| Sept. 12–30 | −29.5 | 19.9 | .0 | | 1119 | | (32) | III, 30 | Belopolsky | Mendenhall |
| Oct. 1–29 | −36.2 | 20.0 | .0 | | 1119 | | (23) | III, 30 | Belopolsky | Mendenhall |
| 1917, Sept. 16–Oct. 31 | −22.5 | 18.0 | .07 | 305 | 1489.311 | 47 | 32 | III, 30 | Belopolsky[37] | Belopolsky |
| 1918, May 15 | − 6.3 | 18.3 | .07 | 305 | 1729.837 | 48 | 6 | III, 11 | Henroteau; Hobe; and Mendenhall | Mendenhall |
| 1919, Sept. | − 9.9 | 13.3 | | | | | 32 | | Boothroyd | Van Arnam[38] |
| 1919 and 1922 | − 9.8 | 11.3 | .05 | 59 | 2219.820 | 29 | | | Boothroyd | Van Arnam[38] |
| 1920, Jan. 19–Feb. 3 | −10.5 | 14.3 | .14 | 230 | 2343.741 | 37 | 71 | 33 | Henroteau[39] and Henderson | Mendenhall |
| 1921, Nov. 6 | − 3.6 | 14.8 | .05 | 78 | 3000.622 | 39 | 16 | III, 11 | Hobe and Mendenhall | Mendenhall |
| 1922, Feb. 16–Apr. 18 | −14.2 | 13.1 | .09 | 82 | 3102.733 | 34 | 155 | 33 | Henroteau[40] | Mendenhall |
| 1923, Mar. 5–28 | − 6.8 | 10.6 | .00 | | 3484 | 28 | 68 | 33 | Henroteau[40] | Mendenhall |
| 1926, June 23–Dec. 38 | − 5.1 | 15.4 | .05 | 274 | 4796.832 | 39 | 95 | III, 11 | Mendenhall | Mendenhall |
| July 19–Aug. 14 | − 6.3 | 13.8 | .08 | 220 | 4796.755 | 36 | 17 | III, 22 | Mendenhall | Mendenhall |
| 1928, Sept. 10 | − 5.0 | 17.0 | .02 | 12 | 5500.874 | 45 | 8 | III, 11 | Mendenhall | Mendenhall |

Belopolsky[41] has combined the results of all the observations of $\beta$ *Cephei* made at Poulkovo in 1914, 1916, and 1917 into a single set of elements by making allowance for the change in $\gamma$ and $\omega$, but he has evidently made an error of an even day at one point in the process.[42] His result for $\gamma$ is given as $-19$ km. $-13$ km. $\sin\{19^\circ(1911.32-t)\}$ but the epoch 1911 is probably a misprint. The 19-year period may be correct although the observational data in Figure 5 will admit of any period whatever that is greater than about 15 years.

[32] *Ap. J.* **15**, 340, 1902.

[33] *Ap. J.* **24**, 260, 1906.

[34] *Ap. J.* **64**, 68, 1926.

[35] *Pub. Detroit Obs.* **2**, 144, 1916.

[36] *Bull. de l'Obs. Central de Russie*, Poulkovo, **7** No. 80.

[37] *Bull. de l'Académie des Sciences de Russie*, 1918.

[38] *Pop. Ast.* **36**, 347, 1928. Definitive elements by Van Arnam appeared in *Publ. DAO.* **4**, 171, 1929 after present paper was in press.

[39] *Pub. Dom. Obs.* **5**, 77, 1922.

[40] *Pub. Dom. Obs.* **9**, 36, 1925.

[41] *Bull. de l'Académie des Sciences de Russie*, 1918.

[42] *Id.* 1918, page 1798, line 8.

*The Semi-amplitude.*—The values of the semi-amplitude for each curve given by the different series of observations of $\beta$ *Cephei* in the interval from 1906 to 1928 have been plotted in Figure 6, together with a smooth curve drawn through them. The departure of the plotted points from the curve as drawn is in every case less than 3 km./sec. and is comparable with the accidental errors involved in the determination of the exact value of $K$. The plot is suggestive of a 20-year period for the variation, but there is little reason to conclude that the period is not many times greater.

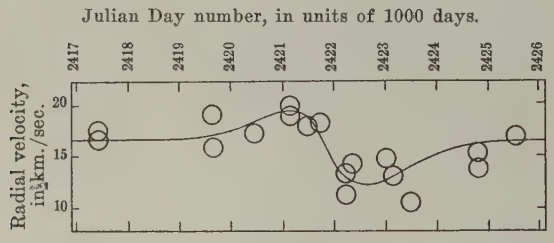



Fig. 6. Values of the semi-amplitude, $K$, of the velocity variation.

Henroteau[43] has found a short period (28-day) variation in $K$ with an amplitude of 20 km./sec. from his observations of the velocity of $\beta$ *Cephei* in 1922. The Lick observations of 1926–27 fail to show either this period or this range for $K$. The latter observations cover the period completely at seven different epochs. For these the minima remain roughly fixed, while the heights of the maxima vary little more than 5 km./sec. in a manner suggestive of a period of the order of 180 days. This period would also approximately represent the values of $K$ determined from Crump's observations of 1912, but the range of possible periods is large in both cases.

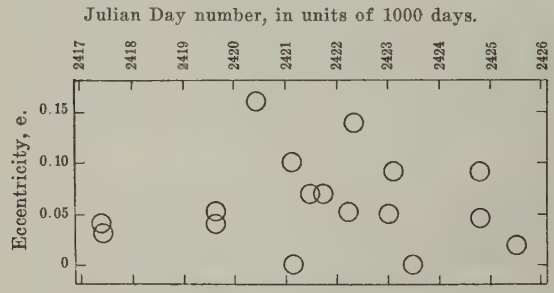



Fig. 7. Values of the orbital eccentricity, $e$.

*The Eccentricity.*—The fifteen determinations of eccentricity made in the interval from 1906 to 1928 and plotted in Figure 7 give results ranging from 0.0 to 0.16. A comparison of the various velocity curves leaves no doubt as to the reality of a variation in their shape greater than could be accounted for by accidental errors in the velocities or in the drawing of the mean curves. Occasional errors in the computation of $e$ may account for the apparent lack of regularity in its variation. The published value[44] of $e$ for Belopolsky's observations of 1916 is 0.10 whereas no eccentricity is discernible in a plot of his velocities for this date. It may be significant that the largest values of $e$ appear during the interval covered by the rapid variation in $\gamma$ and $K$.

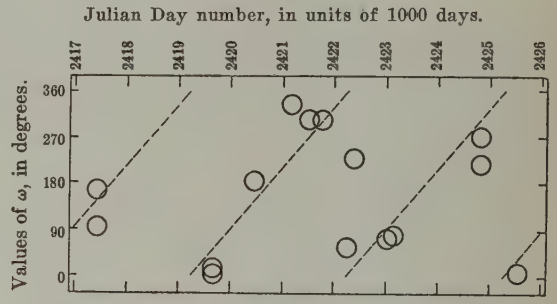



Fig. 8. Values of the longitude of periastron, $\omega$. The broken line represents a continuous variation of $\omega$ through 360° in about 8.2 years.

*The Longitude of Periastron.*—The tabulated values of the angle $\omega$ have been plotted in Figure 8 and at first sight are apparently without system. The small values of the eccentricity result in large uncertainties in the determination of $\omega$ and $T$ so that probably only the quadrant in which $\omega$ is found has any real significance. The lines in Figure 8 represent a continuous change of $\omega$ through 360° in 3000 days or about 8.2 years. The majority of the points lie within 90° of these lines, which would be improbable if the distribution of the angles were entirely the result of accidental errors. It is necessary to conclude that a force is acting which produces an effect upon the velocity curves corresponding to a rotation of the line of apsides in a binary system.

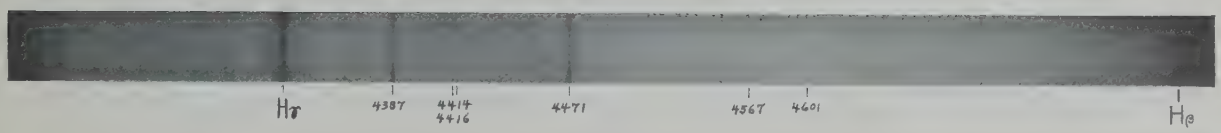
Belopolsky[45] gives $\omega$ equal to $0°+76°.5\,(1917.19-t)$, but a period of $4\frac{1}{2}$ or 5 years does not seem plausible in view of the data obtained since 1917.

In general, there seems to be some correlation between the accuracy of the observations and the variation of the orbital elements derived from them but the variability is greater and more regular than can reasonably be accounted for by accidental errors. The general character of the variation of $P$, $\gamma$, and $K$ over long periods of time is fairly well established by the approximate consistency of the plotted values although several outstanding discrepancies remain unexplained. The nature of the variation in $\omega$ is somewhat more doubtful, while the apparent random

[43] *Pub. Dom. Obs.* **9**, 36, 1925.

[44] *Bull. de l'Académie des Sciences de Russie*, 1918.

[45] *Bull. de l'Académie des Sciences de Russie*, 1918.



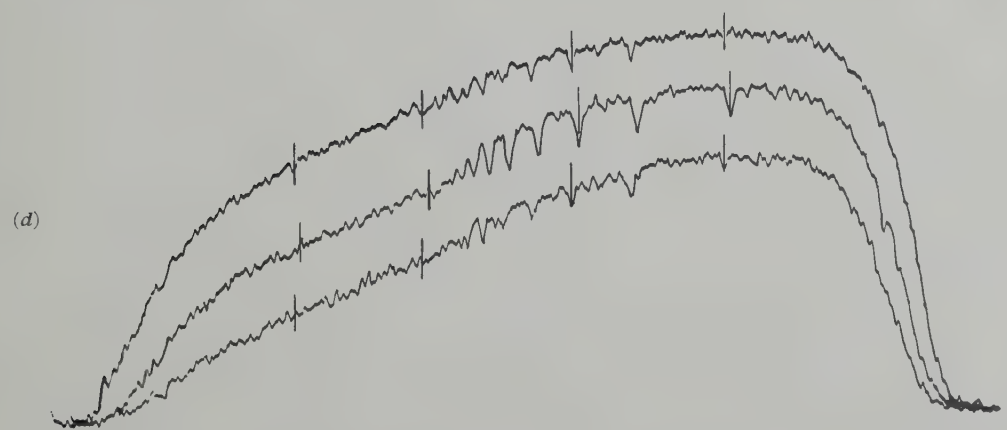



Fig. (*a*). Spectrum of β Cephei obtained with the III-prism violet spectograph and 36-inch refractor.

Fig. (*b*). Spectrum of β Cephei obtained with the Mills III-prism spectograph and 36-inch refractor.

Fig. (*c*). Spectrum of β Cephei between spectra of comparison star 19 Cephei obtained with a slitless quartz spectograph.

Fig. (*d*). Microphotometer tracings of the above spectra. Short vertical lines indicate regions that were compared in intensity.

variation in $e$ is probably not real. The complexity of some of the velocity curves, such as the one derived from the measures of the Mills plates of 1926 and 1927 and shown in Figure 1, undoubtedly affects the accuracy of the foregoing results.

### Level Effect

Interesting results have been obtained by recent investigators[46] from a study of the relative displacements of the individual lines, in the spectra of the Cepheid variables, according to a method first proposed by Rufus.[47] The method consists in dividing the observed lines into from two to four groups, each of which corresponds to a low, intermediate, or high level in the Sun's atmosphere as given in Mitchell's[48] list of chromospheric heights combined with Rowland's intensities. The mean velocity curves for these groups are then compared and differences in phase, shape, or amplitude noted.

From an investigation of this type for *S Sagittae*, using 124 spectrograms made at the Detroit Observatory in the interval from 1913 to 1923, Aldrich[49] has found a lag in phase and a larger amplitude for the velocity curves derived from lines representing high levels relative to the curves given by lower levels. He moreover points out that the relative atmospheric motions would alternately produce a compression and an expansion of the star's outer atmosphere and that the observed epochs of light maximum and light minimum agree closely with the predicted times of maximum compression and maximum expansion, respectively.

Corresponding results were found by Rufus[50] from a similar investigation of $\eta$ *Aquilae*. The prominent "Stillstand" in the mean velocity curve is shown to be emphasized in the high-level and low-level curves and to be absent for the intermediate levels. Curves of velocity differences were drawn, one of which—hydrogen minus intermediate—had an amplitude fully half as great as the mean velocity variation. This same difference curve resembles the photometric light curve of the star and also the curve for spectral variation.

In the case of *W Sagittarii*, R. H. Curtiss[51] has observed marked differences between the curves for various groups of lines originating at different chromospheric heights. However, he finds the range of velocity variation to be less for greater chromospheric heights instead of the reverse, as found for $\eta$ *Aquilae* and *S Sagittae*.

[46] *Pop. Ast.* **32**, 471, 1924.
[47] *Pop. Ast.* **32**, 22, 1924.
[48] *Ap. J.* **38**, 407, 1913.
[49] *Pop. Ast.* **32**, 218, 1924.
[50] *Pop. Ast.* **32**, 228, 1924.
[51] *Pop. Ast.* **32**, 547, 1924.

Velocity curves of $\zeta$ *Geminorium* have been studied by Rufus[52] and these have likewise yielded systematic differences for displacements of lines from different atmospheric levels. In general the high level lines were found to give a higher and sharper maximum in the velocity curve than the maximum derived from the low level lines. Maximum velocity also occurs earlier for the lines of higher level than for the low level lines. Hydrogen was found to show a very high maximum and greater phase lag in its velocity curve than the other high level lines. Velocity difference curves—high minus low, and high minus intermediate—have indicated a sudden change from negative to positive values near the time of light minimum. The amplitude of these curves is found to be greater than 1/3 that of the velocity curve itself.

These investigators have found a correlation between the times of maximum and of minimum compression of the radiating gases and the times of light maximum and minimum. Their conclusions are that pulsations originate deep within the star and travel outward in the form of successive compressions and expansions of concentric layers of the star's atmosphere and that these are sufficient to produce both the observed radial motion of the absorbing gases and the observed variation in the rate of radiation due to the resulting change in temperature and absorbing power of the stellar atmosphere.

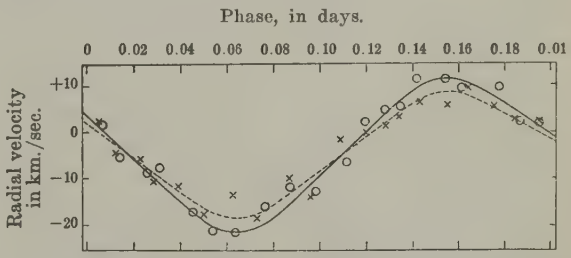



Fig. 9. Velocity curves derived from individual lines measured on the Mills spectrograms of 1926 and 1927. Each plotted point represents the mean of determinations of velocity from 4 or 5 plates with the He I line at 4387A (represented by crosses and the broken line curve) and the Si III line at 4567A (indicated by circles and the continuous curve.

The measures of the 95 plates of $\beta$ *Cephei* obtained during the summer, fall, and winter of 1926–27 with the new Mills III-prism spectrograph were made the basis for a study of the relative displacements of the spectral lines. The eleven italicized lines given in Table VIII were measured with sufficient consistency throughout the series of 95 plates to yield individual velocity curves for the different lines. A superposition of these plots shows that, with a single exception, no certain difference in phase, amplitude, or shape exists between the curves for these lines. In the case of

[52] *Pop. Ast.* **34**, 242, 1926.

the He I line at λ4387, the semi-amplitude is smaller by 3 or 4 km./sec. than that given by the curves for the other lines. The curves for this He line and for the Si III line at λ4567A are shown in Figure 9. Each normal place plotted in Figure 9 covers 0.01 day on the average and consists of the mean of measures of the displacement, on 4 or 5 plates, of the line λ4387A, (indicated by crosses) and of the Si III line at λ4567A, (indicated by circles). The measures of the latter line are typical of results obtained from the other ten good lines as far as the general characteristics of the velocity curves are concerned. The measures are not sufficiently numerous or reliable to merit a consideration of the deviations of normal points from the smooth curves as drawn, although the indentation in the curve suggested by the points for the Si line immediately following minimum is probably real.

Six other lines give such widely scattered measures, due to their faintness and diffuseness, as to render them of no value in this study except to show that no large differences exist in the character of the velocity curves. Table VIII gives the wave-length and identification of the 17 lines studied, together with the heights, where available, at which these lines originate in the solar chromosphere.

TABLE VIII

LINES INVESTIGATED FOR RELATIVE DISPLACEMENTS ON THE MILLS PLATES

| λ I. A. | Elem. | Height |
|---|---|---|
| *4387.85* | He I | 2000 kms. |
| 95.94 | O II | |
| *4414.88* | O II | |
| *16.90* | O II | |
| 37.49 | He I | 750 kms. |
| *47.01* | N II, O II | |
| 52.51 | O II | |
| 69.66 | O II | |
| *71.52* | He I | 7500 kms. |
| *81.23* | Mg II | 400 kms. |
| *4552.59* | Si III, N II | |
| *67.84* | Si III | |
| *74.80* | Si III | |
| *90.97* | O II | |
| *96.18* | O II | |
| 4601.65 | N II | |
| 07.12 | N II | |

The region of the spectrum in good focus on the plates obtained with the violet spectrograph is about twice as extensive as that on the Mills plates, resulting in a corresponding increase in the number of available chromospheric lines. The same set of 35 lines was, in general, measured on each of the 17 violet spectrograms and the velocities derived from each line were plotted. Small differences between the velocity curves for the various lines are indicated but the measures are not of sufficient precision or number to permit an accurate determination of the relative displacements of the individual lines. Table IX contains the lines that were investigated for relative displacements. The ionization potential is given for the absorbing atoms, and the estimated intensity is included in parentheses after each wavelength.

TABLE IX

LINES INVESTIGATED FOR RELATIVE DISPLACEMENTS ON VIOLET SPECTROGRAMS

| H I, I.P. 14 volts | | N II, I.P. 30 volts | |
|---|---|---|---|
| 4340.47 | (15) | 4241.79 | (2) |
| 4101.74 | (16) | 4041.34 | (1) |
| 3970.08 | (20) | 4035.05 | (2) |
| | | 3994.99 | (3) |
| **O II, I.P. 35 volts** | | **Mg II, I.P. 15 volts** | |
| 4416.97 | (2) | 4481.23 | (4) |
| 4414.89 | (3) | | |
| 4366.91 | (3) | | |
| 4351.27 | (3) | **He I, I.P. 24 volts** | |
| 4349.44 | (3) | 4471.48 | (7) |
| 4347.43 | (2) | 4437.55 | (2) |
| 4345.57 | (1) | 4387.93 | (8) |
| 4319.65 | (2) | 4143.76 | (5) |
| 4317.16 | (2) | 4009.33 | (4) |
| 4189.79 | (2) | 3964.73 | (3) |
| 4185.45 | (2) | | |
| 4169.23 | (2) | | |
| 4153.31 | (2) | **Si IV, I.P. 45 volts** | |
| 4119.22 | (3) | 4116.13 | (2) |
| 4104.90 | (2) | | |
| 4075.87 | (3) | | |
| 4072.16 | (3) | **C II, I.P. 24 volts** | |
| 4069.79 | (4) | 4267.16 | (3) |
| 3973.27 | (3) | | |

These lines were combined into groups and their velocity differences were plotted against the time. Several bases for grouping the lines were tried, including ionization potential and solar chromospheric height. The latter was available for the H and He lines only. These preliminary groupings and difference curves indicated that the hydrogen lines were in general displaced by the same amount as the oxygen lines. Figure 10 includes a plot of the velocity differences of hydrogen relative to oxygen. Although rather scattered, the points show that little or no relative motion exists. Therefore, these 22 lines were combined into a single group. Comparison of the other lines with the hydrogen and oxygen group showed that within the errors of observation the He, N, Mg, C, and Si lines could be combined into a single group in view of their similar displacements. This was done and the resulting group compared with the hydrogen and oxygen group. Figure 10 includes a plot of the velocity differences derived from these two groups.

It is obvious from this plot that no large relative motion exists between the gases responsible for the lines in the two groups. No larger relative displacements were found by means of other groupings. The plotted points are few in number and quite scattered,

which results in considerable uncertainty in the curve as drawn. Two maxima appear in the curve, but their reality is very questionable. In fact it would be possible to remove all appearance of relative displacement by shifting only one or two points by amounts no greater than the possible errors of observation.

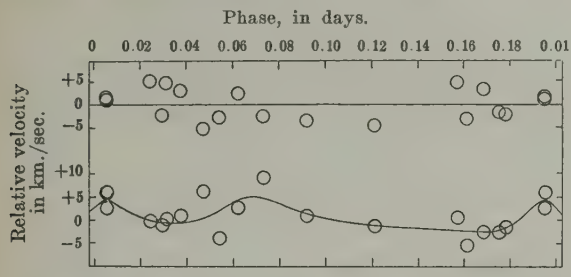



Fig. 10. Upper curve: Velocity differences, H ($\gamma$, $\delta$, $\epsilon$) minus O (19 II lines) from measures of the 17 violet spectrograms.

Lower curve: Velocity differences, O and H minus He, N, Mg, C, and Si (13 lines), from measures of the violet spectrograms.

The following results have been derived from the present investigation of the relative displacements of lines in the spectra of *β Cephei* obtained in 1926 and 1927: (1) Among eleven accurately measured lines on the 95 Mills plates, only one indicates relative motions as large as 5 km./sec. (2) Among certain groups of lines measured on the 17 violet spectrograms, relative displacements of very doubtful reality appear, corresponding to velocities as high as 5 km./sec.

Thus it appears that no definite relative displacements of the spectral lines exist in the case of *β Cephei* such as have been found in the spectra of other variables by Curtiss, Rufus, and others.

### Discussion of Spectrographic Observations

The spectrographic data considered in the preceding paragraphs, although not so complete as could be desired for a definite decision upon many of the questions raised, appear to justify the following conclusions concerning the system of *β Cephei*.

There is good reason for believing that the observed variation in the radial velocity, having a period of about 0.1905 day is not to be interpreted as arising from simple orbital motion of the star, but probably is to be ascribed to the motion of the stellar atmosphere caused by the star's alternate expansion and contraction. Observed changes in the form of the velocity curve are therefore to be considered as variations in the pulsations of the stellar atmosphere. It is to be noted that on this interpretation $e$ and $\omega$ lose their usual physical significance and indicate merely a change in the form of the velocity curve. On the other hand, $P$ and $K$ refer to the period and semi-amplitude of the pulsation and have a perfectly definite meaning, while $\gamma$ in this case denotes the radial velocity of the star as a whole. That $\gamma$ or the radial velocity of the star is variable, with a period of 18 or more years and with an amplitude of about 32 km./sec. seems to be fairly well established by observations secured in the interval 1903–1928. From this it is concluded that *β Cephei* is the brighter component of a binary system having a period of the order of 20 years. With an assumed period of 20 years, the following approximate elements of the orbit for the brighter star have been derived from the velocity curve:

$$P_1 = 20 \text{ years (assumed)}$$
$$K_1 = 16.3 \text{ km./sec.}$$
$$e_1 = 0.55$$
$$\omega_1 = 180^\circ$$
$$T_1 = 1916.0$$
$$\gamma_1 = -13.0 \text{ km./sec.}$$
$$a \sin i = 9.14 \text{ A.U.}$$
$$\frac{mt^3_2 \sin^3 i}{(m_1+m_2)^2} = 1.9 \text{ Sun's mass.}$$

The data derived in this investigation show that the period $P$ and semi-amplitude $K$ of the pulsations of the brighter star are variable in a period which appears to be of the order of 20 years or the same as that tentatively assumed for the orbital revolution of *β Cephei* in the binary system. It is also to be noted that the maximum amplitude of the velocity curves attributed to pulsations occur at or near the epoch of periastron for the binary orbit. Since this orbit is quite eccentric there seems to be some evidence that the observed changes in the amplitude of pulsation may be the result of the varying tidal action upon the atmosphere of the bright star with its changing distance from the secondary. Variation in the apparent period of pulsation of a star moving in an orbit of the probable dimensions obtained above is readily calculable. One finds that the motion of the bright star would result in an apparent change in the period of 0.000,020 day. The observed variation in the period amounts to about 0.000,026 day. Similarly, the observed times of the short period velocity maxima do not in general agree with the times computed by means of an average value of the period. The observed maxima precede the computed near perihelion and follow near aphelion, the two being assumed coincident at periastron. The total observed difference amounts to 2.12 hours while computation shows that 2.04 hours would be required for the light to travel the distance $b \sin i$ (the minor axis of the large orbit corrected for the foreshortening effect of the orbital inclination). These approximate agreements between observation and computation tend to substantiate the existence and size of the orbit of the bright star described above.

In this connection it is of interest to examine the data for stars having short period variation in velocity with reference to the variability of the systemic velocity, $\gamma$, and to observed changes in the amplitude and period of the velocity curve. Five appear to have been sufficiently observed to furnish definite information on these points. Data for the five are the following:

| | Period | Velocity of system | Amplitude and period |
|---|---|---|---|
| *β Cephei* | 0.190 days | variable | variable |
| *12 Lacertae* | 0.193 | variable | variable |
| *σ Scorpii* | 0.247 | variable | ? |
| *β Canis Majoris* | 0.257 | variable | variable |
| *RR Lyrae* | 0.567 | variable | variable |

Two of these stars, *β Cephei* and *RR Lyrae* are light variables of the Cepheid type. All have variable velocity of the system, that is, they are probably long period binary systems. With the possible exception of *σ Scorpii* the velocity curves of all show changes in period and amplitude.

The following Cepheids of longer periods are definitely known to have variable velocity of the system: *Polaris, S Sagittae, Y Sagittarii.* There is also some evidence that the systemic velocities of *δ Cephei* and *η Aquilae* are variable. The forms of the velocity curves of *S Sagittae* and *Y Sagittarii* vary and changes of amplitude are also observed for *S Sagittae.* The periods of all are probably variable. *Polaris* has been shown to be the brighter component of a binary, revolving in a highly eccentric orbit in a period of approximately 30 years. The velocity curve for its short period variation shows marked changes in form analogous to those found for *β Cephei.*

It is a significant fact that many of the Cepheids are found to be the brighter components of a binary, and that the two which are known to have very eccentric orbits also exhibit changes in form of the short period velocity curves of such a nature as to suggest that these changes may be the result of tidal action.

## Spectrophotometric Observations

All photometric methods which have previously been employed—visual, photographic, and photo-electric—furnish light curves that represent the integrated variations in intensity over a considerable region of the spectrum. In the study of the variation in luminosity of the Cepheid type of variable it is important to know definitely to what wave-lengths of the spectrum the observations refer and to be able to compare the variation in different parts of the spectrum. The spectrophotometric method to be described appears to be well adapted to this purpose. Moreover, it possesses certain advantages over photometric methods previously employed in that it is free from any personal equation of the observer, avoids certain difficulties which enter in the determination of magnitudes from photographic star images, and is not subject to the well-known practical difficulties encountered when the photo-electric cell is used in measuring stellar intensities.

The proposed method may be described briefly as follows: The spectrum of the star whose light intensity is to be investigated is photographed on a series of plates on each of which are obtained two or more apertured spectra of a comparison star of supposedly constant luminosity. Let it be assumed that such a series of spectrograms of a variable star, distributed throughout the period, have been obtained, and that upon each plate two spectra of known intensity difference have been impressed, the apertures of the telescope having been so adjusted that the intensity of the spectrum of the variable is intermediate between the intensities of the two comparison spectra. The exposure time is kept the same for the variable and for the comparison star spectra. On each spectrogram the relative intensities at corresponding points in the three spectra, for example at λ3900A, are then determined from measures of the microphotometer record of the photographic blackening in the three spectral images at the chosen wave-length. Variation in the luminosity of the star in this particular spectral region is given directly as the result of these intensity measures. Where the range in luminosity of the variable is greater than 0.5 magnitude, three or more apertured spectra of the comparison star should be used.

This method is readily adaptable to the study of the ultraviolet region of the spectrum. Such a study was deemed of special interest in the case of *β Cephei* because of its early spectral type. For this investigation the two-prism quartz spectrograph was utilized in connection with the Crossley reflector. The instrument was used in its slitless form since with a slit spectrograph an unknown amount of light would be intercepted by the slit jaws. The amount of light entering the telescope was controlled by means of black cardboard diaphragms which fit over the end of the telescope tube. There is now available for the Crossley a large cardboard disk in which there are sets of four 10-inch, 6.3-inch, 4-inch, 2.5-inch, and 3-inch circular openings which give approximate magnitude intervals of 1, 1, 1, and −0.4 magnitudes respectively.

By choosing comparison stars that are close in the sky to the variable, one is able to eliminate largely the effects of changes in zenith distance and atmospheric transparency. Some difficulty was experienced in finding comparison stars sufficiently close to *β Cephei* both in the sky and in spectral type that are not themselves variable, but this difficulty would not be encountered for the more common spectral types.

Three good comparison stars were found at moderate distances from $\beta$ *Cephei*, namely, 7 *Cephei*, type $B_5$, 14 *Cephei*, $B_0$, and 19 *Cephei*, $O_9$. A comparison was also made with $\alpha$ *Cephei* because of its well known constancy of light. Four series of plates were obtained, each series with a different comparison star and covering more than the 4½-hour period of $\beta$ *Cephei*. The two largest diaphragms were used with the comparison stars except $\alpha$ *Cephei*, for which the 4-inch and 2.5-inch openings were utilized. The 3-inch diaphragm was used for $\beta$ *Cephei* in all cases except with 7 *Cephei*, where the 2.5-inch diaphragm gave the desired intensity. Every spectrum received a 1½-minute exposure except that a 1-minute exposure was used when $\alpha$ *Cephei* was taken as the comparison star. The exposures were accurately timed by means of a stop watch.

The following procedure, adopted in making the exposures with 19 *Cephei*, is typical of that followed throughout. Set telescope on 19 *Cephei;* 1½-minute exposure with 10-inch diaphragm; move tube slightly with slow motion; 1½-minute exposure with 6.3-inch diaphragm; set telescope on $\beta$ *Cephei* so that spectrum will fall between the two comparison spectra already on the plate; 1½-minute exposure with 3-inch diaphragm; substitute unexposed plate; 1½-minute exposure with 3-inch diaphragm; set telescope on 19 *Cephei;* etc. The saving in time resulting from but a single shifting of the telescope (except by slow motion) for each plate was considered to outweigh the possible advantage of taking one of the two comparison spectra before and one after each exposure on $\beta$ *Cephei*. The guiding of the telescope during these short exposures was left entirely to the driving clock of the Crossley which was adjusted for accurate following. It is evident that a considerable number of plates may be obtained by this method during a single epoch of even a short period variable.

TABLE X

Relative Magnitude of $\beta$ *Cephei* from Spectro-photometric Observations

| Date 1928 | J. D. 242+ | Phase | Δ mag. 3900Å | Δ mag. 3600Å | Δ mag. 3300Å |
|---|---|---|---|---|---|
| | Comparison Star, 19 *Cephei* | | | | |
| Aug. 30 | 5489.490 | .051 | +.27 | +.09 | ++.04 |
| | .522 | .083 | +.09 | −.02 | −.14 |
| | .525 | .086 | −.08 | −.06 | −.04 |
| | .552 | .113 | −.12 | −.14 | −.17 |
| | .555 | .116 | −.12 | −.02 | −.01 |
| | .576 | .137 | −.03 | −.06 | −.05 |
| | .592 | .153 | +.07 | +.07 | +.01 |
| | .617 | .178 | −.07 | +.12 | +.02 |
| | .624 | .185 | +.08 | +.08 | +.08 |
| | .646 | .016 | −.23 | −.12 | −.08 |
| | .651 | .021 | +.02 | −.06 | +.06 |
| | .676 | .046 | −.13 | +.05 | +.06 |
| | .683 | .053 | +.02 | .00 | −.04 |
| | .705 | .075 | −.05 | −.08 | −.10 |
| | Comparison Star, 7 *Cephei* | | | | |
| Aug. 31 | 5490.487 | .096 | −.14 | −.05 | −.12 |
| | .509 | .118 | −.34 | −.10 | −.10 |
| | .517 | .126 | −.16 | −.04 | −.05 |
| | .541 | .150 | −.02 | .00 | ..... |
| | .560 | .169 | −.12 | −.04 | −.06 |
| | .567 | .176 | −.04 | +.06 | +.06 |
| | .585 | .003 | −.05 | +.06 | −.01 |
| | .592 | .010 | −.09 | +.05 | −.02 |
| | .610 | .028 | −.05 | +.14 | +.10 |
| | .616 | .034 | +.10 | +.06 | +.02 |
| | .635 | .053 | −.13 | −.06 | −.12 |
| | .642 | .060 | +.01 | +.08 | +.06 |
| | .661 | .079 | −.06 | +.01 | −.01 |
| | .668 | .086 | +.05 | −.04 | ..... |
| | .689 | .107 | +.07 | −.06 | ..... |
| | Comparison Star, 14 *Cephei* | | | | |
| Sept. 2 | 5492.491 | .004 | +.12 | +.03 | .00 |
| | .512 | .025 | −.04 | −.16 | −.10 |
| | .514 | .027 | +.29 | +.11 | +.09 |
| | .532 | .045 | −.02 | −.08 | −.03 |
| | .536 | .049 | +.02 | −.09 | −.06 |
| | .554 | .067 | ..... | ..... | −.13 |
| | .556 | .069 | +.27 | +.08 | +.08 |
| | .577 | .090 | −.05 | −.06 | −.03 |
| | .588 | .101 | −.07 | −.07 | −.04 |
| | .607 | .120 | +.07 | −.06 | −.01 |
| | .609 | .122 | −.07 | −.16 | −.22 |
| | .628 | .141 | −.16 | −.21 | −.30 |
| | .630 | .143 | −.05 | −.19 | −.17 |
| | .652 | .165 | −.27 | −.23 | −.21 |
| | .665 | .178 | −.01 | −.23 | −.28 |
| | .685 | .008 | −.06 | −.16 | −.13 |
| | Comparison Star, $\alpha$ *Cephei* | | | | |
| Sept. 3 | 5493.485 | .045 | −.04 | +.04 | +.04 |
| | .503 | .063 | +.09 | +.13 | +.11 |
| | .505 | .065 | +.06 | −.01 | +.03 |
| | .521 | .081 | +.08 | +.04 | +.08 |
| | .523 | .083 | +.01 | +.06 | +.10 |
| | .537 | .097 | +.09 | +.04 | +.02 |
| | .558 | .118 | +.07 | +.09 | +.08 |
| | .575 | .135 | +.07 | +.07 | ..... |
| | .596 | .156 | +.10 | −.02 | −.10 |
| | .618 | .178 | +.07 | +.02 | ..... |
| | .633 | .003 | −.02 | +.02 | −.01 |
| | .647 | .017 | +.03 | +.05 | ..... |
| | .649 | .019 | +.04 | +.01 | ..... |
| | .665 | .035 | .00 | −.02 | ..... |
| | .666 | .036 | −.03 | −.13 | ..... |
| | .686 | .056 | −.04 | −.04 | ..... |
| | .704 | .074 | −.02 | +.05 | ..... |

The time of the exposures should be arranged so that the stars come to the meridian near the middle of the series, thus reducing to a minimum the change in relative zenith distance of the variable and the comparison star. However, for the present investigation the telescope was not available until $\beta$ *Cephei* was approximately on the meridian. The resulting change in zenith distance together with the rapid change in focus of the reflecting telescope, difficulties not foreseen by the writer, have combined to produce somewhat wider and more diffuse spectra toward the end of each series. While this may have produced a lower precision for the observations than that of which the method is capable, it is believed that neither the general form of the light curve nor the order of the amplitude of the observed light variation has been seriously affected.

The plates were developed in groups of 7 or 8 under as nearly identical conditions as possible. They were then run through the Moll self-registering microphotometer at Mt. Hamilton, with the slit in front of the thermocouple of sufficient length to cover the width of the spectrum and include some clear film on each side. The machine was run at maximum speed since the continuous spectra, rather than the absorption lines, are of interest here. The order in which the plates were run through the photometer was varied so as to eliminate any possible secular effect such as might result from variations in the instrument. The three tracings derived from the three spectra on each plate were recorded on a single half sheet of sensitized paper.

A positive reproduction of one of the plates secured August 30, 1928, is shown in Plate II, Figure (c). The spectrum of $\beta$ *Cephei* appears between the two comparison spectra of 19 *Cephei*. Figure (d) is a reproduction of the microphotometer tracings of these spectra. The four regions at which measures were made are indicated by short vertical lines drawn across the tracings.

The measurement of the tracings consisted in determining the deflection of the galvanometer, or the height of the curve, in terms of the total deflection from clear film to opacity. This process was greatly facilitated by use of a "converging reseau" for which the writer is indebted to N. W. Storer. The reseau consisted of a piece of glass upon which were ruled numerous parallel equidistant lines in one direction and 21 slightly convergent lines in the other direction, the 21st line being perpendicular to the parallel lines. This provided an adjustable scale that could be used to divide the distance from clear film to opacity into 20 equal parts. The heights of the three curves, i.e., the blackening of the photographic plate, were measured to a tenth of a division of the reseau at four points corresponding to the spectral region of H$\gamma$, H$\epsilon$, and $\lambda\lambda$3600, 3300A, and are believed to be accurate to one per cent of the total deflection from clear film to opacity.

In Figure 11, the curve was obtained by plotting photographic blackening of a set of sensitometer images having intensity ratios of 1 : 2 : 4 : 8 : 16 against the logarithms of these relative intensities. For deflections over 20% and less than 80%, the relation of deflection, $\Delta$, to log $I$ is approximately linear. Since magnitude is a linear function of log $I$, it is also a linear function of $\Delta$ over the interval quoted.

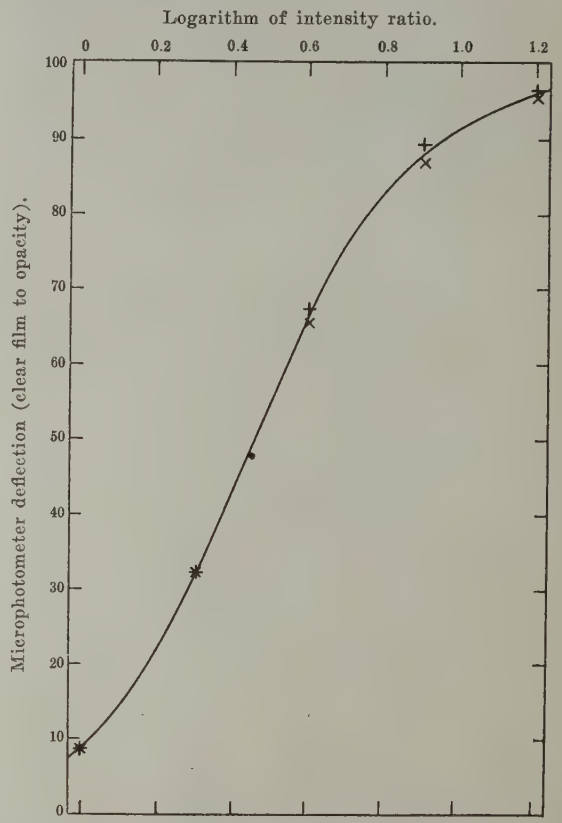



Fig. 11. Relation of photographic blackening to the intensity of the incident light from sensitometer images.

The curve of Figure 11 is not directly applicable to the measures of the spectral tracings because of the clear film included on both sides of the spectra. An infinite exposure would result in a deflection considerably less than 100%. The extent to which this linear relation holds for the tracings of slitless spectra is shown by the plotted measures represented by circles in Figure 12. These are measures of $\Delta$ corresponding to a series of exposures of $\beta$ *Cephei* with the various diaphragms and exposure times indicated on the graph. The range of deflections obtained with these diaphragms and exposures more than covers the range of measures

to be reduced to magnitude intervals. If the relation between deflection and magnitude was perfectly linear for all values of the deflection obtained for the spectra, then Figure 12 would consist of five absolutely straight lines. An obvious deviation from a straight line occurs for small deflections of 5% and less. No measures for such small deflections have been included. A small deviation also appears for deflections between 70% and 85% due to the close approach to infinite blackening. The deflections for the region of Hγ all fall between or near these values, where the method becomes insensitive to small changes in intensity of the incident light. The measures of light variation at Hγ have therefore been discarded. It is evident from these graphs that the assumption of a linear relation between deflection and magnitude will be a very close approximation for deflections between 5% and 70%,

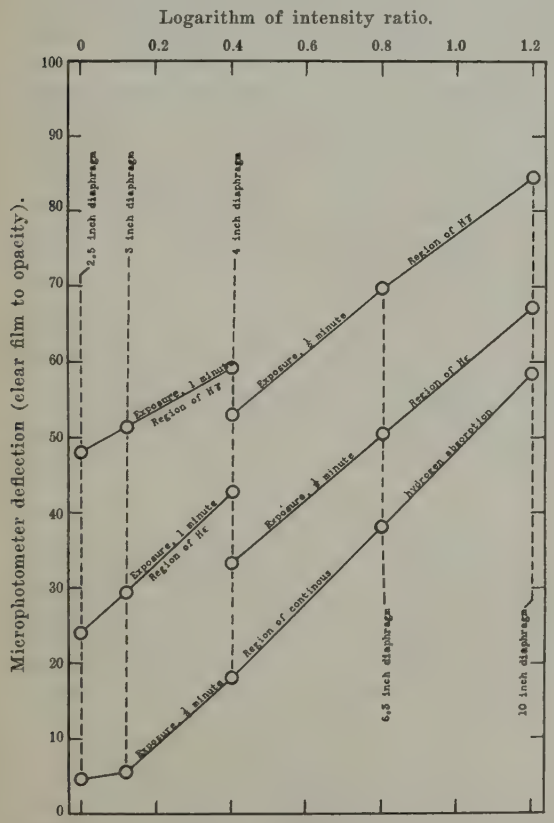



Fig. 12. Relation of photographic blackening to the intensity of the incident light for slitless spectra of *β Cephei*.

particularly where the variation in light is of the order of only 0.05 magnitude. The measures of the tracings have therefore been reduced on the assumption that the relation between magnitude and photographic blackening is linear.

Table X contains the measures of relative magnitude for *β Cephei* obtained in this way for the three spectral regions λλ3900, 3600, 3300A, with reference to the four comparison stars. Each value represents the difference in height of the *β Cephei* spectrum and the weaker comparison spectrum divided by the difference in height of the two comparison spectra.

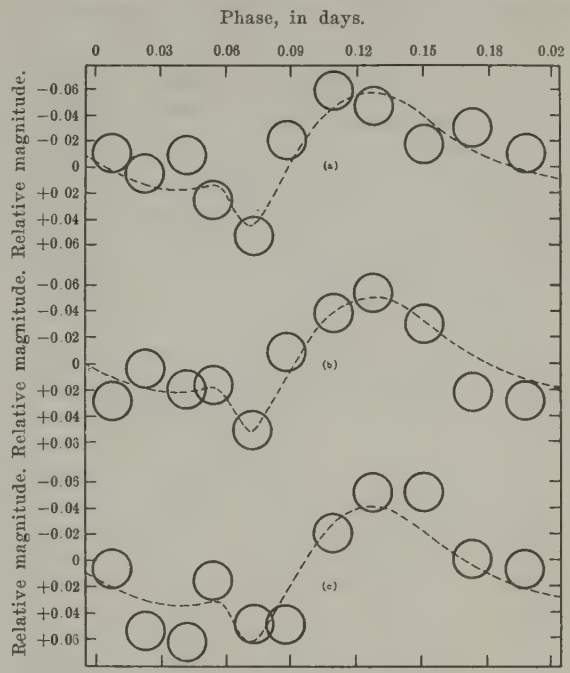



Fig. 13. Light curves at (a) 3900A, (b) 3600A, (c) 3300A. The broken curves represent the mean of the three regions.

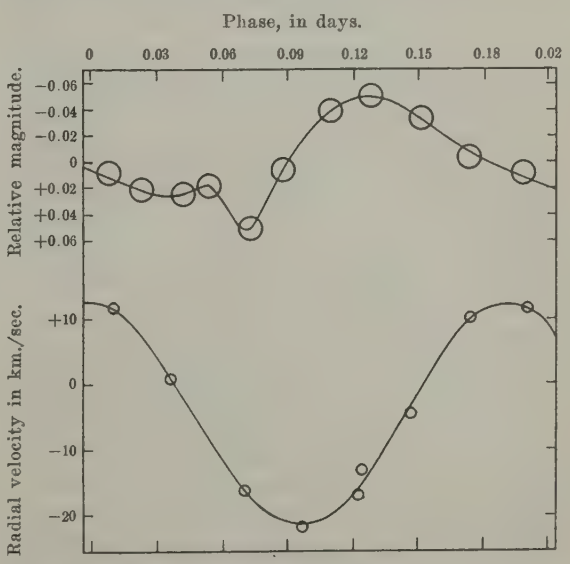



Fig. 14. Upper curve: Light variation in the ultraviolet from spectro-photometric observations.

Lower curve: Velocity curve from Mills spectrographic observations of September, 1928—ten days after photometric observations.

Table XI gives the normal places computed for each region measured after reducing the measures to one epoch with P=0.1904795 day. The corresponding curves for each region are shown in Figure 13. A mean light curve derived from the combined measures for the three regions, to each of which equal weight was assigned, is shown in Figure 14, together with the velocity curve obtained a few days later, reduced to the same epoch. Each normal is the mean of measures of 6 plates and covers about 0.02 day on the average. The mean square error of a single observation is found to be ±0.12 magnitude, and the "probable error" ±0.08 magnitude. The probable error of a normal place containing 18 individual measures comes out ±0.02 magnitudes.

TABLE XI

NORMAL PLACES FOR RELATIVE MAGNITUDE OF β *Cephei*

| Mean phase | 3900A Δ mag. | 3600A Δ mag. | 3300A Δ mag. | Mean Δ mag. |
|---|---|---|---|---|
| .0075 | −0.011 | 0.028 | 0.007 | 0.008 |
| .0227 | 0.005 | 0.004 | 0.054 | 0.021 |
| .0412 | −0.009 | 0.020 | 0.063 | 0.025 |
| .0538 | 0.025 | 0.017 | 0.016 | 0.019 |
| .0722 | 0.053 | 0.051 | 0.049 | 0.051 |
| .0873 | −0.020 | −0.008 | 0.050 | 0.007 |
| .1087 | −0.058 | −0.038 | −0.020 | −0.039 |
| .1273 | −0.047 | −0.053 | −0.051 | −0.050 |
| .1503 | −0.017 | −0.030 | −0.052 | −0.033 |
| .1723 | −0.030 | 0.022 | 0.000 | −0.003 |

The mean light curve in the ultra violet region, which has an amplitude of about 0.1 magnitude, indicates a rapid rise in luminosity, beginning about 0.070 day after the computed time of maximum positive velocity and ending approximately 0.055 day later. This is followed by a gradual decline in luminosity. The secondary variations indicated on the descending branch of the curve are probably real. This mean curve refers to an average wave-length of 3600A while the effective wave-length in the case of the photo-electric methods is about 4500A. Light curves of β *Cephei* have been determined by means of the photo-electric cell by Guthnick and Praeger in 1913–14[53] and 1917,[54] by Miss Cummings[55] in 1921, and by M. Gussow[56] in 1925. The normal points derived by the latter have been plotted in Figure 1. Previously determined light curves are similar to that obtained in 1925 except that the small secondary variations were found to lie on the *ascending* as well as the *descending* branch. The amplitudes of light curves obtained photo-electrically have ranged from 0.042 magnitude in 1925 to 0.063 magnitude in 1914.

The following conclusions may be drawn from the light curves shown in Figures 13 and 14. (1) The light curve of β *Cephei* determined for the short wave-lengths in August and September 1928, differs in form from that in the photographic region obtained by earlier investigators. (2) The amplitude of variation in the ultra-violet is found to be at least 50% greater than in the region measured with the photo-electric cell. (3) There is some indication of a progressive lag in the phase of the maximum point with decreasing wave-length, shown by measures of λ3900, λ3600, and λ3300A.

The probable error of a single observation obtained by the spectro-photometric method is apparently about seven times greater than that given for the Lick photo-electric measures of β *Cephei*,[57] but a single observation in the latter case represents 20 settings, or between 8 and 10 independent comparisons of intensity. Since the precision of the mean increases as the square root of the number of observations, the probable error of a single comparison would be roughly 3 times greater than for a "single observation" or about ±0.036 magnitude. Hence, the precision of the photo-electric method is little more than twice as great as that of the new method. There is no doubt that improvement of the simple technique of observation with the spectrophotometric method at various points suggested by experience will result in measures that rival in accuracy those obtained with the photo-electric cell.

The chief advantages of the new method are its accuracy, the fact that it affords a method of studying light variations in limited and definite spectral regions, including the extreme ultra-violet which is otherwise inaccessible, and the permanency of the record.

## SUMMARY OF RESULTS

(1) A mean radial velocity curve has been determined from measures of 95 Mills III-prism spectrograms of β *Cephei* obtained in 1926 and 1927. The mean square error for the individual measures of these plates is about ±1.5 km./sec. The following orbital elements were estimated from the velocity curve shown in Figure 1:

$$P = 0.1904795 \text{ (assumed)}$$
$$\gamma = -5.14 \text{ km./sec.}$$
$$K = -15.40 \text{ km./sec.}$$
$$e = 0.045$$
$$\omega = 274°$$
$$T = \text{J. D. } 2424796.832$$
$$a \sin i = 39000 \text{ km.}$$

The indentations in the curve near the maximum and minimum points are considered to be real.

[53] *Berlin-Babelsberg* **1**, 23, 1914.
[54] *Berlin-Babelsberg* **2**, 78, 1918.
[55] *L. O. B.* **11**, 115, 1923–24.
[56] *A. N.* 5484, 201, 1925.
[57] *L. O. B.* **11**, 115, 1923–24.

(2) The variation in the shape of the velocity curves derived from different absorption lines, or combinations of lines, is small and of doubtful reality.

(3) A comparison of the orbital elements derived in the interval 1906–1928 indicates that all of them vary in a more or less regular manner over a period of years. The general character of the observed variation of the period, $P$, and of $K$, the semi-amplitude of the velocity curves, are undoubtedly real. Variability in the velocity of the system, $\gamma$, appears to have been established. Comparatively large uncertainties in the determination of the values of $e$ and $\omega$ tend to mask the nature of the true variation in these elements but some regularity seems to exist in the variation of $\omega$.

Interpretation of the observed variation of $\gamma$ has been made in terms of orbital motion of the bright star as a component of a binary system, and a possible connection between this orbital motion and the variation of the amplitude for pulsation of the star's atmosphere has been suggested.

(4) A new method of photometric observation, consisting in the measurement, by means of the Moll recording microphotometer, of the photographic blackening in a series of slitless quartz-prism spectrograms, has been applied to $\beta$ *Cephei*. Light curves have been obtained for the three spectral regions λλ3900, 3600, and 3300A, which show that the amplitude of light variation in the ultra-violet region is about 50% greater than that found for the photographic region at λ4500A.


## Acknowledgments

It is a pleasure to acknowledge my indebtedness to Director W. W. Campbell and Associate Director R. G. Aitken for the opportunity of using the observatory equipment in this investigation. I am, moreover, greatly indebted to the various members and former members of the Lick Observatory personnel and to the members of the Students' Observatory staff who have aided in securing a number of the observations and offered helpful criticism. In particular, to Dr. J. H. Moore, who suggested and outlined the problems, and to Dr. C. D. Shane is extended my most sincere appreciation of the valuable suggestions received from them and patient interest shown by them throughout the course of this study. Finally, this work was made possible, in part, by a grant of the Lick Observatory Fellowship in Astronomy in the University of California.



Lick Observatory,
October, 1929.



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TUCSON, ARIZONA

# LICK OBSERVATORY BULLETIN

NUMBER 419

## ELEMENTS AND EPHEMERIS OF COMET *d* 1929 (WILK)

BY

ERNEST CLARE BOWER AND FRED L. WHIPPLE

Comet *d* 1929 was discovered by Wilk at Cracow, Poland on 1929, Dec. 20, $17^h\ 45^m0$ U. T. The magnitude has been estimated by various observers from 7 to 9. Van Biesbroeck described the comet on December 22 as having a slender tail, extending 20′ in a direction opposite the Sun, without sharp nucleus.

The following observations have been received:

| | 1929–1930 U. T. | α | δ | Observer | Place | Authority |
|---|---|---|---|---|---|---|
| (1) | Dec. 21.7306 | 18 20 15.7 | +35 23 30 | Wolf | Heidelberg | Telegram, H. A. C. No. 101, 1929 |
| (2) | Dec. 22.00154 | 18 23 26.16 | +35 06 55.0 | Van Biesbroeck, Williams Bay; 24″, phot. | | Telegram, H. A. C. No. 101, 1929 |
| (3) | Dec. 22.13244 | 18 24 57.68 | +34 58 43.5 | Jeffers | Mount Hamilton | Telephone, Letter |
| (4) | Dec. 22.71375 | 18 31 42.2 | +34 21 25 | Rydka | Warsaw | B. C. A. Circulaire No. 237, 1929 |
| (5) | Dec. 23.10889 | 18 36 14.38 | +33 55 20.5 | Jeffers | Mount Hamilton | Telegram, H. A. C. No. 101, 1929 |
| (6) | Dec. 23.12866 | 18 36 28.00 | +33 54 01.0 | Jeffers | Mount Hamilton | Telephone, Letter |
| (7) | Dec. 24.09933 | 18 47 24.06 | +32 46 48.3 | Jeffers | Mount Hamilton; 12″, visual | Telephone, H. A. C. No. 102, 1929 |
| (8) | Dec. 26.02271 | 19 08 11.10 | +30 23 14.4 | Van Biesbroeck, Williams Bay; 24″, phot. | | Letter |
| (9) | Dec. 27.00832 | 19 18 18.72 | +29 05 16.1 | Van Biesbroeck, Williams Bay; 24″, phot. | | Letter |
| (10) | Dec. 30.01890 | 19 46 54.12 | +24 54 56.9 | Van Biesbroeck, Williams Bay; 24″, phot. | | Letter |
| (11) | Dec. 31.00102 | 19 55 26.51 | +23 31 02.0 | Van Biesbroeck, Williams Bay; 24″, phot. | | Letter, corrected by telegram |
| (12) | Jan. 2.98876 | 20 19 09.07 | +19 15 12.7 | Van Biesbroeck, Williams Bay; 24″, phot. | | Letter |
| (13) | Jan. 4.01480 | 20 26 30.26 | +17 48 14.8 | Van Biesbroeck, Williams Bay; 40″, visual | | Letter |
| (14) | Jan. 4.98884 | 20 33 08.31 | +16 26 50.1 | Van Biesbroeck, Williams Bay; 24″, phot. | | Letter |
| (15) | Jan. 5.99184 | 20 39 37.88 | +15 03 58.0 | Van Biesbroeck, Williams Bay; 24″, phot. | | Letter |
| (16) | Jan. 7.98733 | 20 51 35.40 | +12 23 21.8 | Van Biesbroeck, Williams Bay; 24″, phot. | | Letter, corrected by telegram |

Observations (1), (2), (3), (5) and (6) were available for a preliminary orbit, which was based on (1), (2) and (6). The residuals of a direct solution without correction by Leuschner's Method were:

| | Δα | Δδ |
|---|---|---|
| (1) | −0″.6 | + 0″.2 |
| (2) | 0.0 | 0.0 |
| (5) | +7.9 | −12.8 |
| (6) | +9.3 | −10.9 |

ELEMENTS, 1929.0

$T$ = 1930 Jan. 22.282 U. T.
$\omega$ = 157° 20′
$\Omega$ = 178 58
$i$ = 124 28
$q$ = 0.6736

This orbit and an ephemeris were telegraphed to Harvard College Observatory and distributed on *H. A. C.* No. 102.

For a second orbit, observations (1) to (11) were available. The preliminary orbit was transferred to (9) as middle place and differentially corrected to represent (1) as first and (11) as third places.

The run of the residuals of all available observations seemed to show that the right ascension of (11) as originally received by letter was in error by about 2′. Accordingly, (10) was adopted as third place instead of (11). A single differential correction, on the parabolic hypothesis, gave:

ELEMENTS, 1930.0

$T = 1930$ Jan. 22.2914
$\omega = 157° 26' 44''.7$
$\Omega = 178\ 58\ 29.4$
$i = 124\ 30\ 56.1$
$q = 0.672436$

$$x = +0.623522\ (1-\tan^2\tfrac{v}{2}) + 0.503161\ \tan\tfrac{v}{2}$$

$$y = +0.039295\ (1-\tan^2\tfrac{v}{2}) - 0.246782\ \tan\tfrac{v}{2}$$

$$z = +0.248691\ (1-\tan^2\tfrac{v}{2}) - 1.222543\ \tan\tfrac{v}{2}$$

The observations are represented as follows:

| | 1929–1930 U. T. | $\Delta\alpha$ | $\Delta\delta$ |
|---|---|---|---|
| (1)* | Dec. 21.73 | 0″ | −6″ |
| (2) | Dec. 22.00 | −2.8 | −3.0 |
| (3) | Dec. 22.13 | −5.2 | −4.3 |
| (4) | Dec. 22.71 | −7 | −8 |
| (5) | Dec. 23.10 | −1.1 | −3.8 |
| (6) | Dec. 23.12 | −0.4 | −2.8 |
| (7) | Dec. 24.09 | −5.6 | −3.0 |
| (8) | Dec. 26.02 | +2.5 | −0.6 |
| (9)* | Dec. 27.00 | 0.0 | 0.0 |
| (10)* | Dec. 30.01 | +0.3 | +0.2 |
| (11) | Dec. 31.00 | −2.4 | +1.2 |
| (12) | Jan. 2.98 | −0.6 | +1.0 |
| (13) | Jan. 4.01 | +0.6 | −2.0 |
| (14) | Jan. 4.98 | +1.0 | +7.0 |
| (15) | Jan. 5.99 | +1.4 | +4.2 |
| (16) | Jan. 7.98 | +4.3 | +3.0 |

* Observations on which the differential correction was based.

Observations (12) to (16) were received after completion of the orbit. In view of the diffuseness of the comet, the outstanding residual of only 6″ in the first declination would not justify at this time the further improvement of the orbit without hypothesis concerning the eccentricity.

EPHEMERIS

| 1930 U. T. | $\alpha$(1930.0) | $\delta$(1930.0) | $\rho$ | Light |
|---|---|---|---|---|
| Jan. 16 | $21^h\ 28^m\ 53^s$ | + 2° 44′.6 | +1.2164 | 1.00 |
| 20 | 21 42 34 | − 1 24.2 | +1.3049 | 0.92 |
| 24 | 21 53 54 | − 5 09.5 | +1.3922 | 0.81 |
| 28 | 22 03 27 | − 8 34.2 | +1.4755 | 0.69 |
| Feb. 1 | 22 11 39 | −11 41.0 | +1.5528 | 0.59 |
| 5 | 22 18 53 | −14 32.0 | +1.6230 | 0.50 |
| 9 | 22 25 25 | −17 9.5 | +1.6853 | 0.42 |
| 13 | 22 31 30 | −19 35.5 | +1.7396 | 0.35 |
| 17 | 22 37 18 | −21 51.8 | +1.7863 | 0.30 |
| 21 | 22 42 56 | −24 00.2 | +1.8255 | 0.26 |
| 25 | 22 48 30 | −26 02.3 | +1.8579 | 0.23 |
| Mar. 1 | 22 54 06 | −27 59.5 | +1.8839 | 0.19 |
| Mar. 1 | $22^h\ 54^m\ 06^s$ | −27° 59′.5 | +1.8839 | 0.190 |
| 9 | 23 05 30 | −31 44.9 | +1.9186 | 0.149 |
| 17 | 23 17 29 | −35 25.4 | +1.9342 | 0.122 |
| 25 | 23 30 22 | −39 08.5 | +1.9349 | 0.102 |
| Apr. 2 | 23 44 27 | −43 00.1 | +1.9247 | 0.087 |
| 10 | 00 00 07 | −47 04.8 | +1.9084 | 0.075 |
| 18 | 00 17 59 | −51 24.9 | +1.8904 | 0.066 |
| 26 | 00 39 00 | −56 00.5 | +1.8756 | 0.059 |
| May 4 | 01 04 44 | −60 48.0 | +1.8686 | 0.053 |

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# PRELIMINARY RESULTS ON THE DISTANCES, DIMENSIONS AND SPACE DISTRIBUTION OF OPEN STAR CLUSTERS

BY

ROBERT J. TRUMPLER

Although the observations of magnitudes and spectral types in open star clusters of the Milky Way undertaken by the writer are still far from being complete, it seemed of interest to utilize the data at present available for a preliminary investigation of the distances and diameters of these clusters and for a study of their space distribution.

## 1. Determination of the Distances of Clusters from Magnitudes and Spectral Types

The dimensions of most clusters are small compared with their distance from us. In any particular cluster we may thus assume that its members are at the same distance and that their absolute magnitudes $M$ differ from their apparent magnitudes $m$ by a constant:

$$m - M = 5 \log r - 5, \qquad (1)$$

where $r$ is the distance of the cluster in parsecs. Plotting the cluster members according to their apparent magnitudes and spectral types we obtain a diagram similar to the Hertzsprung-Russell diagram of giant and dwarf stars. Although the magnitude-spectral class diagrams of individual clusters vary considerably and although they are incomplete for the fainter stars, it is nearly always possible to decide which stars of a cluster belong to the giant or to the dwarf branch. In most clusters, moreover, only B and A type stars are observable and for these such separation is not necessary. If we assign to each cluster star the mean absolute magnitude corresponding to its spectral class and subtract it from the apparent magnitude, we obtain the distance $r$ from formula (1).

The mean absolute magnitudes of the spectral subdivisions have been determined by various observers from trigonometric and moving cluster parallaxes or statistically from proper-motions. The values used are given in Table 1 and are mainly based on the determinations of Adams & Joy[1], Lundmark[2], Malmquist[3], and Hess[4]:

TABLE 1

Adopted Mean Absolute Magnitudes for Spectral Types

| Spectral type | Mean absolute magnitude | | | |
|---|---|---|---|---|
| | Dwarf branch | | Giants | |
| | Vis. | Phtgr. | Vis. | Phtgr. |
| O | −4.0 | −4.3 | | |
| B0 | −3.1 | −3.4 | | |
| B1 | −2.5 | −2.8 | | |
| B2 | −1.8 | −2.1 | | |
| B3 | −1.2 | −1.4 | | |
| B5 | −0.8 | −1.0 | | |
| B8 | −0.2 | −0.3 | | |
| B9 | +0.3 | +0.3 | | |
| A0 | +0.9 | +0.9 | | |
| A2 | +1.7 | +1.7 | | |
| A3 | +2.0 | +2.1 | | |
| A5 | +2.3 | +2.5 | | |
| F0 | +2.9 | +3.2 | +0.5 | +0.9 |
| F2 | +3.2 | +3.5 | | |
| F5 | +3.6 | +4.0 | +0.5 | +1.0 |
| F8 | +4.2 | +4.7 | | |
| G0 | +4.5 | +5.1 | +0.5 | +1.2 |
| G5 | +5.0 | +5.7 | +0.5 | +1.4 |
| K0 | +6.2 | +7.0 | +0.5 | +1.6 |

[1] *Mount Wilson Contr.* Nos. 199, 244, 262.
[2] *Publ. A. S. P.* **34**, 150, 1922.
[3] *Meddel. Lund*, Ser. II, No. 32, 1924.
[4] *Seeliger Festschrift*, p. 265.

The application of this method for determining distances of clusters requires a knowledge of the spectral types and magnitudes for a number of stars in each cluster. Spectroscopic observations have so far been obtained for 57 clusters either with the 1-prism slit spectrograph attached to the 36-inch refractor for the brighter stars, or with the slitless quartz spectrograph attached to the Crossley reflector, or with both. In 43 other clusters, mostly situated too far south for observation from Mount Hamilton, the spectral types of some of the brighter stars are found in the *Henry Draper Catalogue*. This makes a total of 100 clusters for which spectroscopic data are available.

The photographic magnitudes of the stars in each cluster are being determined from photographs taken with a 4-inch Ross objective, comparing each cluster with the North Polar sequence as well as with several other clusters. From the comparisons so far measured and reduced preliminary results for the magnitudes in 38 clusters are available, most of which are based on the North Polar comparison alone. For 16 others we have accurate magnitude determinations by other observers, and in the remaining 46 clusters the magnitudes had to be taken from the *Henry Draper Catalogue*. As it is not possible *à priori* to decide which stars are physical members of the cluster and which are background stars, only the stars in the central part of each cluster, where the percentage of background stars is smallest, were used, and a few stars which did not fit into the giant or dwarf branch of the magnitude-spectral class diagram were omitted. As an illustration of the procedure followed, Table 2 gives the determination of the distance for the cluster N. G. C. 1960 (Messier 36). The first column contains the number of each star according to Hopmann's Catalog[5], the second the apparent photographic magnitude $m$ (mean of Hopmann and Wallenquist[6]), the third the spectral type. The 13 brightest stars were observed with the 1-prism slit spectrograph, the others were classified on plates taken with the slitless quartz spectrograph. In the fourth column are found the mean absolute magnitudes $M$ corresponding to the spectral type, taken from Table 1, and in the 5th, the differences $m$-$M$, which should be the same for all cluster members except for errors of observation and for the natural dispersion in absolute magnitude of the stars of a given spectral type. The 10 brightest stars (8.5–9.5) give a considerably smaller value for $m$-$M$ than the 30 fainter ones. This fact is noted in most clusters in which the observations cover a large magnitude range. It indicates that the brightest cluster members as a rule are of abnormally high luminosity for their spectral class[7]. Some effect of this kind must, of course, be expected as a consequence of the natural luminosity dispersion in any particular spectral class. In Messier 36, for example, B3 and B4 are the spectral types of highest average luminosity and any stars of these types which are more luminous than the average must necessarily be among the

TABLE 2

DETERMINATION OF THE DISTANCE OF MESSIER 36

| Star | App. phtg. magn. $m$ | Spectr. type | Absolute magn. $M$ | $m$–$M$ | Corr. | Corr. $m$–$M$ | Resid. |
|---|---|---|---|---|---|---|---|
| 208a | 8.5 | B3s | −1.4 | 9.9 | +1.0 | 10.9 | −.2 |
| 258 | 8.6 | B3s | −1.4 | 10.0 | +1.0 | 11.0 | −.1 |
| 313 | 8.8 | B3 | −1.4 | 10.2 | +1.0 | 11.2 | +.1 |
| 350 | 8.9 | B4 | −1.2 | 10.1 | +1.0 | 11.1 | .0 |
| 365 | 9.0 | B4e | −1.2 | 10.2 | +1.0 | 11.2 | +.1 |
| 250 | 9.3 | B3 | −1.4 | 10.7 | + .5 | 11.2 | +.1 |
| 162 | 9.3 | B4 | −1.2 | 10.5 | + .5 | 11.0 | −.1 |
| 294 | 9.4 | B5n | −1.0 | 10.4 | + .5 | 10.9 | −.2 |
| 249 | 9.5 | B3n | −1.4 | 10.9 | + .5 | 11.4 | +.3 |
| 238 | 9.5 | B4s | −1.2 | 10.7 | + .5 | 11.3 | +.2 |
| 271 | 9.7 | B4s | −1.2 | 10.9 | | | −.2 |
| 120 | 9.8 | B4 | −1.2 | 11.0 | | | −.1 |
| 173 | 10.1 | B7n | − .5 | 10.6 | | | −.5 |
| 156 | 10.4 | B8 | − .3 | 10.7 | | | −.4 |
| 132 | 10.5 | B9 | + .3 | 10.2 | | | −.9 |
| 130 | 10.5 | B9 | + .3 | 10.2 | | | −.9 |
| 101 | 10.6 | B8 | − .3 | 10.9 | | | −.2 |
| 197 | 10.7 | B8 | − .3 | 11.0 | | | −.1 |
| 189 | 10.7 | B6 | − .8 | 11.5 | | | +.4 |
| 206 | 10.7 | B8 | − .3 | 11.0 | | | −.1 |
| 150 | 10.8 | B8 | − .3 | 11.1 | | | .0 |
| 241 | 10.8 | B8 | − .3 | 11.1 | | | .0 |
| 186 | 11.3 | B9 | + .3 | 11.0 | | | −.1 |
| 118 | 11.3 | B9 | + .3 | 11.0 | | | −.1 |
| 229 | 11.5 | B9 | + .3 | 11.2 | | | +.1 |
| 236 | 11.6 | B9 | + .3 | 11.3 | | | +.2 |
| 297 | 11.7 | B9 | + .3 | 11.4 | | | +.3 |
| 336 | 11.7 | B9 | + .3 | 11.4 | | | +.3 |
| 214 | 11.9 | A0 | + .9 | 11.0 | | | −.1 |
| 246 | 11.9 | B9 | + .3 | 11.6 | | | +.5 |
| 306 | 12.0 | A0 | + .9 | 11.1 | | | .0 |
| 225 | 12.2 | A0 | + .9 | 11.3 | | | +.2 |
| 324 | 12.2 | A1 | +1.3 | 10.9 | | | −.2 |
| 199 | 12.3 | A2 | +1.7 | 10.6 | | | −.5 |
| 310 | 12.3 | B9 | + .3 | 12.0 | | | +.9 |
| 233 | 12.3 | A0 | + .9 | 11.4 | | | +.3 |
| 232 | 12.5 | A2 | +1.7 | 10.8 | | | −.3 |
| 286 | 12.7 | A1 | +1.3 | 11.4 | | | +.3 |
| 305 | 12.7 | A0 | + .9 | 11.8 | | | +.7 |
| 235 | 13.2 | A0 | + .9 | 12.3 | | | +1.2 |
| Mean of 10 brightest stars | | | | 10.36 | | 11.12 | |
| Mean of 30 fainter stars | | | | 11.12 | | 11.12 | |

Mean of all stars: $m-M=11.1=5\log r-5$ ($p.\ e.\pm^m2$)

Distance $r=1660$ parsecs ($p.\ e.\pm10\%$)

[5] *Veröffentl. Bonn*, No. 19, 1924.

[6] *Meddel. Upsala*, No. 32, 1927.

[7] See *Publ. A. S. P.* **40**, 266, 1928.

brightest stars of the cluster. Such selection, however, accounts only for a part of the phenomenon referred to. Suppose we determine the distance of Messier 36 from the 30 fainter stars alone, and use this distance to compute the absolute magnitudes of the brighter stars. We find then that all stars of spectral types B3 and B4 in Messier 36 are on the average $0^m.8$ (resp. $0^m.5$) more luminous than the adopted mean magnitudes of Table 1. The upper end of the dwarf branch between B6 and B3 rises more sharply.

In different clusters this rise is not associated with any particular spectral type, otherwise it would require correction of the adopted mean absolute magnitudes of Table 1. But the rise nearly always occurs at the hottest spectral types present in the cluster or among its brightest members. In the cluster Messier 39 which does not contain any stars of hotter spectral type than A0, the rise takes place between A2 and A0, in the *Pleiades* between B9 and B5.

The abnormally high luminosity of the brightest cluster members is probably due to large mass; it was corrected for empirically by the addition of $+1^m.0$ to the $m$-$M$ for the stars in the first half-magnitude interval and of $+0^m.5$ for the stars in the second half-magnitude interval. These corrections were applied for all clusters which contain mainly stars belonging to the dwarf branch; their omission would be a serious source of error in cases where only a few of the brightest cluster stars are available for the determination of the distance. At the fainter magnitude limit to which the observations extend, the selection due to dispersion in absolute magnitude was neglected. As this dispersion seems to be small in the dwarf branch, and as the observations generally cover a considerable magnitude range, the error thus committed will be small and is partly compensated by the correction applied to the brightest stars.

The mean of the corrected values of $m$-$M$ is then taken and the distance in parsecs is computed according to formula (1). When many stars covering a considerable magnitude interval were observed, as in Messier 36, the probable error of the mean $m$-$M$ should be of the order of $\pm 0^m.2$; this includes the uncertainty of the magnitude scale and of the adopted mean absolute magnitudes (Table 1). It corresponds to a *p. e.* of $\pm 10\%$ in the distance.

In Table 3 are collected the data referring to the determination of the distances and linear diameters of

TABLE 3

DISTANCES AND LINEAR DIAMETERS OF OPEN STAR CLUSTERS

| NGC or I. C. | Other designations | Magnitude sp. class diagr. | Magnitude interval | Spectral types | | $m$-$M$ | Distance in parsecs | | | Diameter in parsecs | | $v'=\lg\frac{D'}{C'}$ | $v''=\lg\frac{D''}{C''}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | From sp. types | | from diam. | | | | |
| | | | | Dwarf br. | $G$ | | Obs. | Corr. | | $D'$ | $D''$ | | |
| 436 | | 1–2b | $11^m.3$–$12^m.2$ | 5 B5–B8 | | 12.4 | 3000 | 1650 | 2290 | 5.2 | 2.9 | −.02 | −.14 |
| 457 | | 1b | 9.7–13.1 | 29 B2–A0 | | 12.5 | 3200 | 1700 | 1260 | 11 | 6.0 | +.25 | +.13 |
| 581 | M103 | 1–2b | 9.2–14.4 | 25 B3–A2 | | 12.3 | 2900 | 1600 | 2320 | 5.5 | 3.0 | −.02 | −.15 |
| 663 | | 1b | 8.9–13.7 | 42 B1–A0 | | 12.7 | 3500 | 1800 | 2540 | 14 | 7.4 | −.01 | −.13 |
| 752 | | 2f | 8.8–12.2 | 39 F0–F8 | 12 | 7.9 | 380 | 340 | 450 | 5.0 | 4.5 | −.20 | −.12 |
| 869 | $h$ Per | 1b | 7.0–13.3 | 62 B1–A0 | | 11.4 | 2200 | 1350 | 1300 | 19 | 12 | +.05 | +.01 |
| 884 | $\chi$ Per | 1–2b | 8.3–13.7 | 40 B2–A0 | | 11.9 | 2200 | 1350 | 1300 | 19 | 12 | +.05 | +.01 |
| I. C. 1805 | | 1–2o | 8.5–13.8 | 33 O6–A1 | | 12.8 | 3600 | 1850 | 1770 | 21 | 11 | +.15 | +.02 |
| ..... | An. 2 | 2a | H. D. | 5 A0–A5 | | *8.9* | *600* | *750* | 650 | *3.1* | *3.9* | −.*14* | +.*07* |
| 1027 | | 1–2b | 9.8–13.7 | 32 B3–A2 | | 12.1 | 2600 | 1500 | 1690 | 16 | 9.1 | +.03 | −.05 |
| 1039 | M34 | 1b–a | 8.0–13.7 | 64 B8–G0 | | 8.6 | 520 | 440 | 460 | 4.5 | 3.8 | −.09 | −.02 |
| ..... | Perseus | 1–2b | 3.0– 7.0 | 31 B1–B9 | | 6.3 | 180 | 170 | (148) | 13 | 12 | −.06 | +.07 |
| ..... | Pleiades | 1b | 2.8–11.5 | 127 B5–K0 | | 6.0 | 160 | 150 | (137) | 5.6 | 5.3 | −.06 | +.04 |
| 1502 | | 1b | 7.3–13.9 | 27 B0–A2 | | 12.0 | 2500 | 1450 | 1460 | 5.8 | 3.4 | +.13 | +.01 |
| 1528 | | 1–2b–a | 10.6–14.1 | 21 B8–A5 | | 11.1 | 1650 | 1100 | 690 | 10 | 7.1 | +.26 | +.21 |
| ..... | Taurus | 2a | 3.6– 8.1 | 63 A2–G7 | 4 | 2.9 | 37 | 37 | (38) | 4.3 | 4.3 | −.13 | −.01 |
| 1647 | | 1b–a | 8.8–12.7 | 31 B7–A7 | | 9.5 | 800 | 630 | 580 | 8.1 | 6.4 | +.01 | +.04 |
| 1662 | | 2a | H. D. | 5 A0–A2 | | *9.3* | *730* | *860* | 830 | *3.0* | *3.5* | −.*15* | +.*04* |
| 1746 | | 2b–a | 8.0–13.2 | 27 B5–A5 | 6 | 10.3 | 1150 | 850 | 890 | 13 | 9.9 | −.06 | −.02 |
| 1807 | | 2a | H. D. | 3 A | 3 | *8.6* | *530* | *670* | 740 | *2.2* | *2.8* | −.*27* | −.*03* |
| 1912 | M38 | 2b–a | 8.3–11.7 | 27 B5–A4 | 5 | 10.2 | 1100 | 820 | 920 | 5.8 | 4.3 | −.05 | −.06 |
| 1960 | M36 | 1b | 8.5–13.2 | 40 B3–A2 | | 11.1 | 1650 | 1100 | 860 | 7.7 | 5.1 | +.15 | +.11 |
| 1981 | | 1b | H. D. | 11 B0–A0 | | 8.9 | 600 | 500 | 470 | 4.4 | 3.6 | +.01 | +.04 |
| 2099 | M37 | 2a | 10.9–12.4 | 22 B9–A5 | 15 | 10.8 | 1450 | 1000 | 630 | 10 | 6.9 | +.20 | +.19 |
| 2168 | M35 | 1–2b | 8.0–12.3 | 34 B4–A2 | | 10.5 | 1250 | 910 | 770 | 11 | 7.7 | +.09 | +.08 |
| 2244 | 12 Mon | 1–2o | 6.6–13.8 | 30 O5–A2 | | 11.7 | 2200 | 1350 | 1320 | 17 | 10 | +.06 | +.01 |
| 2264 | S Mon | 1o | 4.7– 8.7 | 9 O8–B8 | | 8.9 | 600 | 500 | 390 | 5.2 | 4.4 | +.09 | +.12 |
| 2281 | | *1a* | H. D. | 4 A0 | | *9.0* | *630* | *780* | 690 | *2.8* | *3.4* | −.*16* | +.*05* |

TABLE 3—(*Continued*)

| NGC or I. C. | Other designations | Magnitude sp. class diagr. | Magnitude interval | Spectral types | | $m-M$ | Distance in parsecs: From sp. types Obs. | From sp. types Corr. | Distance in parsecs: from diam. | Diameter in parsecs $D'$ | Diameter in parsecs $D''$ | $v'=\lg\frac{D'}{C'}$ | $v''=\lg\frac{D''}{C''}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Dwarf br. | *G* | | | | | | | | |
| 2287 | M41 | 2a | H. D. | 20 A0 | 4 | 8.0 | 400 | 350 | 470 | 3.7 | 3.2 | −.22 | −.14 |
| 2323 | M50 | 1b–a | H. D. | 7 B8–A0 | | *9.0* | *630* | *780* | 860 | *3.0* | *3.6* | *−.26* | *−.04* |
| 2353 | | 1b | H. D. | 7 B1–B9 | | *9.4* | *760* | *890* | 690 | *4.4* | *5.2* | *−.10* | *+.12* |
| 2362 | τ C Ma | 1o | H. D. | 3 O8–B8 | | *9.1* | *660* | *800* | 1470 | *1.3* | *1.6* | *−.50* | *−.27* |
| 2422 | | 1–2b | 5.4–13.5 | 35 B3–F8 | | 8.7 | 550 | 460 | 500 | 4.8 | 4.0 | −.08 | −.03 |
| 2437 | M46 | 1a | 10.1–11.2 | 11 B9–A1 | | 9.8 | 910 | 710 | 610 | 7.2 | 5.6 | +.05 | +.06 |
| 2447 | | *2a* | H. D. | 3 B9–A0 | 1 | *9.3* | *730* | *860* | 840 | *3.8* | *4.5* | *−.22* | *+.01* |
| 2451 | | 1–2b | H. D. | 11 B3–A0 | | 7.5 | 320 | 290 | 280 | 3.5 | 3.1 | −.07 | +.01 |
| 2516 | | 1–2b | H. D. | 24 B3–A3 | | 7.7 | 350 | 310 | 300 | 5.1 | 4.5 | −.08 | .00 |
| 2539 | | 1–2a | 9.3–13.6 | 28 A1–F2 | 6 | 10.1 | 1050 | 790 | 690 | 6.7 | 5.1 | +.07 | +.07 |
| 2546 | | 1b | H. D. | 16 B0–A0 | | 8.9 | 600 | 500 | 340 | 7.8 | 6.5 | +.11 | +.16 |
| 2547 | | 1b | H. D. | 9 B3–A2 | | *8.7* | *550* | *690* | 690 | *2.7* | *3.4* | *−.20* | *.00* |
| 2548 | | 1–2a | 8.4–11.5 | 29 A0–A5 | 3 | 8.6 | 530 | 440 | 500 | 4.7 | 3.9 | −.12 | −.06 |
| 2632 | Praesepe | 2a | 6.3–10.7 | 53 B9–G2 | 9 | 6.0 | 160 | 150 | (168) | 4.2 | 4.0 | −.17 | −.09 |
| I. C. 2391 | | 1b | H. D. | 15 B3–F2 | | 6.2 | 175 | 160 | 260 | 2.3 | 2.2 | −.27 | −.20 |
| I. C. 2395 | | 1b | H. D. | 14 B3–A2 | | 8.5 | 500 | 430 | 580 | 2.9 | 2.5 | −.17 | −.13 |
| ..... | An. 10 | 1b–a | H. D. | 12 B8–A0 | | 7.7 | 350 | 310 | 390 | 3.1 | 2.7 | −.14 | −.05 |
| 2682 | M67 | 2–3a | 10.4–14.5 | 74 B9–G0 | 12 | 9.2 | 690 | 560 | 920 | 3.6 | 2.9 | −.25 | −.22 |
| 3114 | | 2a | H. D. | 30 B9–A0 | 9 | 8.6 | 530 | 440 | 450 | 5.7 | 4.8 | −.05 | −.01 |
| 3228 | | 1–2b–a | H. D. | 12 B8–A0 | 1 | 8.8 | 580 | 480 | 520 | 3.4 | 2.8 | −.08 | −.03 |
| 3293 | | 1b | H. D. | 3 B0 | | *11.7* | *2200* | *1850* | 2060 | *5.1* | *4.3* | *−.10* | *−.09* |
| I. C. 2602 | | 1–2b | H. D. | 22 B0–A2 | | 6.6 | 210 | 195 | 230 | 4.0 | 3.7 | −.16 | −.07 |
| 3532 | | 2b–a | H. D. | 117 B5–A3 | 12 | 8.3 | 460 | 400 | 300 | 7.3 | 6.3 | +.05 | +.11 |
| 3766 | | 1 b | H. D. | 5 B0–B8 | | *10.1* | *1050* | *1150* | 1260 | *3.7* | *4.0* | *−.22* | *−.05* |
| ..... | Coma Ber. | 2a | 4.8– 9.5 | 40 A0–G2 | 2 | 4.6 | 83 | 81 | (39) | 7.2 | 7.1 | +.23 | +.32 |
| 4755 | κ Cru | *1–2b* | H. D. | 3 B2–B3 | | *8.9* | *600* | *750* | 1260 | *2.1* | *2.6* | *−.47* | *−.23* |
| 5316 | | *1–2b–a* | H. D. | 2 B8–B9 | 2 | *8.8* | *580* | *720* | 1060 | *1.9* | *2.3* | *−.35* | *−.15* |
| 5460 | | 1–2b–a | H. D. | 12 B8–A0 | | 8.9 | 600 | 500 | 430 | 6.1 | 5.1 | +.03 | +.07 |
| 5617 | | *2a* | H. D. | 1 A0 | 4 | *9.3* | *730* | *860* | 1080 | *3.0* | *3.5* | *−.31* | *−.10* |
| 5662 | | 1–2b–a | H. D. | 11 B8–A0 | | 9.2 | 690 | 560 | 780 | 3.0 | 2.5 | −.15 | −.13 |
| 5822 | | 1–2b–a | H. D. | 9 B8–A0 | 3 | 9.4 | 760 | 610 | 510 | 8.8 | 7.0 | +.04 | +.08 |
| 6025 | | 1b | H. D. | 5 B3–B8 | | *9.5* | *800* | *930* | 1060 | *2.6* | *3.0* | *−.22* | *−.04* |
| 6087 | | 1–2b | H. D. | 9 B5–A0 | | *9.5* | *800* | *930* | 840 | *4.2* | *4.9* | *−.14* | *+.05* |
| 6124 | | 2b–a | H. D. | 7 B8–A0 | 5 | 9.5 | 800 | 630 | 600 | 5.8 | 4.6 | −.04 | +.01 |
| 6231 | | 1o | 5.9– 8.0 | 9 Oa–B8 | | *9.9* | *960* | *1050* | 940 | *4.5* | *4.9* | *−.15* | *+.04* |
| ..... | An. 24 | 1o | H. D. | 25 O5–B8 | | 9.3 | 730 | 580 | 590 | 13 | 10 | −.06 | +.01 |
| 6242 | | 1–2b | H. D. | 4 B5–B8 | | *10.7* | *1400* | *1350* | 1250 | *4.5* | *4.4* | *−.09* | *+.04* |
| 6322 | | 1b | H. D. | 3 B3–B5 | | *10.0* | *1000* | *1100* | 1300 | *2.6* | *2.9* | *−.22* | *−.06* |
| 6405 | M6 | 1–2b | 6.8– 9.7 | 37 B5–A3 | | 8.7 | 550 | 460 | 580 | 4.2 | 3.5 | −.14 | −.09 |
| 6416 | | 2a | H. D. | 2 A0 | 2 | *9.1* | *660* | *800* | 700 | *4.3* | *5.1* | *−.14* | *+.06* |
| I. C. 4665 | | 1–2b | 6.4–11.4 | 22 B4–F5 | | 7.8 | 360 | 320 | 230 | 5.2 | 4.7 | +.09 | +.15 |
| 6475 | M7 | 1b | 6.0– 9.5 | 30 B5–A3 | | 7.1 | 260 | 240 | 275 | 3.8 | 3.5 | −.16 | −.06 |
| 6494 | M23 | 2a | 9.5–13.6 | 40 B9–F0 | 3 | 10.0 | 1000 | 760 | 560 | 7.8 | 5.9 | +.09 | +.12 |
| 6530 | | 1o | 6.0–12.9 | 25 O5–A1 | | 11.0 | 1600 | 1100 | 1080 | 6.5 | 4.4 | +.05 | .00 |
| 6531 | M21 | 1b | 7.1–12.6 | 34 B0–A2 | | 11.0 | 1600 | 1100 | 860 | 5.6 | 3.7 | +.13 | +.09 |
| 6611 | M16 | 1o | 8.6–13.3 | 24 O7–B8 | | 13.4 | 4800 | 2200 | 1890 | 11 | 5.1 | +.29 | +.07 |
| 6613 | | 1b | H. D. | 3 B2–B3 | | *12.0* | *2500* | *2000* | 1670 | *5.1* | *4.1* | *+.08* | *+.09* |
| 6633 | | 1–2b–a | 7.3–14.3 | 34 B6–G5 | 1 | 8.1 | 420 | 360 | 410 | 3.1 | 2.7 | −.12 | −.05 |
| I. C. 4725 | M25 | 2b | 7.9–13.8 | 47 B4–A5 | | 10.3 | 1150 | 850 | 1110 | 12 | 8.7 | −.14 | −.12 |
| 6645 | | 2a | 10.3–14.4 | 27 B9–A3 | 9 | 11.6 | 2100 | 1300 | 1160 | 7.9 | 4.9 | +.10 | +.04 |
| I. C. 4756 | | 2a | 8.0–12.7 | 35 B9–F5 | 9 | 8.3 | 460 | 400 | 410 | 6.7 | 5.8 | −.07 | −.01 |
| 6694 | M26 | 1–2b–a | 11.1–13.5 | 16 B8–A2 | | 12.2 | 2800 | 1550 | 1680 | 7.3 | 4.1 | +.10 | −.03 |
| 6705 | M11 | 2b–a | 11.3–14.1 | 55 B8–A2 | 27 | 11.7 | 2200 | 1350 | 1320 | 8.0 | 4.9 | +.09 | .00 |
| 6709 | | 1–2b–a | 9.4–13.7 | 37 B8–F0 | | 10.5 | 1250 | 900 | 970 | 4.4 | 3.2 | +.01 | −.02 |
| 6716 | | 1b | H. D. | 3 B5–B8 | | *9.9* | *960* | *1050* | 1460 | *2.2* | *2.5* | *−.29* | *−.13* |
| 6811 | | 2a–f | 11.0–14.5 | 37 A3–G0 | 8 | 9.8 | 910 | 710 | 1190 | 3.4 | 2.7 | −.24 | −.22 |
| ..... | Mel 227 | 2a | H. D. | 12 B9–A5 | 8 | 7.5 | 320 | 280 | 195 | 5.6 | 5.0 | +.12 | +.18 |
| 6866 | | 2a | 10.3–13.7 | 15 A2–F0 | 6 | 10.4 | 1200 | 880 | 1030 | 3.5 | 2.6 | −.08 | −.07 |
| 6871 | | 1o | H. D. | 5 O6–B1 | | *11.7* | *2200* | *1850* | 1090 | *16* | *13* | *+.15* | *+.23* |
| 6882 | | 1–2b–a | 11.0–13.0 | 8 B7–A0 | | 11.8 | 2300 | 1400 | 1290 | 5.3 | 3.2 | +.10 | +.02 |
| 6885 | | 2–3a | 5.9–13.5 | 7 B8–G0 | 11 | 9.2 | 690 | 560 | 700 | 4.4 | 3.6 | −.13 | −.10 |

TABLE 3—(*Concluded*)

| NGC or I C. | Other designations | Magnitude sp. class diagr. | Magnitude interval | Spectral types: Dwarf br. | Spectral types: *G* | *m-M* | Distance in parsecs: From sp. types: Obs. | Distance in parsecs: From sp. types: Corr. | Distance in parsecs: from diam. | Diameter in parsecs: *D'* | Diameter in parsecs: *D''* | $v' = \lg \frac{D'}{C'}$ | $v'' = \lg \frac{D''}{C''}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| I. C. 4996 | | 1b | 8.4–13.1 | 11 B0–B7 | | 13.0 | 4000 | 1950 | 1720 | 7.0 | 3.4 | +.23 | +.06 |
| 6913 | M29 | 1b | 9.7–12.8 | 9 B0–B5 | | 13.1 | 4200 | 2000 | 2210 | 8.6 | 4.1 | +.15 | −.04 |
| 6939 | | 3a | 12.5–13.8 | 2 B8–B9 | 11 | 12.2 | 2800 | 1550 | 2060 | 6.5 | 3.6 | .00 | −.13 |
| 6940 | | 2a | 11.2–13.9 | 28 A2–F2 | 5 | 10.2 | 1100 | 820 | 780 | 8.3 | 6.2 | +.02 | +.02 |
| 7092 | M39 | 1a | 6.8–12.9 | 35 A0–G0 | | 7.5 | 320 | 290 | 370 | 3.0 | 2.7 | −.15 | −.09 |
| ..... | An. 37 | 1–2o | H. D. | 4 O5–B8 | | *9.4* | *760* | *890* | 650 | *13* | *15* | *−.11* | *+.13* |
| 7209 | | 1–2a | 8.9–11.3 | 16 A0–A5 | 2 | 9.2 | 690 | 560 | 585 | 4.0 | 3.3 | −.03 | −.01 |
| 7243 | | 1b | 8.3–13.2 | 29 B6–F0 | | 10.0 | 1000 | 760 | 740 | 6.1 | 4.6 | +.01 | +.02 |
| 7380 | | 1b | 10.4–14.0 | 20 B0–A2 | | 13.0 | 4000 | 1950 | 1720 | 10 | 5.1 | +.22 | +.06 |
| 7654 | M52 | 1b–a | 10.9–13.6 | 33 B7–A3 | | 11.9 | 2400 | 1450 | 1270 | 9.0 | 5.4 | +.14 | +.04 |
| 7789 | | 2–3a | 11.3–12.6 | 7 B9–A4 | 10 | 11.1 | 1650 | 1100 | 1180 | 9.1 | 6.1 | .00 | −.02 |

NOTES.—*NGC* 6231: Spectral types and photographic magnitudes selected from Tables I and II of *H. C. O. Bull.* 846.

*NGC* 6882 and *NGC* 6885, see remark to Table 16.

*NGC* 6705. The observed distance (col. 8) is considerably larger than that (1250 parsecs) given in *L. O. B.* **12**, 14, 1925. This is partly due to slight changes in the mean absolute magnitudes here adopted but mostly to the addition of many fainter stars for which the spectral types are given by Lindblad (*Mt. Wilson Contr.* No. 228, 1922). The present result is in close agreement with Lindblad's determination.

100 open clusters; all data printed in italics are somewhat uncertain or less reliable. In the first column the number of the *New General Catalogue* of Dreyer[8] or of the *Index Catalogues*[9] is inserted and in the second column other designations of the cluster; among these *M* refers to Messier's list, *Mel.* to Melotte's Catalogue[10], *An.* to clusters not listed in any existing catalogue but for which the position is given in Table 16. The 4th column gives the magnitude interval of the stars observed with the spectrograph; an H. D. in this column indicates that the magnitudes and spectral types were taken from the *Henry Draper Catalogue.* The number of stars belonging to the main or dwarf branch and used for the determination of the distance is found in the 5th column together with the range of spectral types. The 6th column gives the corresponding number of red or yellow giant stars. Supergiants of types *F* to *M* have not been utilized, as their mean absolute magnitudes are too uncertain. The mean difference between the apparent and absolute magnitudes of the stars, in column 7, leads by means of formula 1 to the observed value of the distance *r* in parsecs (8th column). The accuracy of these distance results is necessarily uneven, depending on the number of stars used and on their range in magnitude and spectral type. Distances which are based on data taken from the *Henry Draper Catalogue* are as a rule less reliable, and if these are restricted to less than 10 stars the result is printed in italics to mark it as uncertain. For the remaining 76 clusters the probable error of the observed distances should lie between 10% and 20%.

[8] *Mem. R. A. S.* **49**, 1888.

[9] *Mem. R. A. S.* **51**, 185, 1895; **59**, 105, 1908.

[10] *Mem. R. A. S.* **60**, 175, 1915.

## 2. Character of Magnitude-Spectral Class Diagram

The 3rd column of Table 3 contains information concerning the character of the magnitude-spectral class diagram which is obtained by plotting the stars of a cluster according to their spectral types as abscissae and their apparent magnitudes as ordinates. In a former paper[11] attention has been drawn to the fact that the magnitude-spectral class diagrams of open star clusters differ considerably, although they always show some resemblance to the well known Hertzprung-Russell diagram of giant and dwarf stars. In some clusters the giant branch is entirely missing, and the dwarf branch extends unequally far in the direction of the hotter types. A simple classification was proposed which describes, by the combination of a number and a letter, the peculiar character of the magnitude-spectral class diagram of a cluster, and the 3rd column of Table 3 furnishes the class of each cluster on this system. The number is based on the relative frequency of yellow and red giant or supergiant stars. 1: means that the giant branch is entirely missing, all cluster stars belonging to the main branch from O to M. 2: a relatively small number of stars in the giant branch. 3: the majority of the more luminous stars are yellow or red giants. The letter following the number is that of the spectral type of highest temperature reached by the main branch. In addition to the four main types 1b, 1a, 2a, 2f illustrated in the former publication two others have been introduced: 1o for clusters containing O type stars, and 3a for clusters with many red or yellow giants, but very few A type stars. Intermediate steps are also indicated; 1–2 mostly referring to cases

[11] *Publ. A. S. P.* **37**, 307, 1925.

TABLE 4

FREQUENCY OF DIFFERENT TYPES OF MAGNITUDE SPECTRAL CLASS DIAGRAMS

| | *o* | *b* | *b–a* | *a* | *a–f* | *f* | |
|---|---|---|---|---|---|---|---|
| 1 | 7 | 24 | 5 | 3 | | | 39 |
| 1–2 | 3 | 15 | 10 | 3 | | | 31 |
| 2 | | 1 | 5 | 18 | 1 | 1 | 26 |
| 2–3 | | | | 3 | | | 3 |
| 3 | | | | 1 | | | 1 |
| | 10 | 40 | 20 | 28 | 1 | 1 | 100 |

in which it is doubtful whether the few yellow or red stars observed are physical members of the cluster.

Table 4 shows the frequency of the various types; the total number of clusters for which data are available being exactly 100, the numbers also express percentages. The most frequent types are 1b (*Pleiades*) and 2a (*Praesepe*), but there is also a well pronounced transition between them. Table 4 brings out plainly the peculiar feature in the distribution of these types, *i. e.*, a strong concentration along the diagonal from 1b to 2a. In other words open clusters which contain stars of highest temperature (types O and B) contain very few or no red or yellow giant stars (the few being generally supergiants); while clusters, in which types O and B are missing, most frequently contain an appreciable or even a considerable number of stars in the giant branch. The high percentage of clusters (50%) with O and B type stars is undoubtedly exaggerated by selection, as the great luminosity of these stars allows spectroscopic observations of very distant objects while only the nearer ones of type 2a are within reach of the spectroscope.

### 3. DETERMINATION OF LINEAR DIAMETERS (FIRST APPROXIMATION)

Unfortunately the list of clusters included in Table 3 is very incomplete and selective. Owing to more favorable observing conditions in summer and autumn the spectroscopic and photometric observations are more complete for objects in galactic longitudes 330° to 150°, while in the opposite hemisphere practically none of the fainter and more distant clusters have been investigated. A study of the general space distribution of open star clusters should be based on a more complete number of clusters, and for most of these an estimate of the distance can at present be obtained only from the apparent diameter. A careful investigation of the linear diameters of the 100 clusters of known distance was therefore undertaken primarily for the purpose of finding a method to determine the distance of a cluster from its diameter.

The apparent angular diameters found in the 9th column of Table 16 are means of the estimates of various observers, especially Bailey[12] and Melotte[10], to which the writer added a number of estimates made on cluster photographs or reproductions taken with various telescopes (Franklin-Adams Chart, Barnard's Milky Way Photographs[13], Isaac Roberts Photographs[14], Charts of Cordoba Photographs[15], plates taken with the 4-inch Ross Camera, with the Crossley Reflector or with the 30-inch refractor of the Allegheny Observatory). Estimates based on the appearance of a cluster only were utilized, as it is not important that the figures should give the extreme limit of each cluster system, but rather that they should measure the same feature for large and bright as for small and faint clusters. Such large and near clusters as the *Taurus* Cluster, *Coma Berenices*, *Pleiades*, etc. were examined on star charts or small scale photographs, taking special care to estimate their diameters by the same procedure that had to be applied to the small and faint clusters.

The linear diameter $D$ in parsecs is then calculated from the apparent diameter $d$ in minutes of arc and the distance $r$ of the cluster by the formula

$$D = r \sin d = \frac{r}{3438} d \qquad (2)$$

and the result obtained with the observed distance $r$ in the 8th column of Table 3 is given under $D'$ in the 11th column. Even if we disregard the uncertain data (in italics) the observed linear diameters of the remaining 76 clusters show a rather wide range—from 2.3 to 21 parsecs. When we consider the great diversity in appearance and constitution of the open star clusters of the Milky Way it is *à priori* not surprising that these formations should also vary in dimensions. Some of these clusters like Messier 39 are groups of only a few dozen stars, others like *h Persei* contain several hundred or even a thousand members, and we could hardly expect each of these to occupy the same volume of space.

### 4. CLASSIFICATION OF CLUSTERS ACCORDING TO APPEARANCE

Our results undoubtedly discredit the assumption formerly made by some investigators, that all open clusters are nearly of the same dimensions, and that their angular diameters give a direct measure of their distances. At the same time, since clusters are more or less stable systems, it is likely that the dimensions

[12] *H. A.* **60**, 199, 1908.

[13] *Lick Obs. Publ.* **11**, and E. E. Barnard, *A Photographic Atlas of Selected Regions of the Milky Way*, 1927.

[14] Isaac Roberts: *A Selection of Photographs of Stars, Star Clusters and Nebulae.* Vol. I 1893, Vol. II 1899.

[15] *Cordoba Photographs*, by B. A. Gould, 1897.

Fig. 1. NGC 6819, Class I2r.

Fig. 2. NGC 2254, Class I2p.

Fig. 3. NGC 6705, Class II2r.

Fig. 4. NGC 1502, Class II3p.

Fig. 5. NGC 7789, Class III1r.

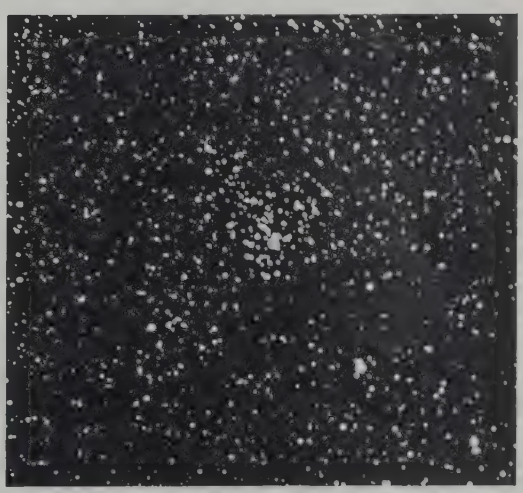

Fig. 6. NGC 1027, Class IV3m.

TYPICAL OPEN CLUSTERS

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of a cluster are governed by its gravitational potential and should therefore be related to its constitution. We evidently must restrict ourselves to the assumption that clusters of similar constitution have similar dimensions. To test this hypothesis the clusters were classified according to certain features which are characteristic of their constitution and which can be ascertained independently of their distances; such as the degree of central concentration of the stars, the range in luminosity of the members, and the number of stars contained in them. For each characteristic 3 or 4 subdivisions were made, designated by a number or a letter and the three characteristic figures were combined to a general symbol.

In the first place the clusters were divided into four main groups:

I. Detached clusters with strong central concentration.

II. Detached clusters with little central concentration.

III. Detached clusters with no noticeable concentration, in which the stars are more or less thinly but nearly uniformly scattered.

IV. Clusters not well detached but passing gradually into the environs, appearing like a star field condensation.

While the first three groups form a series with decreasing central concentration, the fourth defines a quite peculiar class and was added on the basis of the results of Table 3. The linear diameters in the 11th column of this table indicate a distinct group of clusters with abnormally large dimensions. A careful examination of these objects showed that they appear as a rule to be related to the surrounding star field, growing, so to speak, out of a larger star accumulation like a condensation. The double cluster in *Perseus* is a typical example, or the cluster NGC 1027 reproduced from Barnard's *Milky Way Atlas* in figure 6 of Plate I. Often we find in representatives of this group a small well defined densely crowded center (I. C. 1848) which might in itself be taken for a cluster, and occasionally several subsidiary centers of concentration are indicated. It must be admitted that it is sometimes difficult to decide from the appearance alone, whether a faint distant cluster belongs to the last group or to one of the three preceding ones.

In each of the four main groups three subdivisions were made according to the range in the brightness or luminosity of the cluster stars:

1. Most cluster stars nearly of the same apparent brightness (NGC 7789).
2. Medium range in the brightness of the stars.
3. Clusters composed of bright and faint stars; generally a few very bright and some moderately bright stars standing out from a host of fainter ones (*Pleiades*).

The most important distinction is perhaps that according to the number of members contained in a cluster, designated by the following letters:

*p*: Poor clusters with less than 50 stars.

*m*: Moderately rich clusters with 50–100 stars.

*r*: Rich clusters with more than 100 stars.

For most of the brighter clusters the number of stars was taken from the star counts of Raab[16]; for the others, rapid counts were made or estimates by comparison with brighter clusters, taking account of course, in all cases, of the star density of the background.

The classification of the cluster consists in a combination of the three characteristic symbols. Thus I3*m* designates a detached cluster with strong central concentration, composed of 50–100 bright and faint stars. A few clusters typical of the different groups and subdivisions are illustrated in plate I. Peculiarities of a cluster if very pronounced are further indicated by a capital letter added to the symbol: *E* for elongated, *U* for unsymmetrical, *N* for nebulosity involved in the cluster. The adopted classification of each cluster is found in Table 16, column 10.

## 5. Relation of Linear Diameter to Classification

Disregarding, at first, the second subdivision (according to range in luminosity of stars) the mean linear diameters were formed for the twelve subgroups of Table 5. Only the 76 clusters with best determined distances were used for this purpose. It may be assumed that the errors of the linear diameters $D'$ (11th column, table 3) are proportional to their amount (percentage errors) since the same is true for the errors of the distance $r$ and the apparent angular diameter $d$ from which they are derived. The logarithms of the linear diameters are therefore approximately of the same accuracy, and the means of Table 5, accordingly, are geometrical, or logarithmic means.

TABLE 5

Mean Linear Diameters of Subgroups

| Subgroup | Mean obs. diam. $D'$ | | | First approximation | Second approximation | | |
|---|---|---|---|---|---|---|---|
| | Mean log $D'$ | Mean $D'$ | No. of Cl. | Comp. $C'$ | Mean $D'$ | Weight | Adopt $C'$ |
| Ip | 0.63 | 4.3 | 7 | 4.1 | 3.0 | 8.5 | 3.0 |
| m | 0.71 | 5.1 | 4 | 5.5 | 4.0 | 5.5 | 4.0 |
| r | 0.80 | 6.3 | 9 | 6.2 | 4.5 | 11.5 | 4.4 |
| IIp | 0.61 | 4.1 | 13 | 4.3 | 3.4 | 17.0 | 3.4 |
| m | 0.78 | 6.0 | 11 | 5.7 | 4.5 | 11.5 | 4.4 |
| r | 0.80 | 6.3 | 9 | 6.5 | 4.7 | 9.5 | 4.8 |
| IIIp | 0.80 | 6.3 | 6 | 6.0 | 4.3 | 6.5 | 4.5 |
| m | 0.86 | 7.2 | 5 | 8.0 | 5.9 | 5.0 | 5.9 |
| r | 1.00 | 10.0 | 2 | 9.0 | 6.9 | 2.0 | 6.5 |
| IVp | | | | 11.1 | 13.2 | 0.5 | 7.9 |
| m | 1.18 | 15.1 | 7 | 14.9 | 9.9 | 7.0 | 10.3 |
| r | 1.22 | 16.6 | 3 | 16.8 | 11.3 | 3.5 | 11.3 |

[16] *Meddel. Lund.* Ser. II No. 28, 1922.

It is evident that in all of the four groups the mean diameter increases from the poor to the rich clusters, just as was expected. Some of the subgroups have very few representatives and their mean diameters are rather uncertain. In order to reduce the number of constants to be determined from the observational data, the assumption was made, that the rate of increase from poor to medium and rich clusters is the same in all four classes. The mean diameter of any subgroup $G_g$ can then be represented as the product of two factors $D_G$ and $f_g$, the first of which depends only on the group ($G$=I, II, III, IV) the second only on the qualification ($g=p, m, r$). To fix the absolute amount of these factors one of them can be assumed arbitrarily, and we set $f_m=1$. This reduces the number of constants to six: $D_I$, $D_{II}$, $D_{III}$, $D_{IV}$, the mean diameters of medium rich clusters of the four main groups, and $f_p$ and $f_r$, the average ratios between the mean diameters of poor and medium rich clusters and between those of rich and medium rich clusters.

Every subgroup furnishes an equation of condition:

$$\text{Subgroup } G_g\text{: Mean } D'=D_G f_g$$
$$\text{or } \log \overline{D'}=\log D_G+\log f_g$$

from which the six constants, or rather their logarithms, are easily determined by a least squares solution. The results are given in the second column of Tables 6 and 7, as first approximation $D'_G$ and $f'_g$.

TABLE 6

MEAN DIAMETERS OF MEDIUM RICH CLUSTERS OF THE FOUR GROUPS

| | First approximation | | Second approximation | |
|---|---|---|---|---|
| Group | $D'_G$ | No. of clusters | $D''_G$ | No. of clusters |
| I | 5.5 | 20 | 4.0 | 25.5 |
| II | 5.7 | 33 | 4.4 | 38.0 |
| III | 8.0 | 13 | 5.9 | 13.5 |
| IV | 14.9 | 10 | 10.3 | 11.0 |

TABLE 7

RELATIVE SIZES OF POOR, MEDIUM RICH AND RICH CLUSTERS

| Subdivision | First approximation $f_g'$ | Second approximation $f_g''$ | Average number of of stars in cluster | Relative volume | Relative star density |
|---|---|---|---|---|---|
| p | 0.75 | 0.77 | 35 | 0.45 | 78 |
| m | 1.00 | 1.00 | 75 | 1.00 | 75 |
| r | 1.13 | 1.11 | 200 | 1.36 | 147 |

The mean linear diameters $C'$ of the 12 subgroups computed from the 6 constants $D'_G$ and $f'_g$ by the formula

$$C'_{Gg}=D'_G f_g'$$

are found in the 5th column of Table 5 and agree quite closely with the observed means in the third column.

The residuals between the logarithms of observed cluster diameters $D'$ (11th column of Table 3) and the logarithms of these computed mean diameters $C'$:

$$v'=\log D'-\log C'$$

are given in the 13th column of the table.

## 6. Investigation of Systematic Errors

These residuals depend on the one hand on the correctness of our hypothesis that clusters of similar constitution have similar linear diameters everywhere in our stellar system and on the other hand on various sources of errors affecting our observational data (distance determinations, classification of clusters, estimates of angular diameter). If we hold fast to our hypothesis, which seems to have a high degree of probability, it becomes possible to investigate certain systematic errors of observation by means of the residuals $v'$.

In the first place we have to test the homogeneity of our distance determinations. According to the observational material used, we may divide these into 3 classes:

1st: Clusters with photometrically determined magnitudes, and spectral-types observed by the writer.

2nd: Clusters in which magnitudes and spectral types of more than 10 stars were available from the *Henry Draper Catalogue*.

3d: Uncertain distance determinations (given in italics in Table 3) based on the data of the *Henry Draper Catalogue* for only a few stars.

For each class the mean residual $v'$ was formed and is given in Table 8; in the first approximation only clusters for which the observed distance $r'$ is smaller than 1000 parsecs were used, as most of the clusters of the second and third class are relatively near.

TABLE 8

SYSTEMATIC ERRORS OF DISTANCE DETERMINATIONS

| Class | Data | First approximation, clusters within 1000 p. s. | | Second approximation, all clusters | |
|---|---|---|---|---|---|
| | | Average $v'$ | Number clusters | Average $v''$ | Number clusters |
| 1 | Tr | −.07 | 24 | .00 | 58 |
| 2 | H. D. C. | −.06 | 18 | −.01 | 18 |
| 3 | H. D. C. | −.24 | 18 | −.01 | 24 |

The fact that all residuals are negative is due to the selection of relatively near clusters and to a dependence of the residuals on the distance (see Table 10). There is no appreciable systematic difference between the first two classes but the third, which had not been used in deriving the mean linear diameters of Tables 5–7, stands out by a large negative residual. For

clusters whose distances are based on the *Henry Draper Catalogue* data of a few stars the linear diameters come out systematically too small and we must conclude that the distances so derived are systematically too small compared with the much better determined distances of those of the other two classes. This is probably due to the combined effect of two causes; to the selection of stars with luminosity above the average and the omission of stars with luminosity below the average (see page 155-156) which is most serious when only a few of the brightest cluster stars were observed. In this case the empirical correction applied is probably insufficient. As the observed stars of these clusters are mostly fainter than 8th magnitude and mostly of types B and A0, the distance results will also be affected by certain systematic errors in the spectral type estimates of the *Henry Draper Catalogue*. It has been noticed that this catalogue has the tendency to attribute more advanced types to the fainter B type stars than result from well exposed slit-spectrograms. This is probably due to the poor visibility of the faint helium lines in under-exposed spectra. Many of the fainter A0 stars of the *Henry Draper Catalogue* show well defined helium lines on slit-spectrograms and should be classified as B9 or B8. Similarly, many of the fainter B8 stars would be classified B5–B7 on slit-spectrograms. The consequence is that in the clusters of the last class too low luminosities were attributed to the stars and the distances came out too small. An empirical correction of +0.18 was therefore added to the logarithm of the distances and linear diameters of the last class (which corresponds to multiplication by the factor 1.51) and after such correction they were included in the following discussions with half weight.

In studying the relation between the linear diameters of clusters and the classification of their constitution, we have made no use of the second subdivision according to the range in the luminosity of the cluster members. The average residuals $v'$ of the three subdivisions are given in Table 9; no appreciable correlation between range in luminosity and linear diameters is indicated and our procedure appears well justified.

TABLE 9

DEPENDENCE OF LINEAR DIAMETERS ON RANGE IN LUMINOSITY

| Subdivision | First approximation $v'$ | First approximation Weight | Second approximation $v''$ |
|---|---|---|---|
| 1 | −.02 | 9 | −.01 |
| 2 | .00 | 32.5 | .00 |
| 3 | .00 | 46.5 | −.01 |

The clusters of Table 3 were then ordered according to the observed distance (column 8) and group means of the residuals taken. The latter are found in Table 10, column 3. They show a pronounced correlation with the distance of the cluster. For the nearer clusters the observed linear diameters $D'$ are smaller than the average, for the distant clusters larger. The range in log $D'$ is 0.28, which means that the linear diameters of the most distant clusters come out, on the average, nearly twice as large as those of the nearest ones. The following possible causes have to be considered in relation with this phenomenon:

TABLE 10

DEPENDENCE OF LINEAR DIAMETER ON DISTANCE

| Observed distance in parsecs Interval | Mean | Average residual $v'$ | Weight | Cl. with app. diam. 10′–20′ Mean $d$ | Average $v'$ | Weight | Cl. with app. diam. 20′–40′ Mean $d$ | Average $v'$ | Weight | True distance in parsecs Mean | Eff. of abs. $A$ | Residual $v'-A$ | Second approxim. Average $v''$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| <500 | 294 | −.09 | 20 | 20′ | −.17 | 1 | 31′ | −.14 | 5 | 266 | −.09 | .00 | −.01 |
| 500–1000 | 730 | −.05 | 26.5 | 16 | −.10 | 8.5 | 29 | −.02 | 15.5 | 594 | −.04 | −.01 | −.02 |
| 1000–1500 | 1200 | +.01 | 13.5 | 15 | −.01 | 5.5 | 29 | +.03 | 6 | 870 | .00 | +.01 | +.01 |
| 1500–2000 | 1620 | +.08 | 6 | 15 | +.07 | 4.5 | 22 | +.26 | 1 | 1050 | +.03 | +.05 | +.05 |
| 2000–3000 | 2460 | +.06 | 13.5 | 13 | +.11 | 3.5 | 27 | +.05 | 4 | 1500 | +.10 | −.04 | −.03 |
| >3000 | 3850 | +.19 | 8.5 | 15 | +.13 | 3 | 25 | +.33 | 0.5 | 1890 | +.16 | +.03 | +.04 |

1. Selection of residuals: Too small a diameter is obtained if the observed distance is too small; clusters affected with this error will be more numerous in the first group, those affected with the opposite error in the last group. This effect might explain the negative residual in the first, and the positive residual in the last group, but not the steady progress in the intermediate figures.

2. Systematic errors in the distance determination. In the more distant clusters as a rule only B and A type stars have been observed, while in the nearer clusters F and G type dwarf stars have often been included. This source could not account for more than a small fraction of the observed discrepancy.

3. Systematic errors in the classification of near and distant clusters. These need hardly be considered; their effect on the diameters is relatively small and for most of the clusters here considered good photographs were available as well as star counts.

4. Systematic errors in the estimates of the apparent diameters. These must indeed be feared as it is difficult to estimate the diameters of large and near clusters on the same basis as small and distant ones. Our discrepancy would require that the latter have been estimated too large, but this appears *à priori* very unlikely. The fainter cluster members as a rule are scattered over a larger area than the brighter ones, and the omission of the former in the distant clusters should rather make their diameter estimates too small. Owing to the considerable range in the linear diameters of open star clusters the correlation between apparent diameter and distance is not very close; that is, even among the clusters of the same apparent diameter we find a considerable range in distance. It is therefore possible to decide from the residuals themselves whether the observed discrepancy depends on the distance or on the apparent diameter. In Table 11 the clusters are ordered according to their apparent diameters, and the group means show decidedly smaller correlation than in the third column of Table 10. Still more convincing proof that our systematic error depends on the distance and not on the diameter is given in columns 6 and 9 of Table 10 which contain the average residuals $v'$ taken separately for two small intervals of apparent diameter; while the mean diameters (column 5 and column 8) vary very little in each column, the run of the residuals with the distance is just as pronounced as in the third column for all clusters taken together. As none of the observational errors discussed offers a plausible explanation of the observed discrepancy there are only two alternatives left; either to admit an actual change in the dimensions of open clusters with increasing distance or to assume the existence of an absorption of light within our stellar system. In favor of the first alternative it might be argued that the dimensions of clusters are possibly influenced by the star density of the space in which they are situated and that this star density on the whole decreases with greater distance from the center of the local system. If we consider, however, the amount in question it is extremely unlikely that any such influence could be powerful enough to nearly double the dimensions of remote clusters.

TABLE 11

DEPENDENCE OF RESIDUALS ON APPARENT DIAMETER

| Apparent diameter | First approximation | | Second approximation, |
|---|---|---|---|
| | Average $v'$ | Weight | Average $v''$ |
| >160′ | +.01 | 3 | +.13 |
| 80–160 | −.12 | 2 | .00 |
| 40–80 | −.05 | 11.5 | +.02 |
| 20–40 | −.01 | 32 | +.01 |
| 10–20 | −.01 | 26 | −.03 |
| 5–10 | +.08 | 13.5 | −.04 |

The assumption of an absorption of light on the other hand is not only able to give a satisfactory numerical representation of the residuals of Table 10 but receives support also from color-index observations in clusters.

## 7. Absorption of Light in the Milky Way System

Our method of deriving cluster distances from magnitudes and spectral types was based on formula (1) which expresses the law that the apparent brightness of a star diminishes with the square of the distance from the observer. If interstellar space is not perfectly transparent this law does not hold; the apparent brightness decreases more rapidly, our distance results are too large, and the error increases with the distance of the cluster. The linear diameters computed with these distance results are then also too large, and the error also progresses with distance, just like the residuals in column 3 of Table 10.

Let us suppose for the moment that the absorbing material is uniformly distributed throughout the galactic system. The loss of star light in magnitudes caused by absorption is then

$$\Delta m = kr$$

where $r$ is the stars' true distance and $k$ the absorption constant of photographically effective wavelengths. The relation between apparent and absolute magnitude of a star and its distance is accordingly

$$m - M = 5 \log r + kr - 5 \qquad (3)$$

This equation must be substituted for equation (1) in the case of uniform absorption.

Our observed cluster distances $r'$ (Table 3, column 8), on the other hand, were computed with formula (1) and the relation between the true distance $r$ and our observed value $r'$ is given by

$$\log r' = \log r + \frac{k}{5} r \qquad (4)$$

The linear diameter $D'$ (Table 3, column 11) was calculated by formula (2) with the observed distance $r'$, which in logarithmic form is

$$\log D' = -3.536 + \log d + \log r'$$

($d$ = apparent diameter in minutes of arc)

The true linear diameters $D$ would have been obtained from

$$\log D = -3.536 + \log d + \log r$$

Hence the error in the logarithms of our observed linear diameters are

$$\log D' - \log D = \log r' - \log r = \frac{k}{5} r \qquad (5)$$

The residuals $v'$ were obtained by deducting the mean diameter logarithm of the subclass from the individual diameter logarithm of each cluster

$$v' = \log D' - \overline{\log D'} \tag{6}$$

If we take the mean of equation (5) we have

$$\overline{\log D'} - \overline{\log D} = \frac{k}{5}\bar{r}$$

Since we assume the true diameters $D$ of all clusters of the same subclass to be the same,

$$\overline{\log D} = \log D$$

and

$$\overline{\log D'} = \log D + \frac{k}{5}\bar{r} \tag{7}$$

or from (6) and (7): $v' = \log D' - \log D - \frac{k}{5}\bar{r}$ (8)

and combining this with equations (5)

$$v' = \frac{k}{5}r - \frac{k}{5}\bar{r}$$

$$v' = a + br \tag{9}$$

The residual must be a linear function of the true distance $r$ with the two constants $a = \frac{k}{5}\bar{r}$ and $b = \frac{k}{5}$. Equation (6) is strictly true only for the clusters of the same subclass; but since the mean distance $\bar{r}$ is approximately the same for all subclasses, we can also apply it for all clusters taken together.

Our task is now to determine the numerical values of the two constants $a$ and $b$ from the observed residuals $v'$, and to see if the latter are sufficiently well represented by formula (9). Unfortunately the variable $r$, the true distance, is not known, but only the observed distance $r'$. $a$ and $b$ have therefore to be determined from the two equations:

$$\left.\begin{aligned} a + br &= v' \\ \log r + br &= \log r' \end{aligned}\right\} \tag{10}$$

using the data of Table 10 (columns 2–4) by gradual approximation. The first approximation is obtained by considering that for clusters with small distances (first group mean) $br$ is small and $a$ is nearly equal to the mean $v'$ of the first group ($-.09$). Elimination of $b$ $r$ from the two equations (10) leads to

$$\log r = \log r' - v' + a \tag{11}$$

Entering in this equation our approximate value of $a$, we obtain approximate values of $r$ for each group mean of Table 10. We then proceed to determine $a$ and $b$ by least squares solution, applying the first equation (10) as equation of condition to the group means of Table 10. With the results for $a$ and $b$ obtained by this first approximation the true mean distances $r$ of the group means are recomputed in two ways. First, from the second equation (10), and secondly from equation (11).

TABLE 12

| | First approximation | Second approximation | Third approximation |
|---|---|---|---|
| $a$ | $-.133$ | $-.136$ | $-.134$ |
| $1000b$ | $+.141$ | $+.159$ | $+.158$ |

The first corrects the observed spectroscopic distances for absorption; the second corrects them on the basis of uniform linear diameters. The means of both methods are used as the basis for a second approximation. The results for $a$ and $b$ of the successive approximations are found in Table 12; those of the third approximation are finally adopted and furnish for the constant of photographic absorption

$$k = 5b = 0.79 \text{ magnitudes per 1000 parsecs.}$$

In columns 11–12 of Table 10 the resulting true distances of the group means and the corresponding absorption effects $A$ on the observed linear diameters $D'$ are given which were computed from the formula $A = a + br$. The residuals $v' - A$ (col. 13, Table 10) are small and show no systematic arrangement. It may therefore be stated that the apparent increase in the linear diameters of open star clusters is fully removed by the assumption of an absorption of light within our Milky Way system to the amount of 0.79 magnitudes per 1000 parsecs for photographically effective rays.

If the light of the stars in the more distant clusters has been dimmed by the passage through an absorbing material, it seems *à priori* likely that such absorption is selective, and varies with the wave length of the light and thus changes the color of the stars. Since the color of a star depends on its temperature a change of color by absorption can only be detected if its temperature is observable by some other means; *e. g.*, from the spectral types which measure the temperature by means of the ionization and excitation of the atoms in the stellar atmosphere. Spectral type estimates based on comparison of intensities of spectral lines are not affected by a general absorption. For the brighter and nearer stars which are little affected by absorption the relation between color indices and spectral types has been well established, but differs slightly for giant and dwarf stars. From this relation we can find the normal color index corresponding to a given spectral type. The difference between this and the color index actually observed is called the color excess of the star. The existence of large discrepancies between observed color indices and spectral types in open star clusters has been known for some time, but to test our hypothesis of an absorption of light in space it is necessary to show that the average color excesses in various clusters depend on their distance. The spectral types in open clusters determined with the 1-prism slit spectograph and the slitless quartz

spectrograph made it possible to compute the average color excess for a number of clusters in which color-indices have been measured by various observers. Leaving out the few nearest clusters which are of little interest for our purpose and using only color indices determined by direct comparison with the North Polar sequence, seven clusters ranging widely in distance were available. Table 13 gives for each of these the observed distance $r'$ (from magnitude and spectral types), the finally adopted distance (Table 16) which is corrected for absorption, the average color excess of the cluster stars, the number of stars used, and the name of the observer of the color indices.

TABLE 13

COLOR-EXCESS OF OPEN CLUSTERS

| Cluster | | Observed dist. $r'$ | Adopt. corr. dist. | Color excess | Number of stars | Observer of color-ind. | Formula | Obs. —Form |
|---|---|---|---|---|---|---|---|---|
| NGC2682 | Messier 67 | 690 | 740 | $+0^m.26$ | 81 | Shapley[17] | $+^m.24$ | $+^m.02$ |
| 1647 | | 800 | 610 | +0.17 | 33 | Hertzsprung[18] | +.19 | −.02 |
| | | | | +0.19 | 6 | Seares[19] | +.19 | .00 |
| 2099 | Messier 37 | 1450 | 820 | +0.05 | 25 | v. Zeipel and Lindgren[20] | +.26 | −.21 |
| 1960 | Messier 36 | 1650 | 980 | +0.05 | 40 | Wallenquist[21] | +.31 | −.26 |
| 6705 | Messier 11 | 2200 | 1340 | +0.65 | 46 | Shapley[22] | +.43 | +.22 |
| 7654 | Messier 52 | 2400 | 1360 | +0.49 | 43 | Wallenquist[23] | +.43 | +.06 |
| 663 | | 3500 | 2170 | +0.71 | 41 | Wallenquist[23] | +.69 | +.02 |

In all clusters we find a positive excess, and the latter is largest for the 3 most distant clusters. It must be kept in mind, of course, that color indices are subject to systematic errors of observation and especially that errors in the zero point of the photographic and visual magnitude scales may occur, due to changes in the observing conditions between the exposures on the cluster and on the North Pole. It is, however out of question that errors of the order of $0^m.5$–$0^m.7$ could result from this source, especially if the North Polar comparisons have been repeated on different days. Wallenquist, aware of the large discrepancy between his color indices of NGC 663 and the spectral types of the same stars, and unable to account for it by observational errors, concludes: "The most probable explanation is, perhaps, the assumption of selective absorbing clouds within (and in the surroundings of) the cluster NGC. 663." That the effect is not due to an error in method is well illustrated by the fact that Wallenquist observed three clusters by the same method and instrument and that only the two more distant ones show a large color excess; the same is true for the two open clusters investigated by Shapley. In a former publication[24] I drew attention to the large excess of Shapley's color observations in the cluster Messier 11 and, averse to the idea of a general selective absorption in our stellar system, took rather a skeptical attitude concerning the correctnes of Shapley's results until these were confirmed by Wallenquist's observations of two more distant clusters. We are thus led to the assumption of a general absorption of light in our stellar system by two quite independent sets of observations; by the study of the linear diameters of star clusters as well as by color-index observations in such clusters. That the absorption is not caused merely by an absorbing cloud involved in the cluster itself as Ten Bruggencate[25] and Wallenquist suggest is shown by the increase in the color-indices with the distance and by a similar increase in the apparent linear diameters.

If we speak of a general absorption this does not mean that it is necessarily uniform throughout the stellar system; the absorbing material may have many local irregularities but must be so distributed that *on the average* the absorption in magnitudes is approximately proportional to the distance, as this condition seems to be necessary to explain the residuals of the observed linear diameters in Table 10. It is true that the color-excesses of the 7 clusters in Table 13 do not increase quite smoothly with the distance and might suggest local irregularities. If we interpolate these color excesses $E$ by a linear formula

$$E = c\,r \qquad (12)$$

where $r$ is the adopted corrected distance, we find by least squares solution

$$c = +0^m.32 \pm {}^m.03 \text{ (per 1000 parsecs)}$$

[17] *Mount Wilson Contr.* No. 117.
[18] *Mount Wilson Contr.* No. 100, *Ap. J.* **42**, 120, 1915.
[20] *Kgl. Svenska Vet. Akad. Handlingar*, **61**, No. 15, 1921.
[21] *Meddel. Upsala* No. 32, 1927.
[22] *Mount Wilson Contr.* No. 126, *Ap. J.* **45**, 164, 1917.
[23] *Meddel. Upsala*, No. 42, 1929.
[24] *L. O. Bull.* **12**, 12, 1925.
[25] Ten Bruggencate, *Sternhaufen*, 1927, p. 146.

The color excesses computed by formula (12) for the seven clusters are found in the 7th column of Table 13 and the residuals between the observed and computed values, in the 8th column, are in all cases of such small amounts that they can be explained by errors of observation. It would therefore hardly be justifiable to draw from the data of Table 13 a definite conclusion that the absorption is not uniform, although it is rather striking that the two clusters with larger negative residuals are situated in the same region of the sky only a few degrees apart.

To show that the absorption does not depend much on the galactic longitude, the photographic absorption coefficient was determined from the residuals of the cluster diameters for four separate intervals of galactic longitude. These intervals were taken somewhat narrower in those parts of the Milky Way where distant clusters with spectroscopically determined distances are more numerous. The method is similar to that described on page 164, except that the final residuals $v''$, resulting from a second approximation explained later, and the adopted distances in the 11th column of Table 16, both already corrected for absorption, were used to determine corrections $\Delta k$ to the adopted absorption constant. In each longitude interval the clusters more distant than 500 parsecs (true distance) were combined with all nearer clusters for the determination by least squares solution of the constants $\Delta a$ and $\Delta b$ in the equation (derived from (10) ):

$$\Delta a + r\Delta b = v'', \text{ where } \Delta b = \frac{\Delta k}{5}$$

The results are given in Table 14:

TABLE 14

DEPENDENCE OF ABSORPTION CONSTANT ON GALACTIC LONGITUDE

| Galactic longitude | 1000 $\Delta b$ | $\kappa$ | $p.e.$ |
|---|---|---|---|
| 330°– 45° | −.001 | $0^m.79$ | $\pm^m.09$ |
| 45 –110 | −.041 | 0.59 | ±.08 |
| 110 –195 | +.016 | 0.87 | ±.17 |
| 195 –330 | (−.085) | (0.37) | ±.16 |
| All clusters | −.024 | 0.67 | ±.07 |

Every one of the four intervals gives evidence of an absorption effect, and the numerical values of the constant $k$ differ little more than should be expected from their probable errors. The small result for $k$ in the last interval (195°–330°) deserves very little confidence as this interval includes only southern clusters for which no spectroscopic data of the fainter cluster stars are available. The first interval covers the region between *Sagittarius* and *Cygnus* where the Milky Way is divided. This division is sometimes attributed to absorption, and it is perhaps significant that the absorption constant derived from cluster diameters in this region is slightly larger than the average. On the whole, however, there is no convincing observational evidence contrary to the assumption that the absorbing material is of a fairly regular distribution.

It is natural to interpose here the question why such an absorption of light should not have been discovered in the discussion of the diameters of globular clusters which are much more distant, and how it is possible that we still find small color-indices in some globular clusters (such as Messier 13) despite of their great distances. There is only one way which seems to lead out of the dilemma: the hypothesis that the absorbing medium, like the open clusters, is very much concentrated toward the galactic plane. We shall see later that two-thirds of all open clusters lie within 100 parsecs of their plane of symmetry, and it is not improbable that the absorbing material has a similar distribution, thinning out very rapidly at greater distances from the galactic plane and forming so to speak a thin sheet (perhaps 200–300 parsecs thick) extending along the galactic plane to distances of at least 2000 and perhaps 4000 or 6000 parsecs. It is evident that the absorption caused by a material of such distribution is practically negligible (less than $0^m.1$) for objects in high galactic latitudes. Only for objects lying within 8° of the galactic circle could the photographic absorption amount to $0^m.5$ or more and the color excess to $0^m.25$ or more. Of the relatively few globular clusters falling within these limits there are four in which variable stars or magnitudes of the brighter stars have been observed[26]. These four (NGC 6266, 6626, 6638, 6712) do indeed show some effect of absorption, that is, the distance determined from magnitudes is greater than that derived from the diameters. A more thorough investigation of the linear diameters of globular clusters is necessary, however, to draw any definite conclusions.

If our hypothesis is correct, some of the open clusters must also lie outside of this thin layer of absorbing material, and their light should be subject to absorption for part of the distance only. To show that this is the case we take the five clusters of Table 3 for which the distance $Z'$ from the plane of symmetry of the clusters exceeds 200 parsecs and form the average of the final residuals $v''=\log D''-\log C''$ (last column of Table 3). The result is $v''=-0.13$. This means that the corrected linear diameters $D''$ (Table 3, column 12) are too small, and that the same is true for their corrected spectroscopic distances (Table 3, column 8). The absorption correction applied to the latter must have been too large; these clusters seem

[26] *H. C. O. Bull.* No. 869.

to be less affected by absorption than the majority of clusters at the same distance which lie close to the galactic plane.*

We are thus led to the conclusion that some general and selective absorption is taking place in our Milky Way system, but that this absorption is confined to a relatively thin layer extending more or less uniformly along the plane of symmetry of the system. Perhaps this absorbing material is related to interstellar calcium or to the diffuse nebulae which are also strongly concentrated to the galactic plane.

The change in the color of stars with distance exhibited by Table 13 must be due to the fact that the absorption depends on the wave-length of light, being smaller for the longer wave length used in visual observations than for the shorter waves photographically effective. Designating by $k_v$ the absorption constant per 1000 parsecs of visual observations and by $k_p$ that of photographic observations, the change of color $c$, determined on p. 165, gives us the difference of the two absorption constants:

$$c = k_p - k_v = +0^m32$$

Combining this with the photographic absorption constant determined from the cluster diameters, for which we adopt the final value in the last line of Table 14, we have

$$k_p = +0^m67 \text{ per } 1000 \text{ parsecs}$$
$$k_v = +0.35$$

The photographic absorption is nearly twice the visual one. This is not very different from the extinction in the Earth's atmosphere where the ratio of the photographic to the visual extinction is about 2.5.

Defining the absorption coefficient $\kappa$ by the equation

$$I = I_o e^{-\kappa t}$$

where $I_o$ is the intensity of the incident light, $I$ that of transmitted light and $t$ the length of the path in $cm$:

$$\kappa = 0.20 \times 10^{-21} \text{ for } \lambda = 4300 \text{ A}$$
$$= 0.10 \times 10^{-21} \text{ for } \lambda = 5600 \text{ A}$$

## 8. Determination of Distances and Diameters (Second Approximation)

As the investigations of the preceding section give strong evidence of the existence of a general absorption of light within our stellar system it becomes necessary to recompute the cluster distances from the observed $m-M$ (column 7, Table 3) by the formula

$$5 \log r + k\frac{r}{1000} = m - M + 5 \qquad (13)$$

which takes into account the effect of absorption, and in which the numerical value

$$k = +0^m79 \text{ per } 1000 \text{ parsecs}$$

found on page 164 was adopted. The results are given as corrected distances in Table 3, column 9. For the 24 clusters with data printed in italics the systematic error found in Table 8 was also corrected for by the addition of $1^m05$ to the observed $m-M$; this corresponds to the average residual $v'$ of Table 8, reduced by the distance effect of Table 10 (interpolated from the 12th column for the average distance of the clusters).

The corrected distances were adopted as final results of the distance determinations from magnitudes and spectral types; they were used to compute the corrected values $D''$ of the linear diameters in the 12th column of Table 3, which of course are somewhat smaller on the average than the uncorrected results of the preceding column. On this account the calculation of mean linear diameters of the different subgroups of our classification had to be repeated following the same procedure as before except that all clusters were used this time, giving half weight to the 24 clusters with less certain distance determinations. The results are found in Tables 5, 6, 7 under "Second approximation."

The residuals $v''$ in the last column of Table 3 are the differences between the logarithm of the corrected linear diameter $D''$ of each individual cluster and the logarithm of the adopted mean linear diameter $C''$ (last column of Table 5) corresponding to its classification:

$$v'' = \log D'' - \log C''$$

The discussion of the residuals is now repeated in Tables 8, 9, 10 and 11 in second approximation. Table 8 shows that the corrected cluster distances derived from different observational data now form a homogeneous system. In Table 11 a small dependence of the residuals $v''$ on the apparent diameter is perhaps still noticeable, but in the opposite sense than before. There is now an indication that the apparent diameters of small and faint clusters were slightly underestimated, which does not seem unlikely. Disregarding the first group mean, which is entirely due to one large residual, this effect, if real, is of the order of the errors of observation and in no case amounts to more than 10%. It seemed therefore justifiable to neglect it.

* J. P. Van Rhijn (Derivation of the change of color with distance and apparent magnitude; Diss. Groningen 1915) from a discussion of color-indices and spectral types of the Yerkes Aktinometry also found a change of color with distance. That his value of $c$ ($+0^m15$ per 1000 parsecs) is smaller than ours, fits in well with our hypothesis that the absorbing medium is highly concentrated to the galactic plane. The stars investigated by Van Rhijn are situated in high galactic latitudes 50 to 350 parsecs distant from the galactic plane and must lie partly outside of the absorbing medium, or in the region where it is gradually thinning out.

According to the last column of Table 10 no appreciable correlation exists between the residuals $v''$ and the distance; but the determination of a correction to the adopted absorption coefficient from the residuals $v''$ which is mentioned on page 166 yields for this constant a slightly smaller value. It is therefore possible that our final cluster distances are slightly over-corrected for absorption. If the smaller absorption constant ($k=0^m67$) is correct, our final results for the distances and linear diameters of open star clusters are on the average 4% too small. This small difference, however, lies well within the limits of observational uncertainty and it did not seem necessary to correct it by a further approximation.

From the residuals $v''$, which have the character of accidental errors of observation, the probable error $\epsilon$ is calculated, taking into account the eight empirically determined constants. For a cluster of full weight we thus obtain

$$\epsilon = \pm .067, \text{ corresponding to } 16\%$$

This error results from three sources of observational error:

1. Errors in the spectroscopic distance determinations ($\epsilon_r$).
2. Errors in the estimates of angular diameters ($\epsilon_d$).
3. The effects of errors in the classification ($\epsilon_c$). The latter affect the mean linear diameter $C''$ used for forming the residual $v''$.

The following estimates for these three probable errors:

$$\epsilon_r = \pm .05\ (\pm 12\%)$$
$$\epsilon_d = \pm .035\ (\pm 8\%)$$
$$\epsilon_c = \pm .03\ (\pm 7\%)$$

represent rather minimum values. Their combined effect is

$$\epsilon = \pm\sqrt{\epsilon_r^2 + \epsilon_d^2 + \epsilon_c^2} = \pm .068$$

The estimated errors of observation thus fully explain the amount of the residuals $v''$.

Our assumption that clusters of the same constitution have the same linear diameter appears justified if we admit the existence of an absorption of light in our stellar system. It is then possible to determine the distance of a cluster from its apparent diameter and its classification with a probable error of about $\pm 12\%$, *i. e.*, with the same accuracy as our determinations from magnitudes and spectral types.

## 9. The Dimensions of Open Star Clusters

It is of interest to examine our results on the dimensions of open star clusters more closely. In the first place it must be recalled that our figures for apparent or linear diameters are "estimated diameters" and must be distinguished from "limiting diameters" which give the extreme limits of clusters. The latter can be ascertained only from star counts or by segregating the physical members from the background stars by their common proper-motions. The few objects for which such investigations have been carried out indicate that the estimated diameter generally refers to the more concentrated central part of the cluster. This denser central part is surrounded by a region where only very few scattered cluster stars occur which gradually thin out towards the limit and do not detach themselves sufficiently from the background stars. As a rule the limiting diameter of a cluster is 2–3 times larger than the estimated diameter. The distribution of the 100 clusters of Table 3 according to their linear diameter $D''$ (column 12 of Table 3) is given in Table 15 and illustrated in Fig. 1.

TABLE 15

Distribution of Clusters According to Linear Diameters

| Estimated linear diameter $D''$ in parsecs | Number of clusters | |
|---|---|---|
| 0–1 | 0 | |
| 1–2 | 1 | |
| 2–3 | 18 | |
| 3–4 | 25 | 77 |
| 4–5 | 21 | |
| 5–6 | 13 | |
| 6–7 | 7 | |
| 7–8 | 4 | |
| 8–9 | 1 | |
| 9–10 | 4 | |
| 10–11 | 1 | |
| 11–12 | 3 | |
| 12–13 | 1 | |
| 13–14 | 0 | |
| 14–15 | 1 | |
| | 100 | |

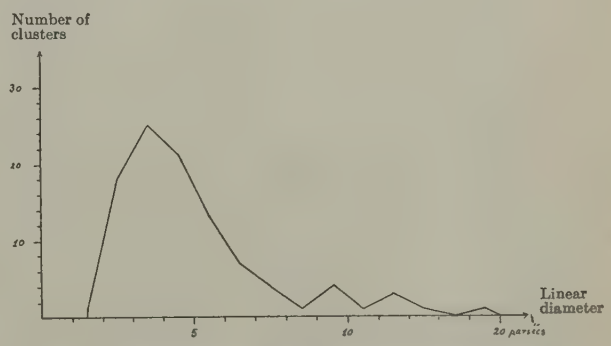


Fig. 1. Distribution of open clusters according to linear diameters.
The ordinate gives the number of clusters with linear diameters falling within the interval plotted as abscissa.

By far the majority of the clusters (77%) have estimated diameters between 2 and 6 parsecs. While the frequency curve of figure 1 drops very sharply

on the side of the smallest diameters it runs out gradually in the direction of increasing diameters; a group of abnormally large clusters (our Class IV) is well indicated although having few members.

The relation between the mean linear diameters and our classification of clusters shown in Tables 5, 6, and 7 (second approximation) is of importance for the study of the laws governing the constitution of star clusters. For clusters containing the same number of members we find the diameter to decrease with increasing central concentration. The difference between groups I and II, however, is small, and it is especially group III, with no noticeable central concentration, which stands out by its larger size.

The increase in the cluster diameters with the number of members is quite natural and was anticipated; this increase is more rapid from the poor to the medium rich clusters and rather small between medium rich and rich ones. In Table 7 the average number of stars for a cluster of each subdivision is given as well as the relative volume which is proportional to the third power of the relative diameter. From these two data the relative star density is calculated. It appears from these figures that in a medium rich and in a poor cluster the star density is on the average the same. The larger number of stars is simply accommodated by a larger volume and diameter. But when we pass from a medium rich to a rich cluster we find the increase in stars no longer compensated by a proportional increase in the volume and the star density in very rich clusters is considerably higher than in the poorer ones.

It is further of interest to compare the dimensions of open clusters with those of globular clusters; if we adopt for the latter Shapley's[27] data we have:

| | Estimated diameter parsecs | Limiting diameter parsecs | Number of stars |
|---|---|---|---|
| Open clusters Gr I–III | 3–7 | 6–20 | 10–2000 |
| Open clusters Gr IV | 8–12 | 15–25 | 10–2000 |
| Globular clusters | 40 | 150 | 5000–500,000 |

The globular clusters evidently distinguish themselves not only by the large number of stars they contain but also by their greater linear dimensions. The star density in globular clusters appears to be of similar order as in open clusters.

## 10. Preliminary Catalogue of Open Star Clusters

The investigations of the preceding chapters have shown that it is possible to determine with a fair degree of accuracy the distance of any cluster for which a good photograph or description is available. It is for this purpose only necessary to classify the cluster according to the system described on page 160 and to estimate its apparent diameter $d$ in minutes of arc. The most probable value $C''$ of its linear diameter is then taken from the last column of Table 5 and the distance $r$ of the cluster in parsecs is calculated by the formula

$$r = 3438\frac{C''}{d} \qquad (14)$$

For the 100 clusters of Table 3 the results so obtained are found in the 10th column.

To study the general space distribution of open star clusters it is desirable to apply this method as far as possible to all open star clusters known. Unfortunately there is no comprehensive catalogue of this kind at present available. The catalogue of Melotte gives diameter estimates as well as descriptions for 162, most of the brighter and more prominent open clusters, but is very incomplete for the smaller and fainter ones. The *New General Catalogue* of Dreyer and the two subsequent *Index Catalogues* offer the most complete list of clusters, but a careful examination of the (more than 600) objects so described showed that a large number of them refer to scarcely noticeable groupings of a few stars which are probably accidental and of little interest, or to rich Milky Way star fields.

It thus became necessary to compile a new list of open clusters suited for distance determination. The purpose was to select all those star groupings which undoubtedly form physical systems (stars situated at the same distance and probably of common origin) and which at the same time are sufficiently rich in stars for statistical investigation. There are probably numerous physical systems of less than a dozen stars, but these were as a rule not included as they should rather be classed with multiple or double stars. It is hardly possible to describe the constitution of such systems by statistical laws of star distribution (estimated diameter, central concentration, etc.) on which our method for determining distances from apparent diameters is based.

The probability of a clustering forming a physical system was judged on the one hand from the denser congregation of the cluster stars as compared with the surroundings, on the other hand from the regularity or symmetry in the formation of the group, from the definiteness of its outline and an eventual central concentration. It is clear that some arbitrariness can not be avoided in drawing the line of demarcation between open star clusters and related formations such as multiple stars, globular clusters and the smaller star clouds of the Milky Way. The rule followed was to include among open clusters only such star formations as appear to be physical systems and contain not less than a dozen and not more than a few thousand stars.

[27] *Mt. Wilson Contr.* No. 152, 1917.

Every object described as a cluster in the *New General Catalogue*, the two *Index Catalogues*, or mentioned by other observers was thus examined on a copy of the *Franklin Adams Chart* covering the whole sky and on the excellent Milky Way photographs by Barnard. The regions of the *Magellanic Clouds*, however, were excluded, and the examination of the fainter objects was restricted to those lying within 20° of the great circle of the Milky Way. A list of the 334 open clusters thus selected is found in Table 16. The first column gives the designation of the cluster; a plain number refers to the *New General Catalogue*, I. C. to the first or second *Index Catalogues*, Mel to Melotte's Catalogue, An to anonymous clusters not previously listed or newly discovered by the writer. The second and third columns contain the right ascension and declination for 1900 computed mostly from the *New General Catalogue*, the fourth and fifth, the galactic longitude $L$ and latitude $B$, the sixth to eighth, the rectangular galactic coordinates

$$x = \cos B \cos L$$
$$y = \cos B \sin L$$
$$z = \sin B$$

For the position of the Galactic North Pole $\alpha = 12^h\ 40^m$, $\delta = +28°.0$ (1900) was used.

TABLE 16

CLASSIFICATION, DIAMETERS AND DISTANCES OF 334 OPEN GALACTIC STAR CLUSTERS

(An asterisk in the first column refers to a remark at the end of the table)

| N.G.C. I.C. | R. A. 1900 | Decl. 1900 | Galactic | | Galactic coordinates | | | Angul. diam. | Class | Adopted distance parsecs | Wt. | Rectangular coordinates | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Long. | Lat. | $x$ | $y$ | $z$ | | | | | $X$ | $Y$ | $Z$ |
| 103 | 0h19m.8 | +60° 47 | 87°.5 | − 1°.1 | +.043 | + .999 | −.020 | 4′ | I 2p | 2580 | ..... | + 110 | +2580 | − 51 |
| 129 | 24.3 | +59 40 | 88.0 | − 2.3 | +.034 | + .999 | −.040 | 13 | IV 2p U | 2090 | ..... | + 70 | +2090 | − 83 |
| 136 | 25.9 | +60 58 | 88.3 | − 1.0 | +.030 | + .999 | −.017 | 2.2 | II 2p | 5320 | ..... | + 160 | +5310 | − 90 |
| 188 | 35.1 | +84 47 | 89.9 | +22.8 | +.002 | + .922 | +.387 | 14 | II 1r | 1180 | ..... | 0 | +1090 | +456 |
| 225 | 37.6 | +61 14 | 89.7 | − 0.7 | +.005 | +1.000 | −.013 | 14 | III 1p | 1100 | ..... | + 10 | +1100 | − 14 |
| 381 | 1 2.1 | +61 3 | 92.7 | − 0.8 | −.047 | + .999 | −.014 | 7 | III 2p | 2210 | ..... | − 100 | +2210 | − 31 |
| 436 | 9.4 | +58 17 | 93.8 | − 3.5 | −.067 | + .996 | −.061 | 6 | I 3m | 1970 | 2 | − 130 | +1960 | −120 |
| 457 | 12.8 | +57 48 | 94.4 | − 3.9 | −.076 | + .995 | −.068 | 12 | I 3r | 1480 | 2 | − 110 | +1470 | −101 |
| 559 | 22.8 | +62 47 | 94.9 | + 1.2 | −.085 | + .996 | +.021 | 6 | I 2m | 2290 | ..... | − 190 | +2280 | + 48 |
| 581 | 26.6 | +60 11 | 95.8 | − 1.3 | −.100 | + .995 | −.023 | 6.5 | II 3m | 1960 | 2 | − 200 | +1950 | − 45 |
| An. 1 | 29.0 | +60 46 | 96.0 | − 0.6 | −.104 | + .994 | −.011 | 4.5 | I 3p | 2290 | ..... | − 240 | +2280 | − 25 |
| 609 | 30.3 | +64 2 | 95.5 | + 2.6 | −.095 | + .995 | +.045 | 3 | I 2p : | 3430 | ..... | − 330 | +3410 | +155 |
| *637 | 36.0 | +63 30 | 96.2 | + 2.2 | −.108 | + .993 | +.038 | 3.5 | II 3p : | 3340 | ..... | − 360 | +3320 | +127 |
| 654 | 37.2 | +61 23 | 96.8 | + 0.1 | −.118 | + .993 | +.002 | 5.5 | I 2p | 1870 | ..... | − 220 | +1860 | + 4 |
| 659 | 37.4 | +60 12 | 97.1 | − 1.0 | −.123 | + .992 | −.018 | 5 | II 2p | 2340 | ..... | − 290 | +2320 | − 42 |
| 663 | 39.2 | +60 44 | 97.2 | − 0.4 | −.125 | + .992 | −.008 | 14 | IV 2m | 2170 | 2 | − 270 | +2150 | − 17 |
| 744 | 51.8 | +54 59 | 100.2 | − 5.6 | −.177 | + .979 | −.098 | 14 | I 2p | 740 | ..... | − 130 | + 720 | − 73 |
| 752 | 51.8 | +37 11 | 105.4 | −22.7 | −.246 | + .889 | −.385 | 45 | III 1m | 390 | 2 | − 100 | + 350 | −150 |
| 869 | 2 12.0 | +56 41 | 102.4 | − 3.1 | −.215 | + .975 | −.054 | 30 | IV 3r | 1330 | 2 | − 290 | +1300 | − 72 |
| 884 | 15.4 | +56 39 | 102.9 | − 3.0 | −.222 | + .974 | −.052 | 30 | IV 3r | 1330 | 2 | − 300 | +1300 | − 69 |
| I.C.1805 | 25.2 | +61 0 | 102.4 | + 1.5 | −.215 | + .976 | +.026 | 20 | IV 3m N | 1810 | 2 | − 390 | +1770 | + 47 |
| 957 | 26.4 | +57 5 | 104.1 | − 2.0 | −.243 | + .969 | −.035 | 9 | II 3m U | 1680 | ..... | − 410 | +1630 | − 59 |
| An. 2 | 30.2 | +55 32 | 105.2 | − 3.2 | −.262 | + .963 | −.056 | 18 | II 3p | 680 | 1.5 | − 180 | + 650 | − 38 |
| 1027 | 35.0 | +61 7 | 103.5 | + 2.1 | −.232 | + .972 | −.037 | 21 | IV 3m | 1590 | 2 | − 370 | +1550 | − 59 |
| 1039 | 35.6 | +42 21 | 111.7 | −14.8 | −.357 | + .899 | −.255 | 30 | I 3m | 450 | 2 | − 160 | + 400 | −115 |
| I.C.1848 | 43.5 | +60 1 | 104.9 | + 1.6 | −.256 | + .966 | +.028 | 22 | IV 3mN | 1610 | ..... | − 410 | +1560 | + 45 |
| 1193 | 59.2 | +43 59 | 114.8 | −11.3 | −.411 | + .890 | −.196 | 3.2 | I 2r | 4720 | ..... | −1940 | +4200 | −925 |
| An. 3 | 3 3.6 | +62 52 | 105.6 | + 5.3 | −.267 | + .959 | +.092 | 17 | II 3p | 690 | ..... | − 180 | + 660 | + 63 |
| 1220 | 4.4 | +52 57 | 110.8 | − 3.2 | −.355 | + .933 | −.055 | 2.0 | I 2p | 5150 | ..... | −1830 | +4800 | −284 |
| 1245 | 7.8 | +46 52 | 114.5 | − 8.0 | −.411 | + .901 | −.140 | 7 | III 2r | 3200 | ..... | −1320 | +2880 | −448 |
| Perseus | 15 | +48 15 | 114.8 | − 6.2 | −.417 | + .903 | −.108 | 240 | IV 3m | 170 | 2 | − 71 | + 154 | − 18 |
| 1342 | 25.2 | +36 59 | 123.0 | −14.3 | −.527 | + .813 | −.248 | 15 | II 2m | 1010 | ..... | − 530 | + 820 | −250 |
| Pleiades | 41.5 | +23 48 | 134.7 | −22.2 | −.651 | + .658 | −.378 | 120 | II 3r N | 150 | 2 | − 98 | + 99 | − 57 |
| 1444 | 41.9 | +52 21 | 115.8 | − 0.4 | −.436 | + .900 | −.006 | 3.5 | II 3p | 3340 | ..... | −1460 | +3010 | − 20 |
| 1502 | 58.7 | +62 3 | 111.3 | + 8.5 | −.358 | + .922 | +.148 | 8 | II 3p | 1450 | 2 | − 520 | +1340 | +215 |
| 1513 | 4 2.6 | +49 15 | 120.3 | − 0.6 | −.505 | + .863 | −.010 | 9 | II 2m U | 1680 | ..... | − 850 | +1450 | − 17 |
| 1528 | 7.8 | +50 59 | 119.7 | + 1.2 | −.495 | + .868 | +.022 | 22 | II 2m | 900 | 2 | − 450 | + 780 | + 20 |
| I.C. 361 | 10.7 | +58 3 | 115.1 | + 6.6 | −.425 | + .900 | +.115 | 6 | III 2m | 3390 | ..... | −1440 | +3050 | +390 |
| Taurus | 14 | +15 23 | 147.0 | −22.4 | −.775 | + .503 | −.382 | 400 | II 3m | 37 | 2 | − 29 | + 19 | − 14 |
| 1545 | 13.4 | +50 0 | 121.1 | + 1.2 | −.516 | + .857 | +.021 | 15 | I 2p | 690 | ..... | − 360 | + 590 | + 14 |
| 1605 | 27.8 | +45 2 | 126.3 | − 0.5 | −.592 | + .806 | −.008 | 4.5 | III 2m | 4520 | ..... | −2680 | +3640 | − 36 |
| 1647 | 40.2 | +18 53 | 148.2 | −15.4 | −.820 | + .507 | −.265 | 35 | III 2m | 610 | 2 | − 500 | + 310 | −162 |
| 1662 | 42.9 | +10 45 | 155.5 | −19.7 | −.857 | + .390 | −.336 | 14 | II 2p | 840 | 1.5 | − 720 | + 330 | −282 |
| 1664 | 43.9 | +43 31 | 129.4 | + 0.7 | −.634 | + .773 | +.013 | 13 | II 2m | 1160 | ..... | − 740 | + 900 | + 15 |
| 1746 | 57.6 | +23 40 | 146.8 | − 9.3 | −.826 | + .540 | −.161 | 40 | IV 2m | 870 | 2 | − 720 | + 470 | −140 |

TABLE 16—(*Continued*)

| N.G.C. I.C. | R.A. 1900 | Decl. 1900 | Galactic Long. | Galactic Lat. | Galactic coordinates $x$ | $y$ | $z$ | Angul. diam. | Class | Adopted distance parsecs | Wt. | Rectangular coordinates $X$ | $Y$ | $Z$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1778 | 5h 1m3 | +36°55′ | 136°6 | − 0°7 | −.726 | + .687 | −.013 | 10′ | II 2p | 1170 | ..... | − 850 | + 800 | − 15 |
| 1807 | 4.9 | +16 24 | 153.9 | −12.1 | −.878 | + .431 | −.209 | 14 | I 2p | 720 | 1.5 | − 630 | + 310 | −151 |
| *1817 | 6.3 | +16 34 | 153.9 | −11.7 | −.879 | + .431 | −.202 | 16 | III 2r | 1400 | ..... | −1230 | + 600 | −283 |
| 1857 | 13.2 | +39 14 | 136.1 | + 2.5 | −.720 | + .693 | +.044 | 9 | II 2m | 1680 | ..... | −1210 | +1160 | + 74 |
| 1883 | 18.5 | +46 27 | 130.6 | + 7.4 | −.646 | + .752 | +.128 | 3 | I 2m | 4580 | ..... | −2960 | +3440 | +586 |
| 1893 | 19.2 | +33 18 | 141.7 | + 0.2 | −.784 | + .620 | +.003 | 15 | I 3m | 920 | ..... | − 720 | + 570 | + 3 |
| 1907 | 21.4 | +35 14 | 140.3 | + 1.6 | −.769 | + .638 | +.028 | 5 | I 2m | 2750 | ..... | −2110 | +1750 | + 77 |
| 1912 | 22.0 | +35 45 | 140.0 | + 2.0 | −.765 | + .642 | +.035 | 18 | II 2r | 870 | 2 | − 670 | + 560 | + 30 |
| 1960 | 29.5 | +34 4 | 142.2 | + 2.4 | −.790 | + .612 | +.042 | 16 | I 3m | 980 | 2 | − 770 | + 600 | + 41 |
| *1981 | 30.2 | − 4 30 | 175.7 | −17.5 | −.951 | + .071 | −.301 | 25 | II 3p | 490 | 2 | − 470 | + 30 | −147 |
| 2099 | 45.8 | +32 31 | 145.3 | + 4.5 | −.820 | + .568 | +.078 | 24 | I 1r | 820 | 2 | − 670 | + 470 | + 64 |
| 2112 | 48.7 | + 0 22 | 173.6 | −11.1 | −.975 | + .110 | −.193 | 9 | I 2m | 1530 | ..... | −1490 | + 170 | −295 |
| 2126 | 55.2 | +49 54 | 130.7 | +14.4 | −.632 | + .734 | +.248 | 5.5 | III 2p | 2810 | ..... | −1780 | +2060 | +697 |
| 2129 | 55.0 | +23 18 | 154.3 | + 1.6 | −.901 | + .433 | +.027 | 7 | II 3p | 1770 | ..... | −1590 | + 770 | + 48 |
| 2141 | 57.5 | +10 26 | 165.8 | − 4.3 | −.967 | + .245 | −.075 | 8 | IV 2r | 4860 | ..... | −4700 | +1190 | −365 |
| An. 4 | 58.9 | +24 0 | 154.2 | + 2.7 | −.899 | + .435 | +.047 | 5.5 | II 2p | 2130 | ..... | −1910 | + 930 | +100 |
| 2158 | 6 1.3 | +24 6 | 154.3 | + 3.2 | −.900 | + .432 | +.056 | 4.5 | I 2r | 3360 | ..... | −3020 | +1450 | +188 |
| 2168 | 2.7 | +24 21 | 154.3 | + 3.6 | −.899 | + .433 | +.063 | 29 | III 3r | 840 | 2 | − 760 | + 360 | + 53 |
| 2169 | 2.8 | +13 58 | 163.3 | − 1.4 | −.958 | + .287 | −.025 | 6 | II 3p | 1950 | ..... | −1870 | + 560 | − 49 |
| 2186 | 6.8 | + 5 28 | 171.2 | − 4.7 | −.985 | + .152 | −.082 | 4.5 | I 3p | 2290 | ..... | −2260 | + 350 | −188 |
| 2192 | 8.2 | +39 53 | 141.0 | +12.0 | −.760 | + .616 | +.207 | 5 | III 2m | 4060 | ..... | −3090 | +2500 | +840 |
| 2194 | 8.2 | +12 50 | 164.9 | − 0.8 | −.966 | + .260 | −.015 | 6 | I 2r | 2520 | ..... | −2430 | + 660 | − 38 |
| 2204 | 11.3 | −18 37 | 193.5 | −14.7 | −.940 | − .226 | −.254 | 9 | I 2m | 1530 | ..... | −1440 | − 350 | −389 |
| 2215 | 16.0 | − 7 15 | 183.6 | − 8.7 | −.986 | − .063 | −.151 | 8.5 | II 2p | 1380 | ..... | −1360 | − 90 | −208 |
| 2236 | 24.3 | + 6 54 | 172.1 | − 0.2 | −.990 | + .138 | −.004 | 6 | I 2m | 2290 | ..... | −2270 | + 320 | − 9 |
| *2243 | 26.0 | −31 13 | 206.9 | −16.8 | −.854 | − .433 | −.290 | 4 | I 2m | 3440 | ..... | −2940 | −1490 | −998 |
| 2244 | 27.0 | + 4 56 | 174.1 | − 0.6 | −.995 | + .102 | −.010 | 27 | IV 3m N | 1340 | 2 | −1330 | + 140 | − 13 |
| 2251 | 29.3 | + 8 26 | 171.3 | + 1.6 | −.988 | + .151 | +.028 | 7 | II 2p | 1770 | ..... | −1750 | + 270 | + 50 |
| 2254 | 30.6 | + 7 45 | 172.1 | + 1.6 | −.990 | + .138 | +.028 | 3.8 | I 2p | 2710 | ..... | −2680 | + 370 | + 76 |
| An. 5 | 31.2 | + 9 31 | 170.6 | + 2.5 | −.986 | + .164 | +.044 | 9 | III 1r | 2490 | ..... | −2460 | + 410 | +110 |
| 2259 | 33.0 | +10 58 | 169.5 | + 3.6 | −.981 | + .182 | +.063 | 3.5 | I 3m | 3920 | ..... | −3850 | + 710 | +247 |
| 2264 | 35.5 | + 9 59 | 170.6 | + 3.7 | −.985 | + .162 | +.064 | 30 | II 3p N | 450 | 2 | − 440 | + 70 | + 29 |
| 2266 | 37.0 | +27 4 | 155.4 | +11.7 | −.890 | + .407 | +.203 | 6 | I 2r | 2520 | ..... | −2240 | +1030 | +512 |
| 2269 | 38.6 | + 4 40 | 175.7 | + 1.9 | −.997 | + .075 | +.033 | 3 | II 2p | 3900 | ..... | −3890 | + 290 | +129 |
| 2281 | 42.3 | +41 10 | 142.5 | +18.4 | −.753 | + .578 | +.316 | 15 | I 3p | 720 | 1.5 | − 540 | + 420 | +228 |
| 2286 | 42.6 | − 3 4 | 183.0 | − 0.9 | −.998 | − .052 | −.015 | 9 | III 2p | 1720 | ..... | −1720 | − 90 | − 26 |
| 2287 | 42.7 | −20 38 | 198.7 | − 9.0 | −.936 | − .316 | −.156 | 32 | I 3r | 410 | 2 | − 380 | − 130 | − 64 |
| 2301 | 46.6 | + 0 35 | 180.3 | + 1.7 | −1.000 | − .005 | +.030 | 15 | I 3m | 920 | ..... | − 920 | 0 | + 28 |
| 2304 | 49.2 | +18 8 | 164.9 | +10.4 | −.950 | + .257 | +.180 | 4 | I 1m | 3440 | ..... | −3270 | + 880 | +619 |
| 2309 | 51.2 | − 7 4 | 187.6 | − 0.9 | −.991 | − .131 | −.015 | 3 | II 2p | 3900 | ..... | −3860 | − 510 | − 59 |
| 2311 | 52.8 | − 4 27 | 185.4 | + 0.7 | −.995 | − .095 | +.013 | 4.5 | II 2p | 2600 | ..... | −2590 | − 250 | + 34 |
| 2323 | 58.2 | − 8 12 | 189.4 | + 0.1 | −.987 | − .163 | +.002 | 16 | I 2m | 830 | 1.5 | − 820 | − 140 | + 2 |
| 2324 | 59.0 | + 1 12 | 181.2 | + 4.8 | −.996 | − .020 | +.083 | 8 | I 2m | 1720 | ..... | −1710 | − 30 | +143 |
| 2335 | 7 1.8 | − 9 56 | 191.3 | + 0.1 | −.981 | − .196 | +.002 | 10 | II 2p | 1170 | ..... | −1150 | − 230 | + 2 |
| 2343 | 3.5 | −10 30 | 192.0 | + 0.2 | −.978 | − .208 | +.004 | 6 | II 3p | 1950 | ..... | −1910 | − 410 | + 8 |
| 2345 | 3.7 | −13 1 | 194.2 | − 1.0 | −.969 | − .246 | −.017 | 11 | II 2p U | 1060 | ..... | −1030 | − 260 | − 18 |
| 2353 | 9.8 | −10 8 | 192.4 | + 1.7 | −.976 | − .215 | +.030 | 20 | I 3m U | 760 | 1.5 | − 740 | − 160 | + 23 |
| 2354 | 10.1 | −25 34 | 206.0 | − 5.6 | −.894 | − .436 | −.098 | 20 | III 1m | 1020 | ..... | − 910 | − 440 | −100 |
| 2355 | 11.3 | +13 57 | 171.1 | +13.3 | −.962 | + .151 | +.230 | 7 | I 2m | 1960 | ..... | −1890 | + 300 | +451 |
| 2360 | 13.2 | −15 27 | 197.5 | − 0.1 | −.954 | − .301 | −.002 | 11 | I 2r | 1370 | ..... | −1310 | − 410 | − 3 |
| 2362 | 14.6 | −24 46 | 205.8 | − 4.4 | −.898 | − .434 | −.076 | 7 | I 3p | 1250 | 1.5 | −1120 | − 540 | − 95 |
| 2367 | 15.9 | −21 45 | 203.3 | − 2.6 | −.918 | − .395 | −.046 | 4.5 | II 3p E | 2600 | ..... | −2390 | −1030 | −120 |
| 2368 | 16.2 | −10 12 | 193.2 | + 3.1 | −.972 | − .229 | +.054 | 4 | II 2p | 2920 | ..... | −2840 | − 670 | +158 |
| 2374 | 19.4 | −13 4 | 196.1 | + 2.3 | −.960 | − .278 | +.041 | 4.5 | II 2p | 2600 | ..... | −2500 | − 720 | +107 |
| 2383 | 20.4 | −20 44 | 202.9 | − 1.2 | −.921 | − .390 | −.022 | 6 | I 2p | 1720 | ..... | −1580 | − 670 | − 38 |
| 2384 | 20.7 | −20 50 | 203.1 | − 1.2 | −.920 | − .392 | −.021 | 4.5 | II 3p | 2600 | ..... | −2390 | −1020 | − 55 |
| *2395 | 21.5 | +13 47 | 172.4 | +15.4 | −.955 | + .128 | +.266 | 14 | III 2p : | 1100 | ..... | −1050 | + 140 | +293 |
| An. 6 | 21.9 | −24 6 | 206.0 | − 2.6 | −.898 | − .438 | −.045 | 5.5 | I 2p | 1870 | ..... | −1680 | − 820 | − 84 |
| An. 7 | 23.1 | −23 50 | 205.9 | − 2.2 | −.898 | − .437 | −.039 | 5.5 | II 3p | 2130 | ..... | −1910 | − 930 | − 83 |
| Mel. 66 | 23.4 | −47 32 | 227.0 | −13.5 | −.663 | − .711 | −.234 | 11 | II 2m | 1370 | ..... | − 910 | − 970 | −321 |
| 2401 | 24.8 | −13 46 | 197.4 | + 3.1 | −.953 | − .299 | +.055 | 2 | I 2p : | 5150 | ..... | −4910 | −1540 | +283 |
| 2414 | 28.7 | −15 14 | 199.1 | + 3.2 | −.943 | − .327 | +.056 | 8 | I 3p | 1200 | ..... | −1130 | − 390 | + 67 |

TABLE 16—(*Continued*)

| N.G.C. I. C. | R. A. 1900 | Decl. 1900 | Galactic Long. | Galactic Lat. | Galactic coordinates x | y | z | Angul. diam. | Class | Adopted distance parsecs | Wt. | Rectangular coordinates X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2420 | 7h32m5 | +21°48′ | 165°8 | +21°1 | −.904 | + .229 | +.360 | 6′ | II 2m | 2520 | ..... | −2280 | + 580 | +907 |
| 2421 | 31.9 | −20 23 | 204.0 | + 1.3 | −.914 | − .406 | +.022 | 9 | II 2p | 1300 | ..... | −1190 | − 530 | + 29 |
| 2422 | 32.0 | −14 16 | 198.7 | + 4.4 | −.944 | − .320 | +.077 | 30 | II 3m | 480 | 2 | − 450 | − 150 | + 37 |
| 2423 | 32.5 | −13 38 | 198.2 | + 4.8 | −.946 | − .312 | +.084 | 19 | I 2m | 720 | ..... | − 680 | − 220 | + 61 |
| Mel. 71 | 32.9 | −11 50 | 196.7 | + 5.8 | −.953 | − .286 | +.102 | 8 | II 2m | 1890 | ..... | −1800 | − 540 | +193 |
| Mel. 72 | 33.7 | −10 27 | 195.6 | + 6.7 | −.956 | − .267 | +.116 | 5 | II 1p | 2340 | ..... | −2240 | − 620 | +272 |
| 2432 | 36.5 | −18 51 | 203.2 | + 3.0 | −.918 | − .394 | +.052 | 4.5 | II 2p | 2600 | ..... | −2390 | −1020 | +135 |
| 2437 | 37.2 | −14 35 | 199.6 | + 5.3 | −.938 | − .334 | +.093 | 27 | II 2r | 660 | 2 | − 620 | − 220 | + 61 |
| 2439 | 37.0 | −31 25 | 214.0 | − 3.4 | −.827 | − .559 | −.059 | 9 | II 3m U | 1680 | ..... | −1390 | − 940 | − 99 |
| 2447 | 40.4 | −23 38 | 207.8 | + 1.3 | −.885 | − .466 | +.023 | 18 | I 3r | 850 | 1.5 | − 750 | − 400 | + 20 |
| 2451 | 41.8 | −37 44 | 220.0 | − 5.8 | −.762 | − .639 | −.102 | 37 | I 3p | 285 | 2 | − 220 | − 180 | − 29 |
| 2453 | 43.6 | −27 0 | 211.0 | + 0.1 | −.857 | − .516 | +.002 | 4 | I 3p | 2580 | ..... | −2210 | −1330 | + 5 |
| 2455 | 44.6 | −21 3 | 206.1 | + 3.5 | −.897 | − .439 | +.060 | 5.5 | III 2p | 2810 | ..... | −2520 | −1230 | +169 |
| 2477 | 48.7 | −38 17 | 221.2 | − 4.9 | −.750 | − .656 | −.086 | 25 | I 2r | 600 | ..... | − 450 | − 390 | − 52 |
| 2482 | 50.7 | −24 2 | 209.4 | + 3.1 | −.870 | − .490 | +.054 | 11 | II 2p | 1060 | ..... | − 920 | − 520 | + 57 |
| An. 8 | 50.8 | −17 33 | 203.9 | + 6.6 | −.908 | − .402 | +.114 | 9 | II 1p | 1300 | ..... | −1180 | − 520 | +148 |
| An. 9 | 51.1 | −25 40 | 210.8 | + 2.3 | −.858 | − .512 | +.040 | 5 | II 3p | 2340 | ..... | −2010 | −1200 | + 94 |
| 2489 | 52.2 | −29 48 | 214.4 | + 0.2 | −.825 | − .565 | +.004 | 7 | I 2m | 1960 | ..... | −1620 | −1110 | + 8 |
| *2506 | 55.2 | −10 31 | 198.4 | +11.2 | −.931 | − .310 | +.194 | 11 | I 2r | 1370 | ..... | −1280 | − 420 | +266 |
| 2509 | 56.3 | −18 48 | 205.7 | + 7.0 | −.895 | − .430 | +.122 | 4.5 | I 2m : | 3050 | ..... | −2730 | −1310 | +372 |
| 2516 | 56.7 | −60 36 | 241.2 | −15.5 | −.464 | − .845 | −.267 | 50 | I 3r | 305 | 2 | − 140 | − 260 | − 81 |
| 2527 | 8 1.1 | −27 53 | 213.9 | + 2.9 | −.829 | − .557 | +.051 | 15 | II 2p | 780 | ..... | − 650 | − 430 | + 40 |
| 2533 | 3.0 | −29 37 | 215.6 | + 2.3 | −.813 | − .581 | +.040 | 4 | I 2p | 2580 | ..... | −2100 | −1500 | +103 |
| 2539 | 6.0 | −12 32 | 201.6 | +12.4 | −.908 | − .360 | +.214 | 22 | II 1m | 740 | 2 | − 670 | − 270 | +159 |
| *2546 | 8.7 | −37 20 | 222.6 | − 1.1 | −.736 | − .676 | −.020 | 45 | III 2p | 420 | 2 | − 310 | − 280 | − 8 |
| 2547 | 7.7 | −48 58 | 232.1 | − 7.9 | −.609 | − .781 | −.138 | 17 | II 3p | 690 | 1.5 | − 420 | − 540 | − 95 |
| 2548 | 8.8 | − 5 30 | 195.8 | +16.7 | −.922 | − .261 | +.287 | 30 | I 2r | 470 | 2 | − 430 | − 120 | +135 |
| 2567 | 14.6 | −30 20 | 217.6 | + 3.9 | −.791 | − .608 | +.069 | 11 | II 2m | 1370 | ..... | −1080 | − 830 | + 95 |
| 2571 | 14.9 | −29 26 | 216.9 | + 4.5 | −.797 | − .598 | +.078 | 10 | I 3p | 1030 | ..... | − 820 | − 620 | + 80 |
| 2588 | 19.2 | −32 39 | 220.0 | + 3.3 | −.764 | − .642 | +.058 | 2.5 | II 2p : | 4680 | ..... | −3580 | −3000 | +271 |
| 2627 | 33.1 | −29 36 | 219.4 | + 7.6 | −.766 | − .629 | +.132 | 8 | II 2m | 1890 | ..... | −1450 | −1190 | +249 |
| 2632 | 34.3 | +20 20 | 173.3 | +34.0 | −.824 | + .096 | +.559 | 90 | I 2r | 150 | 2 | − 123 | + 14 | + 84 |
| 2635 | 34.5 | −34 25 | 223.4 | + 4.8 | −.724 | − .684 | +.084 | 3 | II 2m : | 5040 | ..... | −3650 | −3450 | +423 |
| I.C.2391* | 37.4 | −52 42 | 237.8 | − 6.4 | −.528 | − .842 | −.111 | 45 | II 3p | 210 | 2 | − 110 | − 180 | − 23 |
| 2658 | 39.4 | −32 18 | 222.4 | + 6.9 | −.733 | − .669 | +.121 | 9 | I 2m | 1530 | ..... | −1120 | −1020 | +185 |
| 2659 | 39.2 | −44 36 | 231.8 | − 1.0 | −.618 | − .786 | −.017 | 11 | II 2m | 1370 | ..... | − 850 | −1080 | − 23 |
| I.C.2395 | 40.0 | −47 49 | 234.4 | − 3.0 | −.582 | − .812 | −.052 | 20 | II 3p | 500 | 2 | − 290 | − 410 | − 26 |
| *2669 | 42.0 | −52 36 | 238.3 | − 5.8 | −.523 | − .846 | −.101 | 14 | I 3p | 740 | ..... | − 390 | − 630 | − 75 |
| 2670 | 42.3 | −48 25 | 235.1 | − 3.0 | −.571 | − .819 | −.053 | 11 | I 2p | 940 | ..... | − 540 | − 770 | − 50 |
| 2671 | 42.6 | −41 31 | 229.9 | + 1.5 | −.644 | − .764 | +.026 | 4 | II 2p : | 2920 | ..... | −1880 | −2230 | + 76 |
| An. 10 | 44.2 | −42 7 | 230.5 | + 1.3 | −.635 | − .772 | +.023 | 30 | II 3p | 350 | 2 | − 220 | − 270 | + 8 |
| 2682 | 45.8 | +12 11 | 183.6 | +33.3 | −.834 | − .053 | +.549 | 18 | II 2r | 740 | 2 | − 620 | − 39 | +406 |
| 2818 | 9 12.0 | −36 12 | 229.9 | + 9.3 | −.636 | − .755 | +.161 | 9 | II 2m | 1680 | ..... | −1070 | −1270 | +271 |
| I.C.2488 | 24.6 | −56 32 | 245.4 | − 4.1 | −.415 | − .907 | −.071 | 18 | IV 2m U | 1970 | ..... | − 820 | −1790 | −140 |
| 2910 | 27.0 | −52 28 | 243.0 | − 0.8 | −.454 | − .890 | −.014 | 6 | II 2p | 1950 | ..... | − 890 | −1740 | − 27 |
| 2925 | 30.3 | −53 0 | 243.7 | − 0.9 | −.443 | − .896 | −.016 | 16 | II 2p | 730 | ..... | − 320 | − 650 | − 12 |
| 2972 | 36.7 | −49 52 | 242.4 | + 2.2 | −.462 | − .886 | +.038 | 4.5 | I 2m | 3050 | ..... | −1410 | −2700 | +116 |
| 3033 | 45.4 | −55 57 | 247.2 | − 1.8 | −.387 | − .922 | −.031 | 5 | II 2p | 2340 | ..... | − 910 | −2160 | − 73 |
| 3105 | 57.2 | −54 18 | 247.6 | + 0.5 | −.380 | − .925 | +.009 | 2.5 | II 2p : | 4680 | ..... | −1780 | −4330 | + 42 |
| 3114 | 59.5 | −59 38 | 250.9 | − 3.6 | −.326 | − .943 | −.064 | 37 | II 3r | 440 | 2 | − 140 | − 410 | − 28 |
| An. 11 | 10 1.9 | −61 7 | 252.0 | − 4.7 | −.307 | − .948 | −.082 | 4 | II 2p | 2920 | ..... | − 900 | −2770 | −239 |
| An. 12 | 3.2 | −59 49 | 251.4 | − 3.5 | −.318 | − .946 | −.062 | 4 | I 2m | 3440 | ..... | −1090 | −3250 | −213 |
| 3228 | 17.8 | −51 13 | 248.6 | + 4.8 | −.364 | − .927 | +.084 | 20 | I 2p | 500 | 2 | − 180 | − 460 | + 42 |
| An. 13 | 20.3 | −59 35 | 253.1 | − 2.2 | −.290 | − .956 | −.038 | 4 | II 2m : | 3770 | ..... | −1080 | −3600 | −143 |
| 3255 | 22.9 | −60 10 | 253.7 | − 2.5 | −.280 | − .959 | −.044 | 3 | I 2p : | 3430 | ..... | − 960 | −3290 | −151 |
| I.C.2581 | 23.7 | −57 8 | 252.3 | + 0.1 | −.304 | − .953 | +.002 | 10 | II 3p | 1170 | ..... | − 360 | −1120 | + 2 |
| 3293 | 32.0 | −57 43 | 253.6 | + 0.2 | −.283 | − .959 | +.003 | 8 | II 3r | 1990 | 1.5 | − 560 | −1910 | + 6 |
| 3330 | 34.6 | −53 37 | 251.9 | + 4.0 | −.309 | − .949 | +.069 | 8 | II 3m | 1890 | ..... | − 580 | −1790 | +130 |
| Mel. 101 | 38.6 | −64 34 | 257.4 | − 5.6 | −.217 | − .971 | −.097 | 15 | II 2m | 1010 | ..... | − 220 | − 980 | − 98 |
| I.C.2602 | 39.4 | −63 52 | 257.2 | − 4.9 | −.221 | − .972 | −.085 | 65 | II 3m | 210 | 2 | − 50 | − 200 | − 18 |
| An. 14 | 40.1 | −59 2 | 255.1 | − 0.5 | −.257 | − .966 | −.009 | 4 | I 3m N | 3440 | ..... | − 880 | −3320 | − 31 |
| An. 15 | 40.8 | −58 50 | 255.1 | − 0.3 | −.257 | − .967 | −.005 | 3.5 | I 3p | 2940 | ..... | − 760 | −2840 | − 15 |

TABLE 16—(*Continued*)

| N.G.C. I. C. | R. A. 1900 | Decl. 1900 | Galactic Long. | Galactic Lat. | Galactic coordinates $x$ | $y$ | $z$ | Angul. diam. | Class | Adopted distance parsecs | Wt. | Rectangular coordinates $X$ | $Y$ | $Z$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| An. 16 | $10^h41^m2$ | −59°11′ | 255°3 | − 0°6 | −.254 | − .967 | −.010 | 10′ | IV 3m NU | 3550 | ..... | − 900 | −3430 | − 36 |
| An. 17 | 52.2 | −58 41 | 256.4 | + 0.5 | −.236 | − .972 | +.008 | 5.5 | I 3p | 1870 | ..... | − 440 | −1820 | + 15 |
| 3496 | 55.8 | −59 48 | 257.2 | − 0.4 | −.221 | − .975 | −.007 | 7 | III 2m | 2900 | ..... | − 640 | −2830 | − 20 |
| 3532 | 11 2.2 | −58 8 | 257.4 | + 1.5 | −.218 | − .975 | +.026 | 55 | II 2r | 350 | 2 | − 80 | − 340 | + 9 |
| 3572 | 6.2 | −59 42 | 258.4 | + 0.2 | −.201 | − .980 | +.003 | 5 | II 3p | 2340 | ..... | − 470 | −2290 | + 7 |
| An. 18 | 7.2 | −60 8 | 258.6 | 0.0 | −.198 | − .982 | .000 | 12 | IV 3m | 2960 | ..... | − 590 | −2910 | 0 |
| An. 19 | 10.0 | −57 3 | 258.0 | + 2.8 | −.208 | − .977 | +.050 | 13 | IV 2m | 2730 | ..... | − 570 | −2670 | +136 |
| 3603 | 10.8 | −60 43 | 259.3 | − 0.6 | −.186 | − .983 | −.010 | 2 | I 3m : | 6880 | ..... | −1280 | −6760 | − 69 |
| I.C.2714 | 13.6 | −62 10 | 260.1 | − 1.8 | −.172 | − .984 | −.032 | 11 | II 2r | 1500 | ..... | − 260 | −1480 | − 48 |
| Mel. 105 | 15.2 | −62 58 | 260.5 | − 2.5 | −.164 | − .985 | −.044 | 4 | I 3m | 3440 | ..... | − 560 | −3390 | −151 |
| 3680 | 20.9 | −42 41 | 254.9 | +17.0 | −.249 | − .923 | +.292 | 12 | I 2p | 860 | ..... | − 210 | − 790 | +251 |
| 3766 | 31.5 | −61 3 | 261.8 | − 0.1 | −.142 | − .990 | −.002 | 12 | I 2r | 1220 | 1.5 | − 170 | −1210 | − 2 |
| *3960 | 45.9 | −55 8 | 262.3 | + 6.1 | −.134 | − .986 | +.106 | 7 | I 2m | 1960 | ..... | − 260 | −1930 | +208 |
| *4052 | 56.8 | −62 38 | 265.1 | − 1.0 | −.086 | − .996 | −.018 | 10 | II 2m E | 1510 | ..... | − 130 | −1500 | − 27 |
| 4103 | 12 1.5 | −60 41 | 265.3 | + 1.0 | −.082 | − .997 | +.017 | 9 | I 3m | 1530 | ..... | − 130 | −1530 | + 26 |
| 4230 | 11.8 | −54 45 | 265.9 | + 7.0 | −.071 | − .980 | +.122 | 4.5 | II 2p : | 2600 | ..... | − 180 | −2550 | +317 |
| 4337 | 18.5 | −57 34 | 267.1 | + 4.2 | −.050 | − .996 | +.073 | 4 | I 2p | 2580 | ..... | − 130 | −2570 | +188 |
| 4349 | 19.0 | −61 20 | 267.5 | + 0.6 | −.044 | − .999 | +.010 | 17 | II 2r | 970 | ..... | − 40 | − 970 | + 10 |
| Coma | 20 | +26 40 | 195.6 | +85.2 | −.078 | − .022 | +.997 | 300 | II 3p | 81 | 2 | − 6 | − 2 | + 81 |
| 4439 | 22.9 | −59 32 | 267.8 | + 2.4 | −.038 | − .998 | +.042 | 3.5 | II 2p | 3340 | ..... | − 130 | −3330 | +140 |
| 4463 | 24.3 | −64 14 | 268.3 | − 2.3 | −.030 | − .999 | −.040 | 5 | I 3p | 2060 | ..... | − 60 | −2060 | − 82 |
| An. 20 | 33.9 | −60 3 | 269.2 | + 1.9 | −.013 | − .999 | +.034 | 10 | III 2r | 2240 | ..... | − 30 | −2240 | + 76 |
| 4609 | 36.5 | −62 25 | 269.6 | − 0.4 | −.007 | −1.000 | −.007 | 4.5 | II 2p | 2600 | ..... | − 20 | −2600 | − 18 |
| 4755 | 47.7 | −59 48 | 270.9 | + 2.2 | +.017 | − .999 | +.038 | 12 | I 3r | 1090 | 1.5 | + 20 | −1090 | + 41 |
| 4815 | 51.8 | −64 25 | 271.3 | − 2.4 | +.022 | − .999 | −.043 | 4 | I 3m | 3440 | ..... | + 80 | −3440 | −148 |
| 4852 | 54.1 | −59 4 | 271.8 | + 2.9 | +.032 | − .998 | +.050 | 11 | II 2m | 1370 | ..... | + 40 | −1370 | + 68 |
| 5138 | 13 20.9 | −58 29 | 275.3 | + 3.1 | +.093 | − .994 | +.054 | 8 | II 2p | 1460 | ..... | + 140 | −1450 | + 78 |
| 5168 | 24.6 | −60 25 | 275.5 | + 1.1 | +.095 | − .995 | +.019 | 3.2 | I 3m | 4300 | ..... | + 410 | −4280 | + 82 |
| An. 21 | 25.5 | −62 17 | 275.3 | − 0.7 | +.092 | − .996 | −.013 | 5 | I 3p | 2060 | ..... | + 190 | −2050 | − 27 |
| 5281 | 39.7 | −62 24 | 276.8 | − 1.2 | +.119 | − .993 | −.021 | 4 | I 3m | 3440 | ..... | + 410 | −3420 | − 72 |
| 5288 | 41.6 | −64 11 | 276.7 | − 3.0 | +.116 | − .992 | −.052 | 3 | II 2p | 3900 | ..... | + 450 | −3870 | −203 |
| 5316 | 46.9 | −61 22 | 277.9 | − 0.4 | +.138 | − .990 | −.007 | 11 | II 2p | 950 | 1.5 | + 130 | − 940 | − 7 |
| 5460 | 14 1.2 | −47 50 | 283.8 | +12.0 | +.233 | − .950 | +.208 | 35 | II 3m | 470 | 2 | + 110 | − 450 | + 98 |
| 5606 | 20.5 | −59 11 | 282.5 | + 0.4 | +.217 | − .986 | +.006 | 2.2 | II 2p : | 5310 | ..... | +1150 | −5240 | + 32 |
| 5617 | 22.3 | −60 16 | 282.4 | − 0.7 | +.214 | − .977 | −.013 | 14 | I 2r | 1010 | 1.5 | + 220 | − 990 | − 13 |
| An. 22 | 23.7 | −60 43 | 282.3 | − 1.2 | +.214 | − .977 | −.021 | 7 | III 2p | 2210 | ..... | + 470 | −2160 | − 46 |
| 5662 | 28.0 | −56 7 | 284.7 | + 2.8 | +.253 | − .966 | +.049 | 15 | II 3p E | 670 | 2 | + 170 | − 650 | + 33 |
| 5715 | 36.1 | −57 7 | 285.3 | + 1.4 | +.264 | − .964 | +.025 | 9 | II 2m | 1680 | ..... | + 440 | −1620 | + 42 |
| 5749 | 41.8 | −54 6 | 287.4 | + 3.8 | +.297 | − .952 | +.066 | 8 | II 2p | 1460 | ..... | + 430 | −1390 | + 96 |
| 5764 | 46.5 | −52 16 | 288.8 | + 5.1 | +.321 | − .943 | +.088 | 2.2 | I 2p : | 4690 | ..... | +1510 | −4420 | +413 |
| 5822 | 57.9 | −53 57 | 289.5 | + 2.8 | +.333 | − .942 | +.049 | 40 | III 1m | 560 | 2 | + 190 | − 530 | + 27 |
| 5823 | 58.3 | −55 12 | 288.9 | + 1.7 | +.324 | − .946 | +.029 | 10 | III 2m | 2030 | ..... | + 660 | −1920 | + 59 |
| 5925 | 15 20.2 | −54 10 | 292.1 | + 0.8 | +.377 | − .926 | +.015 | 20 | III 2m | 1020 | ..... | + 380 | − 940 | + 15 |
| 5999 | 44.3 | −56 10 | 293.7 | − 2.8 | +.401 | − .915 | −.049 | 6 | I 2r | 2520 | ..... | +1010 | −2310 | −123 |
| 6005 | 47.8 | −57 8 | 293.4 | − 3.8 | +.397 | − .916 | −.067 | 3.5 | I 2m | 3920 | ..... | +1560 | −3590 | −263 |
| An. 23 | 52.8 | −53 14 | 296.5 | − 1.4 | +.446 | − .894 | −.024 | 5 | III 2p | 3090 | ..... | +1380 | −2760 | − 74 |
| 6025 | 55.2 | −60 13 | 292.1 | − 6.8 | +.374 | − .920 | −.119 | 11 | II 3 p | 1020 | 1.5 | + 380 | − 940 | −121 |
| 6031 | 59.8 | −53 47 | 296.9 | − 2.5 | +.452 | − .891 | −.043 | 2 | II 2p : | 5850 | ..... | +2650 | −5210 | −252 |
| 6067 | 16 5.4 | −53 57 | 297.4 | − 3.2 | +.460 | − .886 | −.055 | 16 | I 2r | 940 | ..... | + 430 | − 830 | − 52 |
| 6087 | 10.6 | −57 39 | 295.3 | − 6.3 | +.425 | − .898 | −.110 | 18 | II 3m | 870 | 1.5 | + 370 | − 780 | − 96 |
| 6124 | 18.8 | −40 26 | 308.6 | + 4.8 | +.621 | − .779 | +.084 | 25 | I 3r | 620 | 2 | + 390 | − 480 | + 52 |
| 6134 | 20.3 | −48 55 | 302.6 | − 1.2 | +.539 | − .842 | −.022 | 8 | II 2r | 2060 | ..... | +1110 | −1730 | − 45 |
| 6152 | 24.9 | −52 24 | 300.6 | − 4.2 | +.507 | − .859 | −.072 | 20 | I 2m | 690 | ..... | + 350 | − 590 | − 50 |
| 6167 | 26.8 | −49 23 | 303.1 | − 2.4 | +.544 | − .836 | −.041 | 14 | I 3m | 980 | ..... | + 530 | − 820 | − 40 |
| 6178 | 28.5 | −45 25 | 306.1 | + 0.1 | +.589 | − .808 | +.002 | 4 | II 3p | 2920 | ..... | +1720 | −2360 | + 6 |
| 6192 | 33.3 | −43 10 | 308.4 | + 1.0 | +.621 | − .784 | +.017 | 7 | I 3m | 1960 | ..... | +1220 | −1540 | + 33 |
| 6204 | 39.1 | −46 50 | 306.3 | − 2.2 | +.591 | − .806 | −.038 | 6 | II 2m | 2520 | ..... | +1490 | −2030 | − 96 |
| 6208 | 41.5 | −53 38 | 301.3 | − 6.8 | +.515 | − .848 | −.119 | 20 | IV 2m U | 1770 | ..... | + 910 | −1500 | −211 |
| *6216 | 42.2 | −44 33 | 308.4 | − 1.1 | +.620 | − .784 | −.020 | 3.8 | II 2m | 3970 | ..... | +2460 | −3110 | − 79 |
| 6231 | 47.0 | −41 38 | 311.2 | 0.0 | +.658 | − .753 | .000 | 16 | I 3r | 980 | 1.5 | + 640 | − 740 | 0 |

TABLE 16—(*Continued*)

| N.G.C. I.C. | R. A. 1900 | Decl. 1900 | Galactic Long. | Galactic Lat. | Galactic coordinates x | y | z | Angul. diam. | Class | Adopted distance parsecs | Wt. | Rectangular coordinates X | Y | Z |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| An. 24 | 16h48m8 | −40°33′ | 312°2 | + 0°4 | +.672 | − .740 | +.008 | 60′ | IV 3m N | 580 | 2 | + 390 | − 430 | + 5 |
| 6242 | 48.8 | −39 20 | 313.2 | + 1.2 | +.684 | − .729 | +.021 | 11 | I 3m | 1280 | 1.5 | + 880 | − 930 | + 27 |
| 6249 | 50.4 | −44 37 | 309.2 | − 2.3 | +.632 | − .774 | −.041 | 7 | II 2p | 1770 | ..... | +1120 | −1370 | − 73 |
| 6253 | 51.2 | −52 33 | 303.0 | − 7.3 | +.541 | − .832 | −.127 | 5 | I 2m | 2750 | ..... | +1490 | −2290 | −349 |
| 6259 | 53.5 | −44 31 | 309.6 | − 2.7 | +.637 | − .769 | −.047 | 16 | IV 2r | 2430 | ..... | +1550 | −1870 | −114 |
| 6268 | 55.2 | −39 35 | 313.7 | + 0.1 | +.691 | − .722 | +.001 | 7 | II 2p | 1770 | ..... | +1220 | −1280 | + 2 |
| 6281 | 58.0 | −37 45 | 315.6 | + 0.8 | +.714 | − .700 | +.013 | 9 | II 2p | 1300 | ..... | + 930 | − 910 | + 17 |
| 6318 | 17 10.8 | −39 20 | 315.7 | − 2.2 | +.716 | − .697 | −.038 | 5 | II 2m | 3020 | ..... | +2160 | −2100 | −115 |
| *6322 | 11.3 | −42 50 | 312.9 | − 4.3 | +.679 | − .730 | −.075 | 9 | II 3p | 1230 | 1.5 | + 840 | − 900 | − 92 |
| I.C.4651 | 16.9 | −49 50 | 307.6 | − 9.0 | +.603 | − .782 | −.157 | 14 | II 2r | 1180 | ..... | + 710 | − 920 | −185 |
| An. 25 | 17.8 | −38 54 | 316.9 | − 3.1 | +.729 | − .683 | −.054 | 5 | I 2p | 2000 | ..... | +1460 | −1370 | −108 |
| An. 26 | 21.9 | −29 23 | 325.3 | + 1.5 | +.821 | − .569 | +.027 | 7.5 | II 2p | 1560 | ..... | +1280 | − 890 | + 42 |
| 6383 | 28.2 | −32 30 | 323.4 | − 1.3 | +.802 | − .596 | −.023 | 5.5 | II 3p | 2130 | ..... | +1710 | −1270 | − 49 |
| An. 27 | 29.6 | −33 25 | 322.7 | − 2.0 | +.795 | − .605 | −.036 | 8 | I 2p | 1290 | ..... | +1030 | − 780 | − 46 |
| An. 28 | 30.2 | −32 25 | 323.7 | − 1.6 | +.805 | − .592 | −.028 | 7.5 | II 2p | 1560 | ..... | +1260 | − 920 | − 44 |
| 6396 | 31.5 | −34 56 | 321.7 | − 3.2 | +.783 | − .619 | −.056 | 4 | I 2p | 2580 | ..... | +2020 | −1600 | −144 |
| 6400 | 32.7 | −36 53 | 320.2 | − 4.4 | +.766 | − .638 | −.078 | 7.5 | I 2p | 1370 | ..... | +1050 | − 870 | −107 |
| 6404 | 33.0 | −33 11 | 323.3 | − 2.5 | +.802 | − .597 | −.044 | 5 | II 3m | 3020 | ..... | +2420 | −1800 | −133 |
| 6405 | 33.5 | −32 9 | 324.3 | − 2.0 | +.811 | − .584 | −.036 | 26 | II 3m | 520 | 2 | + 420 | − 300 | − 19 |
| An. 29 | 34.7 | −40 3 | 317.7 | − 6.5 | +.734 | − .669 | −.112 | 14 | II 2p | 830 | ..... | + 610 | − 560 | − 93 |
| 6416 | 37.8 | −32 18 | 324.6 | − 2.9 | +.814 | − .578 | −.051 | 22 | III 2p | 730 | 1.5 | + 590 | − 420 | − 37 |
| 6425 | 40.5 | −31 29 | 325.6 | − 3.0 | +.824 | − .564 | −.052 | 10 | II 1p | 1170 | ..... | + 960 | − 660 | − 61 |
| I.C.4665 | 41.4 | + 5 45 | 358.2 | +15.6 | +.962 | − .030 | +.269 | 50 | II 2p | 280 | 2 | + 270 | − 10 | + 75 |
| 6451 | 44.3 | −30 11 | 327.2 | − 3.0 | +.839 | − .542 | −.052 | 6 | II 2m | 2520 | ..... | +2110 | −1370 | −131 |
| 6469 | 46.9 | −22 19 | 334.2 | + 0.5 | +.900 | − .435 | +.009 | 15 | IV 2m | 2370 | ..... | +2130 | −1030 | + 21 |
| 6475 | 47.3 | −34 47 | 323.5 | − 5.9 | +.800 | − .592 | −.102 | 50 | I 3m | 255 | 2 | + 200 | − 150 | − 26 |
| An. 30 | 49.7 | −35 18 | 323.3 | − 6.6 | +.796 | − .594 | −.114 | 15 | IV 3m | 2370 | ..... | +1890 | −1410 | −270 |
| 6494 | 51.0 | −19 0 | 337.6 | + 1.4 | +.924 | − .382 | +.024 | 27 | I 2r | 660 | 2 | + 610 | − 250 | + 16 |
| An. 31 | 53.6 | −28 10 | 329.9 | − 3.7 | +.864 | − .500 | −.065 | 4.5 | II 2m | 3360 | ..... | +2900 | −1680 | −218 |
| 6520 | 57.1 | −27 54 | 330.5 | − 4.3 | +.868 | − .490 | −.074 | 4.5 | II 3m | 3360 | ..... | +2920 | −1650 | −249 |
| 6530 | 58.6 | −24 20 | 333.8 | − 2.8 | +.896 | − .440 | −.049 | 14 | II 2m N | 1090 | 2 | + 980 | − 480 | − 53 |
| 6531 | 58.6 | −22 30 | 335.4 | − 1.9 | +.909 | − .416 | −.033 | 12 | I 3p | 980 | 2 | + 890 | − 410 | − 32 |
| *6546 | 18 1.2 | −23 19 | 335.0 | − 2.8 | +.905 | − .422 | −.049 | 13 | IV 3r | 3000 | ..... | +2720 | −1270 | −147 |
| 6568 | 6.8 | −21 37 | 337.1 | − 3.1 | +.920 | − .389 | −.055 | 15 | III 1m | 1350 | ..... | +1240 | − 530 | − 74 |
| 6583 | 9.8 | −22 10 | 337.0 | − 4.0 | +.918 | − .390 | −.070 | 3.8 | I 2m | 3620 | ..... | +3320 | −1410 | −253 |
| An. 32 | 11.9 | −13 23 | 344.9 | − 0.2 | +.966 | − .260 | −.004 | 5.5 | I 2p | 1870 | ..... | +1810 | − 490 | − 7 |
| 6603 | 12.6 | −18 27 | 340.6 | − 2.8 | +.942 | − .333 | −.049 | 4.5 | I 2r | 3360 | ..... | +3160 | −1120 | −165 |
| 6604 | 12.5 | −12 16 | 346.0 | + 0.2 | +.970 | − .243 | +.004 | 2.8 | I 3p | 3680 | ..... | +3570 | − 890 | + 15 |
| 6611 | 13.2 | −13 49 | 344.7 | − 0.7 | +.964 | − .264 | −.012 | 8 | II 3m N | 2050 | 2 | +1980 | − 540 | − 25 |
| 6613 | 14.1 | −17 10 | 341.8 | − 2.5 | +.949 | − .311 | −.043 | 7 | II 3p | 1780 | 1.5 | +1690 | − 550 | − 77 |
| An. 33 | 18.8 | −19 44 | 340.1 | − 4.7 | +.937 | − .339 | −.082 | 5.5 | I 3p | 1870 | ..... | +1750 | − 630 | −153 |
| 6631 | 21.6 | −12 6 | 347.2 | − 1.7 | +.975 | − .222 | −.029 | 4.5 | I 2p | 2290 | ..... | +2230 | − 510 | − 66 |
| 6633 | 22.7 | + 6 30 | 3.8 | + 6.8 | +.991 | + .065 | +.119 | 25 | I 2p E | 380 | 2 | + 380 | + 20 | + 45 |
| I.C.4725 | 25.8 | −19 19 | 341.3 | − 6.0 | +.942 | − .320 | −.104 | 35 | IV 3r | 980 | 2 | + 920 | − 310 | −102 |
| 6645 | 26.8 | −16 58 | 343.5 | − 5.1 | +.955 | − .284 | −.088 | 13 | I 2r | 1230 | 2 | +1170 | − 350 | −108 |
| 6649 | 27.9 | −10 28 | 349.4 | − 2.3 | +.982 | − .184 | −.040 | 7.5 | I 2m | 1830 | ..... | +1800 | − 340 | − 73 |
| 6664 | 31.3 | − 8 18 | 351.7 | − 2.0 | +.989 | − .145 | −.035 | 20 | IV 2m | 1770 | ..... | +1750 | − 260 | − 62 |
| I.C.4756 | 34.0 | + 5 22 | 4.0 | + 3.8 | +.995 | + .070 | +.067 | 50 | III 1m | 405 | 2 | + 400 | + 30 | + 27 |
| An. 34 | 34.4 | − 8 34 | 351.8 | − 2.8 | +.988 | − .143 | −.048 | 10 | II 2p | 1170 | ..... | +1160 | − 170 | − 56 |
| An. 35 | 37.7 | − 4 14 | 356.0 | − 1.4 | +.997 | − .070 | −.025 | 5 | II 2m | 3020 | ..... | +3010 | − 210 | − 76 |
| 6694 | 39.8 | − 9 30 | 351.6 | − 4.4 | +.986 | − .146 | −.077 | 9 | II 2m | 1610 | 2 | +1590 | − 240 | −124 |
| 6704 | 45.5 | − 5 19 | 355.9 | − 3.7 | +.995 | − .071 | −.065 | 5 | I 2p | 2060 | ..... | +2050 | − 150 | −134 |
| 6705 | 45.7 | − 6 23 | 355.0 | − 4.3 | +.994 | − .087 | −.074 | 12.5 | II 2r | 1340 | 2 | +1330 | − 120 | − 99 |
| 6709 | 46.7 | +10 14 | 9.8 | + 3.3 | +.984 | + .170 | +.058 | 12 | II 2p | 940 | 2 | + 920 | + 160 | + 55 |
| 6716 | 48.6 | −20 1 | 343.1 | −11.1 | +.939 | − .285 | −.192 | 8 | II 3p | 1320 | 1.5 | +1240 | − 380 | −253 |
| 6717 | 49.1 | −22 50 | 340.5 | −12.4 | +.921 | − .325 | −.214 | 3 | II 2p | 3900 | ..... | +3590 | −1270 | −835 |
| 6755 | 19 2.8 | + 4 4 | 6.3 | − 3.1 | +.992 | + .109 | −.054 | 15 | IV 2m | 2370 | ..... | +2350 | + 260 | −128 |
| 6756 | 3.7 | + 4 31 | 6.8 | − 3.1 | +.992 | + .118 | −.054 | 4 | I 2p | 2570 | ..... | +2550 | + 300 | −139 |
| 6802 | 26.2 | +20 4 | 23.0 | − 0.3 | +.920 | + .391 | −.005 | 5 | III 1m E | 4060 | ..... | +3740 | +1590 | − 20 |
| 6811 | 35.2 | +46 20 | 46.9 | +11.2 | +.671 | + .716 | +.194 | 13 | III 1p | 950 | 2 | + 640 | + 680 | +184 |
| 6819 | 37.9 | +39 57 | 41.5 | + 7.6 | +.742 | + .657 | +.132 | 6 | I 2r | 2520 | ..... | +1870 | +1660 | +332 |

TABLE 16—(*Concluded*)

| N.G.C. I. C. | R. A. 1900 | Decl. 1900 | Galactic Long. | Galactic Lat. | Galactic coordinates $x$ | $y$ | $z$ | Angul. diam. | Class | Adopted distance parsecs | Wt. | Rectangular coordinates $X$ | $Y$ | $Z$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6823 | 19h38m9 | +23° 4′ | 27°1 | − 1°3 | +.890 | + .456 | −.022 | 7′ | IV 3p | 3890 | ..... | +3460 | +1770 | − 86 |
| 6830 | 46.8 | +22 50 | 27.9 | − 3.0 | +.883 | + .467 | −.052 | 10 | IV 2m | 3550 | ..... | +3140 | +1660 | −185 |
| 6834 | 48.2 | +29 9 | 33.4 | + 0.2 | +.835 | + .550 | +.003 | 7 | I 2m | 1960 | ..... | +1640 | +1080 | + 6 |
| Mel. 227 | 59 | −79 36 | 281.4 | −31.0 | +.170 | − .840 | −.516 | 60 | II 2p | 240 | 2 | + 40 | − 200 | −124 |
| 6866 | 20 0.5 | +43 43 | 47.0 | + 6.0 | +.679 | + .727 | +.105 | 10 | I 2p | 960 | 2 | + 650 | + 700 | +101 |
| 6871 | 2.1 | +35 30 | 40.3 | + 1.2 | +.762 | + .647 | +.021 | 25 | IV 3p | 1340 | 1.5 | +1020 | + 870 | + 28 |
| An. 36 | 7.0 | +40 55 | 45.3 | + 3.5 | +.702 | + .710 | +.060 | 6 | III 1r | 3740 | ..... | +2630 | +2660 | +224 |
| *6882 | 7.5 | +26 15 | 33.3 | − 5.0 | +.832 | + .548 | −.087 | 8 | I 2p | 1350 | 2 | +1120 | + 740 | −117 |
| 6883 | 7.5 | +35 33 | 41.0 | + 0.3 | +.755 | + .656 | +.005 | 15 | IV 3p | 1810 | ..... | +1370 | +1190 | + 9 |
| *6885 | 7.8 | +26 11 | 33.3 | − 5.1 | +.832 | + .547 | −.089 | 22 | III 2p | 630 | 2 | + 520 | + 340 | − 56 |
| I.C.4996 | 12.8 | +37 20 | 43.0 | + 0.5 | +.731 | + .682 | +.008 | 6 | I 3p | 1840 | 2 | +1340 | +1250 | + 15 |
| 6910 | 19.5 | +40 27 | 46.3 | + 1.3 | +.690 | + .723 | +.022 | 13 | IV 3p | 2090 | ..... | +1440 | +1510 | + 46 |
| 6913 | 20.3 | +38 12 | 44.6 | − 0.2 | +.711 | + .702 | −.004 | 7 | III 3p | 2100 | 2 | +1490 | +1470 | − 8 |
| 6939 | 29.4 | +60 18 | 63.3 | +11.9 | +.440 | + .874 | +.207 | 8 | II 1r | 1800 | 2 | + 790 | +1570 | +373 |
| 6940 | 30.4 | +27 58 | 37.8 | − 8.1 | +.783 | + .606 | −.141 | 26 | III 1m | 800 | 2 | + 630 | + 480 | −113 |
| 7031 | 21 4.1 | +50 26 | 59.0 | + 1.8 | +.515 | + .856 | +.031 | 7 | II 3p U | 1770 | ..... | + 910 | +1510 | + 55 |
| I.C.1369 | 8.7 | +47 20 | 57.3 | − 0.9 | +.540 | + .842 | −.016 | 3 | II 2p | 3900 | ..... | +2110 | +3280 | − 62 |
| 7044 | 9.2 | +42 5 | 53.7 | − 4.7 | +.590 | + .803 | −.082 | 5 | I 2r | 3020 | ..... | +1780 | +2420 | −248 |
| 7062 | 19.6 | +45 57 | 57.6 | − 3.2 | +.535 | + .844 | −.056 | 6 | II 2p | 1950 | ..... | +1050 | +1650 | −110 |
| 7063 | 20.4 | +36 4 | 51.0 | −10.6 | +.618 | + .764 | −.183 | 8 | II 2p | 1460 | ..... | + 900 | +1120 | −267 |
| *7067 | 20.6 | +47 35 | 58.8 | − 2.3 | +.518 | + .854 | −.040 | 2.5 | II 2p | 4680 | ..... | +2420 | +4000 | −187 |
| 7086 | 27.1 | +51 9 | 62.1 | − 0.2 | +.468 | + .884 | −.003 | 7.5 | I 2m | 1840 | ..... | + 860 | +1630 | − 6 |
| 7092 | 28.6 | +48 0 | 60.2 | − 2.7 | +.496 | + .867 | −.048 | 32 | II 2p | 330 | 2 | + 160 | + 280 | − 16 |
| An. 37 | 35.9 | +57 2 | 66.9 | + 3.5 | +.392 | + .918 | +.060 | 60 | IV 3r N | 730 | 1.5 | + 290 | + 670 | + 44 |
| 7128 | 40.6 | +53 15 | 65.0 | + 0.1 | +.422 | + .907 | +.002 | 3.2 | II 2p | 3650 | ..... | +1540 | +3310 | + 7 |
| 7142 | 43.5 | +65 20 | 72.9 | + 9.3 | +.291 | + .943 | +.162 | 11 | II 1m | 1370 | ..... | + 400 | +1290 | +222 |
| 7160 | 50.9 | +62 8 | 71.5 | + 6.3 | +.314 | + .943 | +.110 | 10 | I 3p | 1030 | ..... | + 320 | + 970 | +113 |
| 7209 | 22 1.2 | +46 0 | 63.4 | − 7.7 | +.444 | + .886 | −.134 | 20 | II 2p | 570 | 2 | + 250 | + 500 | − 76 |
| 7226 | 6.9 | +54 55 | 69.1 | − 0.8 | +.356 | + .934 | −.014 | 2.5 | II 2p | 4680 | ..... | +1670 | +4370 | − 66 |
| I.C.1434 | 6.7 | +52 20 | 67.7 | − 3.0 | +.379 | + .924 | −.052 | 7 | III 2m | 2900 | ..... | +1100 | +2680 | −151 |
| 7235 | 9.0 | +56 47 | 70.4 | + 0.6 | +.335 | + .942 | +.011 | 4.5 | II 3p | 2600 | ..... | + 870 | +2450 | + 29 |
| 7243 | 11.3 | +49 23 | 66.7 | − 5.8 | +.394 | + .914 | −.101 | 21 | III 2p | 750 | 2 | + 300 | + 690 | − 76 |
| 7245 | 11.5 | +53 50 | 69.1 | − 2.1 | +.356 | + .934 | −.036 | 4 | II 2p | 2920 | ..... | +1040 | +2730 | −105 |
| 7261 | 16.8 | +57 35 | 71.7 | + 0.7 | +.314 | + .949 | +.012 | 7 | II 2p | 1770 | ..... | + 560 | +1680 | + 21 |
| 7296 | 24.2 | +51 47 | 69.7 | − 4.8 | +.346 | + .935 | −.084 | 4 | II 2p | 2920 | ..... | +1010 | +2730 | −245 |
| 7380 | 43.0 | +57 34 | 74.8 | − 0.9 | +.262 | + .965 | −.015 | 9 | III 2p | 1840 | 2 | + 480 | +1770 | − 28 |
| 7419 | 50.3 | +60 18 | 76.8 | + 1.1 | +.228 | + .973 | +.014 | 2.2 | II 2p : | 5320 | ..... | +1210 | +5180 | + 74 |
| 7510 | 23 7.3 | +60 2 | 78.7 | + 0.1 | +.197 | + .980 | +.001 | 3 | II 2m U | 5040 | ..... | + 990 | +4940 | + 5 |
| 7654 | 19.8 | +61 3 | 80.4 | + 0.5 | +.166 | + .986 | +.009 | 13 | II 2r | 1360 | 2 | + 230 | +1340 | + 12 |
| 7686 | 25.4 | +48 34 | 77.5 | −11.6 | +.211 | + .956 | −.202 | 13 | I 3p | 790 | ..... | + 170 | + 760 | −160 |
| 7762 | 45.0 | +67 28 | 84.7 | + 6.0 | +.091 | + .990 | +.105 | 11 | I 1m U | 1250 | ..... | + 110 | +1240 | +131 |
| 7788 | 51.7 | +60 50 | 84.1 | − 0.6 | +.102 | + .995 | −.011 | 11 | IV 3p | 2470 | ..... | + 250 | +2460 | − 27 |
| 7789 | 52.0 | +56 10 | 83.3 | − 5.2 | +.116 | + .989 | −.091 | 19 | III 1r | 1140 | 2 | + 130 | +1130 | −104 |
| 7790 | 52.0 | +60 40 | 84.1 | − 0.8 | +.102 | + .990 | −.014 | 4.5 | II 2p | 2600 | ..... | + 270 | +2570 | − 36 |

## NOTES TO THE TABLE

NGC 637.—Position corrected by $\Delta\alpha = +1^m1$, $\Delta\delta = -2'$.

NGC 1817.—The cluster Melotte 29 should undoubtedly be identified with this and not with NGC 1807.

NGC 1981.—Position of NGC corrected.

NGC 2243.—R. A. corrected according to I. C. II Notes and Corrections.

NGC 2314.—This cluster was omitted from the list on account of its high galactic latitude. It may be a loose globular system similar to NGC 5053.

NGC 2395.—Position corrected according to Wolf, *A. N.* **231**, 231, 1927.

NGC 2506.—Position corrected according to I. C. II Notes and Corrections.

NGC 2546.—Position of center corrected according to chart of cluster in *Cordoba Photographs* p. 119.

I. C. 2391.—Position corrected so as to refer to center of cluster.

NGC 2669.—Declination of NGC corrected by +1°.

NGC 3960.—R. A. of NGC corrected by $-3^m6$, it must be the same cluster as Melotte 108.

NGC 4052.—R. A. of NGC corrected.

NGC 6216.—Melotte 152 should be identified with this and not with NGC 6222.

NGC 6322.—Position corrected.

NGC 6546.—Declination corrected.

NGC 6882, 6885.—In the region around 20 *Vulpeculae* two clusterings seem to be superposed: A loose clustering of 20–30 mostly faint stars, about 8′ in diameter, with center at $20^h\ 7^m\ 28^s$ $+26°\ 15'.0$ (1900) was identified with NGC 6882. The coarse group of a few bright stars clustered around 20 Vulpec, with a diameter of about 22′ was identified with NGC 6885. In each of the two clusters a physical relationship of the stars is indicated by the magnitude spectral class diagram, but the two clusters are evidently at different distances.

NGC 7067.—R. A. corrected according to I. C. II, Notes and Corrections.

The apparent diameter in minutes of arc (9th column) and the classification (10th column) were estimated as described on pages 159 and 160. Column 11 gives the finally adopted distance. For those clusters contained in Table 3 this is the mean of the spectroscopic determination (Table 3, column 9) and of the determination from apparent diameter and classification (Table 3, column 10). The result of each method normally received weight 1 but spectroscopic distances printed in italics were given half weight. The weight of the adopted distance in the 12th column of Table 16 is the sum of the weights of the two determinations. When no weight is entered in this column the adopted distance is based entirely on the diameter method, and the weight is 1. For the five nearest clusters the spectroscopic method is given double weight. In these cases the result of the diameter method is rejected as uncertain, owing to the difficulty of estimating the angular diameters of such large clusters. The probable error of unit weight is about $\pm 12\%$.

Thirty-seven of the clusters in Table 16 were not found in any previous cluster catalogue although some of them have been mentioned by other observers. Table 17 gives a brief description of these objects and of four others contained in the *New General Catalogue* but not correctly described as open clusters.

TABLE 17

| | |
|---|---|
| NGC 136 | Described in N. G. C. as globular cluster. A photograph taken with the Crossley Reflector ($40^m$, H. D. Curtis) shows it to be a small loose open cluster of 40–50 stars $13–18^m$. |
| An 1 | I. Roberts draws attention to a group of four $10–11^m$ stars in a straight line. These form the center of a well defined cluster of about 40 stars $10–15^m$. |
| An 2 | Mentioned by Barnard (*Bd. Atl.* pl. 1). Pretty well defined clustering of bright and faint stars, not rich, not quite regular. |
| NGC 1193 | Described in NGC as nebula. Photograph with Crossley Reflector ($90^m$, Curtis) shows it to be a typical open cluster with slightly over 100 stars $14–19^m$. |
| An 3 | Mentioned by Barnard (*Bd. Atl.* pl. 2). Very loose open cluster not rich, but regular in outline and structure. |
| NGC 2141 | Described in NGC as nebula. Photograph with Crossley Reflector ($30^m$) shows a well resolved thin but rich cluster of exclusively faint stars ($15–18^m$). |
| An 4 | Mentioned by Barnard (*Bd. Atl.* pl. 8). Loose and somewhat irregular group of about 30 stars $12–15^m$. |
| An 5 | Found on pl. 28 and 29 of *Publ. Lick Obs.* vol. 11, where it looks like a nebulosity. Photograph with Crossley Reflector ($30^m$) shows this to be a fairly large, thin cluster of even star distribution and regular outline, very rich in extremely faint stars ($>17^m$). |
| An 6 | Found on *Bd. Atl.* pl. 10. Pretty well defined clustering of faint stars, not rich, but regular in structure and outline. |
| An 7 | Mentioned by Barnard (*Bd. Atl.* pl. 10). Well defined dense clustering of 20–30 stars $10–15^m$, slightly irregular. |
| An 8 | Found on *F. A. Chart.* Pretty well marked thin cluster of faint stars, regular outline and structure. |
| An 9 | Found on *F. A. Chart.* Pretty well marked small group of about 15 bright and faint stars, slightly irregular. |
| An 10 | A large coarse cluster of a few bright and medium bright stars, observed by Dunlop (490) and Bailey. Both observations have been erroneously identified with the cluster NGC 2671, a small cluster of faint stars near by; it is probably for this reason that the cluster was omitted from the NGC. |
| An 11 | Found on *F. A. Chart.* Pretty well defined small cluster, not rich, but regular in outline. |
| An 12 | Found on *F. A. Chart.* Small cluster of regular outline and structure and marked central concentration. Pretty rich in faint stars. |
| An 13 | Found on *F. A. Chart.* Small, somewhat irregular, but dense clustering of faint stars. |
| An 14 | Observed by Gould (*Cordoba Photogr.* pl. 20) and shown in *H. A.* **26**, pl. 8, fig. 2. On a plate taken with the $37''$ reflector of the Chile Station by H. D. Curtis this appears as a typical open cluster of bright and faint stars, medium rich, with strong central concentration, slightly unsymmetrical. |
| An 15 | Noticed on Gould's chart of the $\eta$ *Carinae* region as well as on *H. A.* **60**, VIII pl. 3. Well defined small cluster with considerable concentration regular structure and outline. |
| An 16 | Cluster including $\eta$ *Carinae*, with its center somewhat south of this star, noted by many observers. NGC 3372 only refers to the nebula, and does not mention the existence of this cluster. |
| An 17 | Found on *F. A. Chart.* A small cluster of bright and faint stars, much concentrated at center. |
| An 18 | Found on *F. A. Chart.* Not very well defined clustering of bright and faint stars close to NGC 3572 and NGC 3590, but rather better marked than these. |
| An 19 | Found on *F. A. Chart.* Medium rich clustering of faint stars with regular outline, passes gradually into surroundings. |
| An 20 | Found on *F. A. Chart.* Rich cluster of regular outline composed exclusively of very faint stars nearly evenly scattered. |
| An 21 | Found on *F. A. Chart.* Small dense group of a few stars around two stars of $10^m$. Not well resolved on the chart. |
| An 22 | Found on *F. A. Chart.* Fairly well marked cluster of $12^m$ stars thinly but uniformly scattered with regular outline. |
| An 23 | Found on *F. A. Chart.* Thin cluster of very faint stars, not very conspicuous but regular in outline and structure. |
| An 24 | Mentioned by Barnard (*Bd. Atl.* pl. 16) as a group of stars about 1° in diameter. I. C. 4628 refers to nebulosity involved in the northern part of the cluster. |
| An 25 | Found on *F. A. Chart.* Pretty well defined small regular cluster of faint stars with noticeable central concentration. |

TABLE 17—(*Concluded*)

| | |
|---|---|
| An 26 | Mentioned by Barnard (*Bd. Atl.* pl. 18, 21, 22) as "a small group of considerable stars." |
| An 27 | Found on plate 22 of *Bd. Atl.* as a small somewhat irregular cluster of faint stars. |
| An 28 | Mentioned by Barnard (*Bd. Atl.* pl. 24). Fairly dense group of stars 11–14$^m$ with well defined outline but slightly unsymmetrical. |
| An 29 | Found on *F. A. Chart:* Curious cluster of triangular outline and nearly vacant center around which bright and faint stars are densely crowded. |
| An 30 | Mentioned by Barnard (*Bd. Atl.* pl. 24) as "a scattering cluster of smaller stars." |
| An 31 | Found on pl. 26 of *Bd. Atl.* as a small pretty dense cluster of well defined somewhat triangular outline in the Great *Sagittarius* cloud. |
| An 32 | Found on pl. 34 of *Bd. Atl.* A small, regular cluster of about 50 faint stars. |
| An 33 | Mentioned by Barnard (*Bd. Atl.* pl. 32) as "a small group of a few bright stars." |
| An 34 | Mentioned by Barnard (*Bd. Atl.* pl. 35) as "a small cluster or intensification of the Milky Way." |
| An 35 | Mentioned by Barnard (*Bd. Atl.* pl. 36) as "a very small detached mass of stars." |
| An 36 | Found on *Bd. Atl.* (pl. 44). According to a photograph with the Crossley Reflector (40$^m$) this is a very rich beautiful cluster of exclusively faint stars (17–19$^m$) densely but evenly scattered with regular circular outline. |
| An 37 | Found on *Bd. Atl.* (pl. 49): Large loose somewhat irregular cluster of bright and faint stars, triangular figure, involved in nebulosity. |
| NGC 7226 | Given in NGC as nebula in cluster. A photograph with the Crossley Reflector (1$^h$ 45$^m$, Curtis) shows only a small loose cluster of a few dozen faint stars. |

Table 16 should not be considered as a complete catalogue of open clusters, as only those were included which seemed to be suited for a determination of the distance. A number of the fainter or smaller clusters not well shown or unresolved on the photographic charts used had to be omitted for lack of adequate classification or diameter estimate. The writer has a more complete catalogue of star clusters in preparation which will be issued with descriptions for each object when the reobservation of the many doubtful objects has been completed.

For judging the completeness of our list as far as small and faint distant clusters are concerned it is of interest to state that only 34 objects of this class given in the *New General Catalogue* or the *Index Catalogues* with galactic latitudes of less than 20° had to be omitted for insufficient observations, while for about half a dozen globular clusters it seemed doubtful whether they should not be included as open clusters. It is also possible that some of the objects described as nebulae may turn out to be resolved into clusters when photographed with sufficiently large instruments, but the cases of this kind so far discovered are rather rare. It is surprising that despite the valuable help of modern photographic methods very few faint clusters in low galactic latitudes have been added to our knowledge during the last fifty years. In the two *Index Catalogues* containing the discoveries between 1888 and 1907 about 50 small clusters are listed (including the numerous cases marked as doubtful). It is remarkable, however, that these objects do not show the concentration toward the Milky Way so characteristic of open clusters, but that they are actually more numerous in high galactic latitudes. Most likely many of these are accidental groupings of three or four faint stars offering a nebulous appearance in a small telescope while some are perhaps globular clusters.

These various considerations make it probable that nearly all of the open star clusters of the Milky Way are at present known, and our list of Table 16 should be representative of the more prominent formations of this kind except for a deficiency of perhaps 30–50 among the more distant ones. We have thus for the first time a fairly adequate knowledge of the distances for a class of galactic objects and a study of their space distribution should furnish valuable information on the dimensions and structure of our Milky Way system.

## 11. The Apparent Distribution of Open Star Clusters

TABLE 18

Apparent Distribution of Open Clusters According to Galactic Longitude

| Galactic longitude | Number of of clusters | Constellations |
|---|---|---|
| 0°– 20° | 5 | Aquila. |
| 20 – 40 | 7 | Vulp.-Cygn. |
| 40 – 60 | 15 | Cygnus. |
| 60 – 80 | 19 | Cygn.-Ceph.-Lac. |
| 80 –100 | 21 | Cassiop. |
| 100 –120 | 19 | Camel.-Perseus |
| 120 –140 | 11 | Pers.-Auriga |
| 140 –160 | 17 | Auriga-Taur.-Gem. |
| 160 –180 | 19 | Monoceros. |
| 180 –200 | 27 | Monoc.-Can. Major |
| 200 –220 | 27 | Puppis |
| 220 –240 | 14 | Puppis-Vela. |
| 240 –260 | 29 | Vela-Carina |
| 260 –280 | 22 | Crux-Cent. |
| 280 –300 | 19 | Circin.-Norm. |
| 300 –320 | 22 | Norma-Scorp. |
| 320 –340 | 22 | Scorp.-Sag.-Ophiuch. |
| 340 –360 | 19 | Sagittar.-Scutum. |
| | 334 | |

TABLE 19

APPARENT DISTRIBUTION OF OPEN CLUSTERS ACCORDING TO GALACTIC LATITUDE

| Galactic latitude | | Longitude 90°–270° Number clusters | Longitude 90°–270° Per 1000 sq. degr. | Longitude 270°–90° Number clusters | Longitude 270°–90° Per 1000 sq. degr. | Total Number clusters | Total Per 1000 sq. degr. |
|---|---|---|---|---|---|---|---|
| −90° to | −30° | 0 | 0 | 1 | 0.1 | 1 | 0.1 |
| −30 | −20 | 3 | 0.9 | 0 | 0 | 3 | 0.9 |
| −20 | −15 | 5 | 2.9 | 0 | 0 | 5 | 2.9 |
| −15 | −10 | 8 | 4.5 | 4 | 2.3 | 12 | 6.8 |
| −10 | − 8 | 4 | 5.6 | 2 | 2.8 | 6 | 8.4 |
| − 8 | − 6 | 3 | 4.2 | 8 | 11.2 | 11 | 15.4 |
| − 6 | − 4 | 12 | 16.8 | 16 | 22.4 | 28 | 37.7 |
| − 4 | − 2 | 19 | 26.5 | 38 | 52.9 | 57 | 79.4 |
| − 2 | 0 | 34 | 47.3 | 28 | 38.9 | 62 | 86.2 |
| 0 | + 2 | 34 | 47.3 | 27 | 37.6 | 61 | 84.8 |
| + 2 | + 4 | 27 | 37.6 | 10 | 13.9 | 37 | 51.5 |
| + 4 | + 6 | 12 | 16.8 | 2 | 2.8 | 14 | 19.5 |
| + 6 | + 8 | 9 | 12.6 | 5 | 7.0 | 14 | 19.6 |
| + 8 | +10 | 2 | 2.8 | 1 | 1.4 | 3 | 4.2 |
| +10 | +15 | 7 | 4.0 | 3 | 1.7 | 10 | 5.7 |
| +15 | +20 | 4 | 2.3 | 1 | 0.6 | 5 | 2.9 |
| +20 | +30 | 1 | 0.3 | 1 | 0.3 | 2 | 0.6 |
| +30 | +90 | 3 | 0.3 | 0 | 0 | 3 | 0.3 |
| | | 187 | | 147 | | 334 | |

We shall first consider the apparent distribution in the sky of the 334 open clusters of Table 16. The distribution according to galactic longitude shown in Table 18 and Fig. 2 has a number of irregularities.

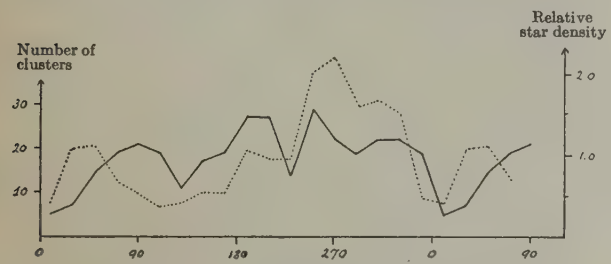


Fig. 2. Distribution of open clusters according to galactic longitude.

The full line gives the number of clusters per interval of 20° in galactic longitude. The dotted line represents the relative star density (mean of galactic latitudes +5°, 0°, −5°) according to Seares. The average star density of this belt is taken as unit and drawn in the same dimension as the average number (18.5) of clusters per 20° interval.

In the first place there is a deep minimum between longitude 0° and 50°, in the constellations *Aquila*, *Vulpecula* and *Cygnus*; *i. e.*, in the region where the dark division of the Milky Way is most pronounced. This seems to lend support to the hypothesis that the division is produced by a cloud of totally absorbing material which prevents us from seeing a part of the clusters in this region. Two less pronounced minima at longitudes 130° (*Perseus-Auriga*) and 230° (*Puppis-Vela*) also fall in somewhat poorer parts of the Milky Way. There is undoubtedly some similarity between the distribution in longitude of the star clusters and the fainter Milky Way stars; this is well exhibited by Fig. 2 in which the average density of stars brighter than 18$^m$ is drawn on the same scale by the dotted line. The data for the star density are taken from Seares[28] and are the means of galactic latitudes −5°, 0, and +5°. Strictly speaking, the two curves are not comparable because the number of star clusters in each longitude interval also depends on the width of the Milky Way structure while the star densities do not. Some of the differences of the two curves must be attributed to this cause, as between longitudes 60° and 210° the open clusters show a larger scattering in galactic latitude than between 240° and 360°. But the excess of clusters over stars in the *Cassiopeia* region (80°–110°) and their deficiency in the *Cygnus* region (20°–60°) are undoubtedly real.

The distribution of clusters in galactic latitude is given in Table 19 and Fig. 3 separately for the two

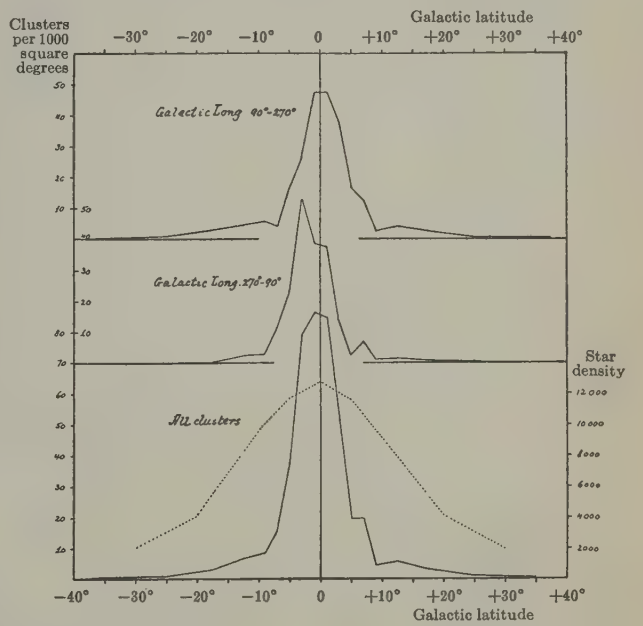


Fig. 3. Distribution of open clusters according to galactic latitude.

Abscissae are galactic latitudes, ordinates give the number of clusters per 1000 square degrees. The first curve applies to clusters falling between galactic longitudes 90° and 270°, the second to clusters between 270° and 90° longitude, the third to all clusters combined. The dotted curve represents the average density per square degree of stars brighter than 18th magnitude as a function of galactic latitude.

hemispheres, as they exhibit some notable differences. In longitudes 90°–270° the open clusters are arranged very nearly symmetrically to the adopted galactic plane and the great majority lie within 7.5° of it covering thus a zone about 15° wide. Between longitudes 270° and 90° the large majority of clusters lie

[28] *Mt. Wilson Contr.* No. 346, Table XVe.

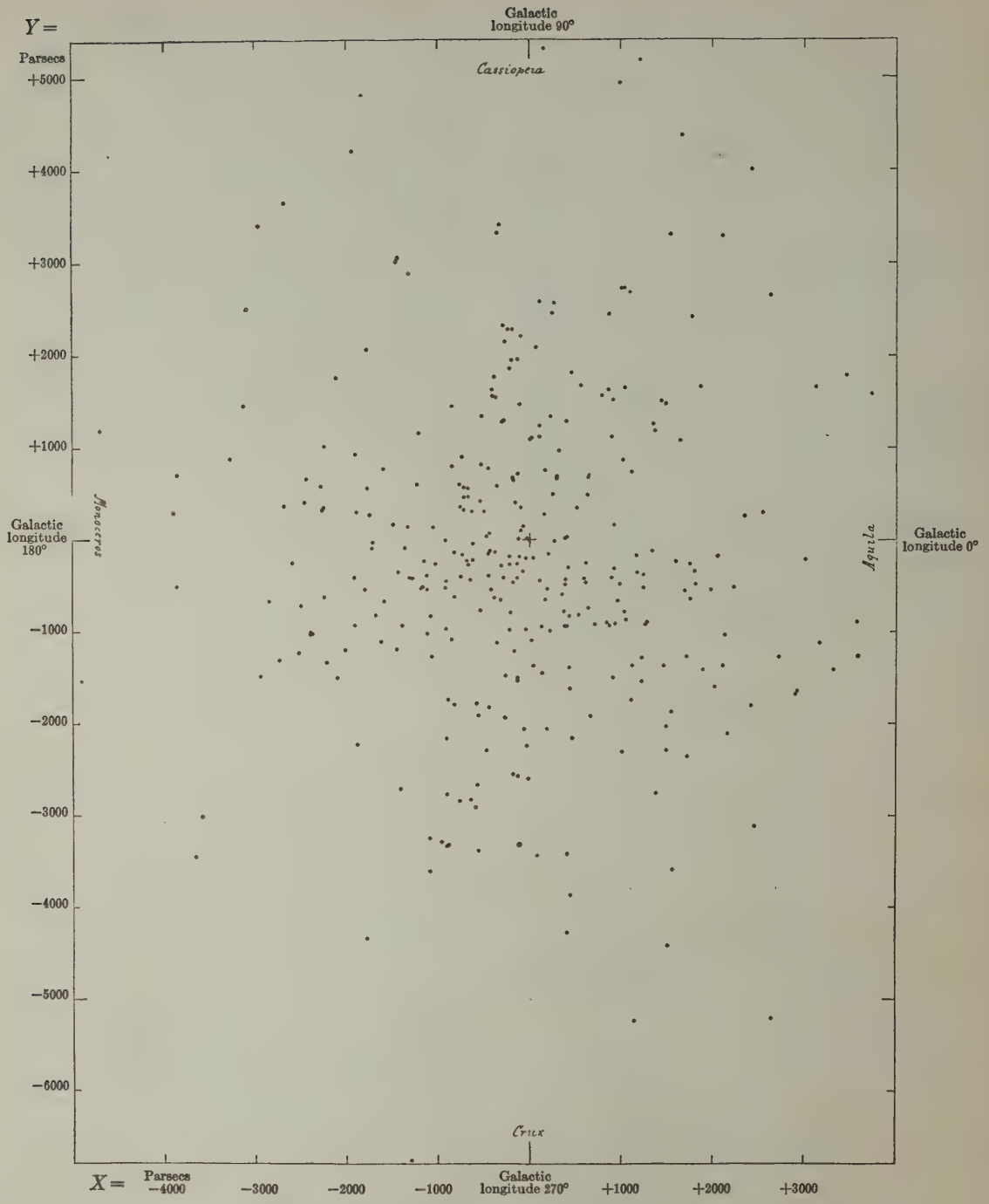


Fig. 4. Space distribution of 334 open clusters.

The figure gives the projection of the clusters on the galactic plane. The few clusters more distant from this plane than 500 parsecs are plotted as open circles. The position of the Sun is marked by the cross at the center; the scale of the $X$ and $Y$ coordinates (table 16, columns 13–14) is given in parsecs. On this scale the dots represent the correct (limiting) dimensions for the largest clusters, but are about twice too large for the majority of the clusters.

south of the galactic plane and the cluster zone is only about 11° wide. The smaller width of the cluster zone in the second hemisphere has already been mentioned; it is the more surprising since the Milky Way structure is unusually wide in the *Sagittarius-Ophiuchus* region. It is remarkable that practically all open clusters in this part of the sky lie in the southern (*Sagittarius-Scutum*) branch of the Milky Way and extremely few in the *Ophiuchus* branch. The unsymmetrical distribution of the clusters with respect to the galactic plane we shall later show to be due to the fact that the plane of symmetry of the more distant open clusters is somewhat inclined to the galactic plane here adopted.

The distribution of all clusters taken together is still slightly unsymmetrical, which indicates that the observer (Sun) is situated north of the galactic plane as defined by the clusters. The dotted line in Fig. 3 gives the average density per square degree of the stars brighter than 18$^m$ for different galactic latitudes according to Seares[29]. This figure brings out the extremely high galactic concentration of open clusters as compared with the fainter stars; it also shows that, in the Milky Way, stars brighter than 18th magnitude are about 200,000 times more numerous than clusters. Only a very small proportion of the stars are organized in cluster formations.

## 12. The Space Distribution of Open Star Clusters

In the three last columns of Table 16 the space coordinates in parsecs of each cluster are given in a coordinate system $X\ Y\ Z$ so oriented that the $Z$ axis is directed toward the adopted galactic pole (RA 12$^h$ 40$^m$, Decl. +28°), while the $X\ Y$ plane coincides with the galactic plane, the $+X$ axis in the direction of galactic longitude 0° (RA 18$^h$ 40$^m$ Decl. 0), the $+Y$ axis in galactic longitude 90°. The space coordinates $X\ Y\ Z$ of each cluster are obtained by multiplying $x\ y\ z$ of columns 6–8 by the adopted distance $r$ (column 12). In Fig. 4 each cluster is plotted in its projection on the galactic plane; those which are more distant than 500 parsecs from the galactic plane are marked by open circles. Fig. 5 gives a projection of the clusters on the $X\ Z$ plane which is perpendicular to the galactic plane, the clusters within 1000 parsecs of the center being marked as open circles. A comparison of the two figures shows that the clusters fill a system which has the shape of a flat disk or grindstone. Its thickness is about 1000 parsecs (if we disregard a few isolated objects) while the diameter of the disk in the galactic plane is about 9000 parsecs in the direction of galactic longitude 0° and 180° and about 11,000–12,000 parsecs in the direction of galactic longitude 90° and 270°.

The disk thus appears slightly elliptical, but of course its outline is rather difficult to define on account of the few scattered objects at the circumference. That the plane of symmetry of the cluster system is somewhat inclined to the adopted galactic plane was already noted from the apparent distribution of the clusters. This fact is quite marked in Fig. 5. By least squares solution we find the inclination to be 2°.3 and the pole of the plane of symmetry to fall in galactic longitude 352°. In order to fit the space distribution of the open clusters, the galactic North Pole should be placed at RA 12$^h$ 50$^m$.4 Decl.+27°.7 (1900).

Seares[30] has drawn attention to the fact that if the galactic pole is determined from the star distribution its position changes gradually as we include fainter stars. He gives the distance $p$ and the longitude $L_O$ of the determined pole referred to Gould's Pole (1900 : $\alpha = 190°.6$ $\delta = +27°.4$). His data are collected in Table 20.

TABLE 20

Position of the Galactic Pole

| | | $p$ | $L_O$ |
|---|---|---|---|
| Stars brighter than | 9$^m$ | 8°.1 | 275° |
| | 11 | 6.8 | 296 |
| | 13.5 | 8.0 | 319 |
| | 16 | 4.1 | 357 |
| | 18 | 2.7 | 350 |
| Open star clusters | | 1.7 | 10 |

[29] *Mt. Wilson Contr.* No. 346, Tables XIV and XVII.

[30] *Mt. Wilson Contr.* No. 347, 26, 1928.

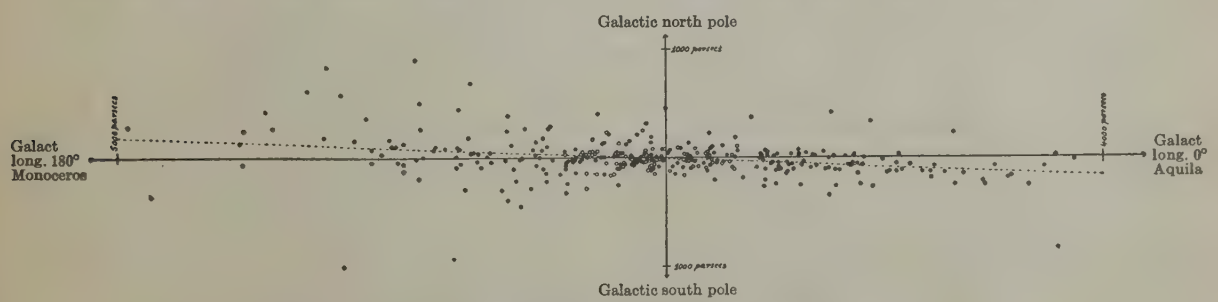


Fig. 5. Space distribution of 334 open clusters.

The figure gives the projection of the clusters on the $XZ$ plane which is perpendicular to the galactic plane and intersects the latter at longitudes 0° and 180°. Clusters within 1000 parsecs of the Sun are plotted as open circles. The dotted line marks the plane of symmetry of the open clusters.

Our result for the open clusters (also referred to Gould's pole) closely follows that for the 18$^m$ stars in the progression of decreasing $p$ and increasing $L_0$. Seares interprets this change in the position of the galactic pole as being due to a difference between the plane of symmetry of the local system and that of the more remote parts of the Milky Way. The brighter stars belong to the local system and furnish its plane of symmetry; as we proceed to fainter stars the proportion of stars belonging to the local system decreases and we gradually approach the position of the pole of the Milky Way system as a whole. The fact that our result for the open clusters follows in line after the 18$^m$ stars suggests that with our catalogue of open clusters we reach farther out into space than the average distance of 18$^m$ stars. This is quite plausible; a star of spectral type $A_o$ at a distance of 5000 parsecs would be of apparent photographic magnitude 17.8, if we take into account the effect of absorption; while the average luminosity of 18$^m$ stars is probably much less than that corresponding to type $A_o$.

The clusters plotted in Fig. 4 show a strong central concentration and a gradual thinning out with increasing distance. In fact, the whole figure appears very much like an open cluster. This concentrated central part seems to represent what is generally called the local cluster; but it is evident from our figure that it hardly deserves to be treated as a separate unit. It passes over very gradually into the more irregularly scattered Milky Way regions. It would be difficult to draw any limit for this local system except that it is of considerable dimensions, about 2000–4000 parsecs in diameter.

The center of the cluster system is found by taking the mean of the $X$ $Y$ $Z$ coordinates of all the 334 clusters. This gives:

$$X_c = -117 \quad Y_c = -201 \quad Z_c = -6$$

The center of gravity (giving equal weight) thus lies 6 parsecs south of the adopted galactic plane; if we take into account the fact that the plane of symmetry of the clusters is inclined to the galactic plane we find the Sun situated 10 parsecs north of this plane of symmetry. For the $X$ and $Y$ coordinates this method for determining the center of the cluster system is somewhat unfavorable, because it gives very high weight to clusters at the circumference which are the least complete. The addition of one cluster at the outline will change the coordinates of the center by 15 parsecs. It seems preferable to define the center of the cluster system as the median point, *i. e.* the intersection of two lines drawn parallel to the $X$ and $Y$ axes (in Fig. 4) so that they divide the clusters into two halves of equal number; in this way clusters at the circumference receive no more weight than those near the center. Since the latter are much more numerous, the median point should coincide more nearly with the center of the concentrated central part (local system). It falls at:

Median point: $X_m = -130$ parsecs $Y_m = -305$ parsecs

or at a distance of 350 parsecs in galactic longitude 247°. This is in good agreement with other observers: Charlier[31] finds the center of the B type stars in galactic longitude 244°; Stromberg[32], on the assumption that the preferential motions of the brighter F, G, K, M type stars are rotational in character, finds 257°; Kapteyn[33], from the stream motion, 257° or 77°. Shapley[34] and Seares[35], however, find the longitude of the center to increase with the magnitude of the stars included and the latter gives for stars brighter than 18$^m$: $L = 305°$ (Galactic latitudes $-5°$ to $+5°$). We would conclude from this that the local system has an eccentric location compared with the stellar system as a whole and that the center of the latter lies near galactic longitude 325°. The distribution of open clusters does not seem to confirm this view, but the question will be discussed later.

Since the space distribution of the clusters is fairly symmetrical around the center, it is possible to study the decrease in the density of clusters with increasing distance by counting the number of clusters in successive circles drawn about the median point. The spread of clusters perpendicular to the plane of symmetry can for this purpose be neglected, as the thickness of the disk-shaped cluster system is approximately uniform. The data are collected in Table 21, in which

TABLE 21

DISTRIBUTION OF CLUSTERS IN GALACTIC PLANE

| Limits of ring in parsecs | Area of projection in square parsecs | Number of of clusters | Density of clusters per $10^6$ square parsecs |
|---|---|---|---|
| 0–1000 | $3.1 \times 10^6$ | 88 | 28.4 |
| 1000–2000 | 9.4 | 108 | 11.5 |
| 2000–3000 | 15.7 | 77 | 4.9 |
| 3000–4000 | 22.0 | 38 (48) | 1.7 (2.1) |
| 4000–5000 | 28.1 | 16 (36) | 0.6 (1.3) |
| >5000 | | 7 | |

the first column gives the limits of the rings, the second, their area in square parsecs, the 3d, the number of clusters counted in each ring; this number divided by the area of the ring found in the 4th column is the density of clusters per $10^6$ square parsecs in the projection on the galactic plane. In Fig. 6 these densities are plotted as a function of the distance

[31] *Lund Meddel.* II No. 34, 1926.
[32] *Mt. Wilson Contr.* No. 144, *Ap. J.* **47**, 7, 1918.
[33] *Mt. Wilson Contr.* No. 230, *Ap. J.* **55**, 302, 1922.
[34] *Mt. Wilson Contr.* No. 157, *Ap. J.* **49**, 311, 1919.
[35] *Mt. Wilson Contr.* No. 347, *Ap. J.* **67**, 1927.

from the median point, showing the rapid, but continuous thinning out of clusters with increasing distance. At 4500 parsecs from the center the clusters are about 50 times less numerous per unit volume than near the center. Even if we added about 30 clusters in the last two rings as having possibly escaped detection or as being omitted from our list for insufficient description, this result would not be materially changed, as the figures in brackets of Table 21 and the dotted line in Fig. 6 show.

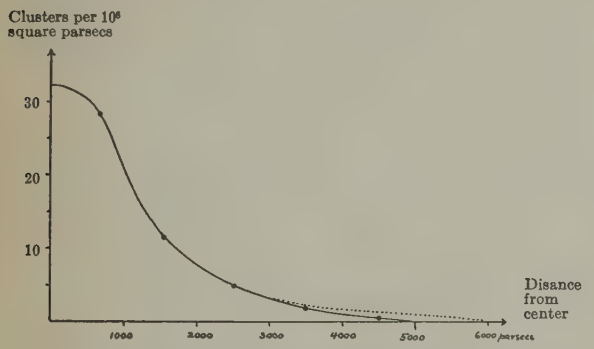


Fig. 6. Distribution of open clusters in the galactic plane.

Abscissae are distances from the median point of the cluster system, measured in the projection on the galactic plane. Ordinates are the numbers of clusters per $10^6$ square parsecs of the projection, showing the rapid thinning out of clusters with increasing distance from the center. The full line gives the distribution of the clusters actually observed, the dotted line the probable correction for omitted or undiscovered clusters.

The distribution of clusters perpendicular to the plane of symmetry is shown in Table 22 and Fig. 7. For each cluster the coordinate $Z'$ perpendicular to the plane of symmetry was computed and the numbers of clusters counted for successive intervals of 100 parsecs in $Z'$ are entered in column 3-5 of Table 22 and plotted in Fig. 7 as a function of the distance $Z'$ from the plane.

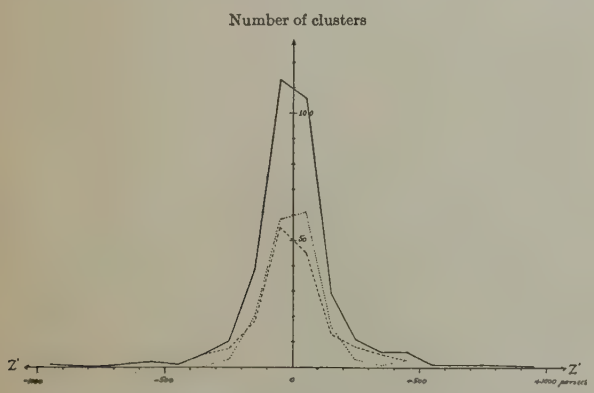


Fig. 7. Space distribution of open clusters perpendicular to their plane of concentration.

Abscissae: Distances $Z'$ from the plane of concentration, measured in parsecs, positive in the direction of the galactic North Pole. Ordinates: Numbers of clusters per interval of 100 parsecs. Dotted line: Hemisphere centered at galactic longitude 0° ($X > -130$). Broken line: Hemisphere centered at galactic longitude 180° ($X < -130$). Full line: All clusters taken together.

TABLE 22

Distribution of Clusters Perpendicular to Plane of Concentration

| $Z'$ | | Mean $Z'$ | Number of clusters per layer of 100 parsecs thickness | | |
|---|---|---|---|---|---|
| | | | $X<-130$ | $X>-130$ | All |
| | $<-800$ | | 2 | 0 | 2 |
| $-800$ to | $-700$ | $-750$ | 0 | 0 | 0 |
| $-700$ | $-600$ | $-650$ | 0 | 1 | 1 |
| $-600$ | $-500$ | $-550$ | 2 | 0 | 2 |
| $-500$ | $-400$ | $-450$ | 1 | 0 | 1 |
| $-400$ | $-300$ | $-350$ | 5 | 0 | 5 |
| $-300$ | $-200$ | $-250$ | 7 | 3 | 10 |
| $-200$ | $-100$ | $-150$ | 18 | 20 | 38 |
| $-100$ | 0 | $-50$ | 55 | 58 | 113 |
| 0 | $+100$ | $+50$ | 45 | 61 | 106 |
| $+100$ | $+200$ | $+150$ | 13 | 16 | 29 |
| $+200$ | $+300$ | $+250$ | 8 | 3 | 11 |
| $+300$ | $+400$ | $+350$ | 5 | 1 | 6 |
| $+400$ | $+500$ | $+450$ | 3 | 3 | 6 |
| $+500$ | $+600$ | $+550$ | 0 | 1 | 1 |
| $+600$ | $+700$ | $+650$ | 1 | 0 | 1 |
| $+700$ | $+800$ | $+750$ | 1 | 0 | 1 |
| | $>+800$ | | 1 | 0 | 1 |
| | | | 167 | 167 | 334 |

The counting was done separately for the hemisphere centered at galactic longitude 180° ($X = -130$ to $-5000$ parsecs) and for the opposite one ($X = -130$ to $+4000$). The curves show that in the former hemisphere the clusters are slightly more widely scattered, a fact which is well illustrated by Fig. 5 and was already noted in the apparent distribution (page 178). The high concentration of open clusters toward their plane of symmetry is very striking; 68% of them lie within 100 parsecs of this plane. In this respect the open clusters resemble the O and B type stars. The frequency law of clusters for different distances $Z'$ from the plane of symmetry does not follow a normal frequency law (Gaussian error law); it has a considerable positive excess, but is very nearly symmetrical.

While Figs. 6 and 7 describe the general statistical features of the cluster system, there are, of course, many local irregularities. To these must be counted certain groups of clusters which, as a rule, stand out not only by the close proximity in space of the clusters (except for some scattering in distance due to errors of observation) but often also by similarity in structure and spectral types, or by association with a prominent star cloud. In the latter case it is then possible to derive the probable distance of such star cloud from the clusters contained in it. Six of these

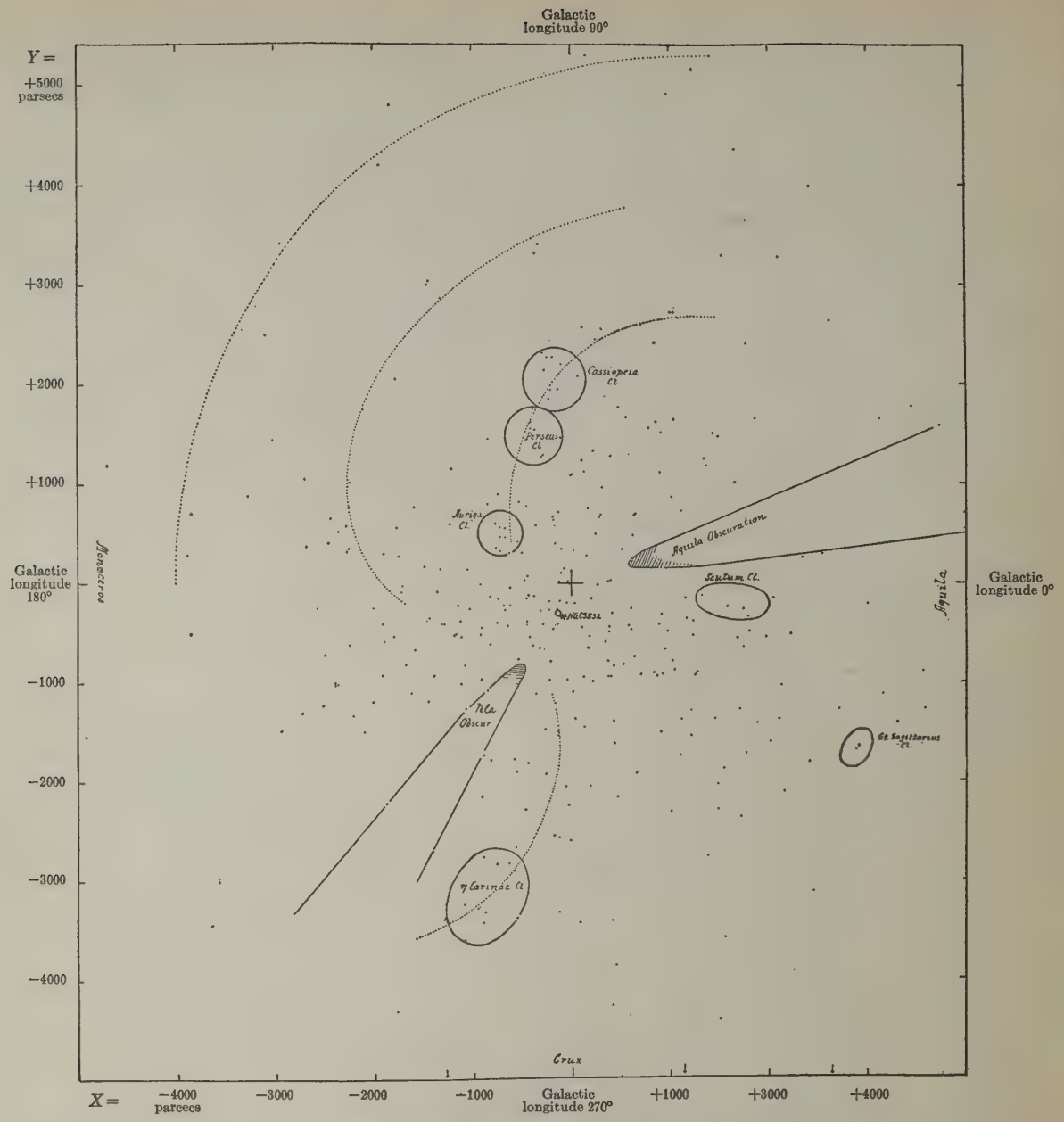


Fig. 8. Special features of the cluster system.

This figure gives a projection of the clusters on the galactic plane like Fig. 4. The position of the Sun is marked by a cross, that of the median point of the system by an open circle, the cluster NGC 3532 by an asterisk. Some traces of spiral structure are indicated by the dotted lines; the large open circles or ovals represent the probable location of cluster groups or star clouds, the shaded areas, dark clouds of absorbing material with their sectors of obscuration.

groups are outlined in Fig. 8 while the clusters composing it are listed in Table 23, together with the mean coordinates and the mean distance of the group and the classification of the magnitude spectral class diagrams (Table 3, column 3) of those clusters which have spectroscopic observations.

Remarkable is the great distance of $\eta$ *Carinae* with its surrounding star cloud and the nebula, in which two clusters of the fourth group are directly involved. The distance of the *Scutum* cloud derived from the clusters is considerably smaller than that obtained by C. J. Krieger[36] from magnitudes and color indices of faint stars in six areas. The latter result is based on the assumption that there is no absorption of light in interstellar space; if Krieger's result (2800 parsecs)

[36] *L. O. Bull.* **14**, 95, 1929.

TABLE 23

GROUPS OF OPEN CLUSTERS

| Group | Clusters | Mean rectangular coordinate in parsecs X | Y | Z | Number of clusters | Mean distance in parsecs | Magnitude spectral class diagram |
|---|---|---|---|---|---|---|---|
| Cassiopeja | 129, 381, 436, 457, 559, 581, An. 1, 654, 659, 663 | − 170 | +2060 | − 41 | 10 | 2070 | 1b (2)<br>1–2b (2) |
| Double cluster Perseus | 869, 884, IC1805, 957, 1027, IC1848, 1502 | − 380 | +1490 | + 7 | 7 | 1540 | 1–2o (1)<br>1b (2)<br>1–2b (2) |
| Auriga | 1893, 1912, 1960, 2099, 2168 | − 720 | + 510 | + 38 | 5 | 880 | 1b (2)<br>2a (2) |
| $\eta$ Carinae cloud | An. 11, An. 12, An. 13, An. 14, 3255, An. 15, An. 16, 3496, An. 18 | − 870 | −3140 | − 94 | 9 | 3220 | |
| Large Sagittarius cloud | An. 31, 6520 | +2910 | −1660 | −234 | 2 | 3360 | |
| Scutum cloud | 6649, 6664, An. 34, 6694, 6704, 6705 | +1610 | − 210 | − 91 | 6 | 1630 | 1–2 b–a (1)<br>2 b–a (1) |

is corrected for absorption ($0^m.67$ per 1000 parsecs) it becomes 1700 parsecs which is in close agreement with ours.

Aside from such groupings, Fig. 4 exhibits some vacancies in the distribution of open clusters. In the outer parts of the figure where clusters are rather thinly scattered, vacancies of considerable size may be due to chance; only those reaching into the denser central part can claim real significance. If in addition they have the shape of a sector, with limits radiating from the Sun, they strongly suggest obscuration by dark matter. Two such cases are marked in Fig. 8: the most striking is that in *Aquila* (galactic longitude 15°) the other in *Vela* (galactic longitude 236°). Both coincide with minima in the apparent distribution of clusters (Fig. 2) and stars and are noticeable as dark spots in the Milky Way. The space distribution of open clusters indicates that the obscuring matter probably lies at a distance of about 500 parsecs in *Aquila* and within 1000 parsecs in *Vela*. There are many other less conspicuous regions of obscuration indicated in Fig. 4.

## 13. THE STRUCTURE OF THE MILKY WAY SYSTEM

By the Milky Way system we wish to designate that large system which comprises practically all the stars visible to the naked eye or observable in large telescopes and which is particularly characterized by the dense accumulations of faint stars forming the conspicuous features of the Milky Way. To anybody who closely examines the beautiful Milky Way photographs of Barnard or the Franklin Adams chart it is quite apparent that the open clusters must be related to the star clouds of the Milky Way in which they often appear imbedded like condensations. We have already drawn attention to the similarity in the apparent distribution of stars and clusters except for the greater galactic concentration. Most convincing of all is perhaps the close agreement between the plane of symmetry of open clusters and that of faint stars.

In view of these facts it seems quite justifiable to make the hypothesis that the space distribution of open clusters is similar to that of stars in general and that a study of open star clusters may give us some information concerning the structure of the Milky Way system. As every test seems to indicate that our list of clusters is not much short of completeness we may expect figures 4 and 5 to represent a general outline of the Milky Way system. The features we brought out in the discussion of the open clusters should then for the most part apply also to the Milky Way system, and in fact they are generally in good agreement with the results of the statistical investigations of Seeliger[37], Kapteyn[38] and others who describe the stellar system as a flattened lens shaped system 10,000–15,000 parsecs in diameter and 3000–4000 parsecs in thickness, with the stars concentrated toward the center and thinning out toward the edge. The only difference is that the clusters seem to be more strongly concentrated toward the galactic plane than the stars in general.

The close analogy between these views concerning the structure of our Milky Way system and the main features of many spiral nebulae led to the conclusion that the Milky Way system belongs to this class of objects. Our results concerning the open clusters quite support this conclusion. In some of the nearer spirals especially *M* 51 and *M* 101 we find numerous small nuclei not as well defined as star images which have all the earmarks of open star clusters and in *M* 33 some clusters are even partly resolvable into stars. The size of our Milky Way system suggested by the open clusters (10,000–12,000 parsecs in diameter) is well

[37] *Sitzungsber. d. Münch. Akad. d. Wiss.* 1920, p. 87

[38] *Mt. Wilson Contr.* No. 230, 1922.

comparable with the dimensions of the *Andromeda* Nebula (13,000 parsecs) and Messier 33 (4600 parsecs) according to Hubble's[39] investigations.

On the other hand it is true that the space arrangement of open clusters as illustrated in Fig. 4 shows hardly any indication of spiral structure. The errors of observation in the cluster distances (p. e. 10–12%) will of course have the tendency to blur any existing spiral structure, and local obscuration by dark matter may at many places have interrupted the spiral arms. Despite of these disturbing influences it seems hardly possible to account for our diagram unless we assume that our Milky Way system is an extremely resolved spiral, even more resolved than Messier 33. Nevertheless there are some traces of spiral structure noticeable which have been drawn in Fig. 8. While they are not sufficiently prominent to prove the spiral structure of the Milky Way system, they may, once we admit the hypothesis of spiral structure, give an indication of its direction. The different fragments of branches especially between galactic longitude 60° and 180° indicate a right-handed spiral as seen from the galactic North Pole. The best marked of these branches is undoubtedly that joining the cluster groups in *Auriga*, *Perseus* and *Cassiopeia*.

It is well known that most spiral nebulae have a pronounced central nucleus, which in the case of the *Andromeda* nebula, for example, produces a nearly star like image with a short exposure. On this account there is probably some significance attached to the fact that one of the richest and most remarkable open clusters: NGC 3532 (RA = $11^h\,2^m.2$, Decl = $-58°\,8'$) falls by our distance determination quite close to the median point of the cluster system (center of local system). This cluster which is marked in Fig. 8 by an asterisk contains, according to Raab, more than 100 stars brighter than magnitude 10 (mostly of types B5–A0) and over 300 stars brighter than magnitude 12, and it is imbedded in a region which is also exceptionally rich in stars of magnitude 8–10. It thus seems not impossible that NGC 3532 and the surrounding star field represent the remainder of a central nucleus.

While our results on the space distribution of open clusters support in every way the older views concerning the structure of our Milky Way system and its similarity to a spiral nebula, they disagree entirely with the more recent conclusions by Shapley and Seares that the Sun and its surrounding star concentration (local system) are quite a secondary formation in a much larger galactic system over 100,000 parsecs in diameter, the center of which is situated in the direction of *Sagittarius* (galactic longitude 325°) at a distance of 20,000 to 40,000 parsecs. These conclusions are based mainly on three facts of observation:

1. The distribution of globular clusters (Shapley).
2. The asymmetry in the distribution of faint stars (Seares).
3. The results obtained for galactic rotation.

On the other hand there is no noticeable feature in the distribution of open star clusters which suggests a considerable extension of the Milky Way system in the direction of *Sagittarius*. A careful examination of Barnard's excellent Milky Way photographs of this region, which reach at least to the $17^m$, did not reveal any appreciable number of small distant undiscovered star clusters. It is hardly possible that every one of scores or hundreds of such distant clusters should be hidden from our view through absorption by dark matter. But even if we should admit such an assumption, we should still expect the visible parts of the open cluster system to show some arrangement concentric with the distant *Sagittarius* center. We should, for example, expect the limit of the cluster system in the opposite direction (galactic longitude 60°–120°), where there is not so much evidence of dark clouds, to be a segment of a circle of large radius centered in the *Sagittarius* direction; or we should expect the cluster system to widen out in the direction of the center with many distant clusters in galactic longitudes 200°–270° and 0°–70°. None of these expectations is fulfilled, and the hypothesis of a distant galactic center would leave the observed space distribution of open clusters quite unintelligible. But if we examine a little more closely the three facts of observation on which this hypothesis is based we find their evidence not quite convincing.

Taking up the second point first, it is true that the star counts based on the *Durchmusterung* of the selected areas reveal an asymmetry in galactic longitude. If the star density is represented by a simple harmonic of the longitude the maximum of star density gradually shifts from about 260° to 320° as we include fainter stars. Seares interprets this feature as being produced by an eccentric position of the local system compared with the middle of the larger Milky Way system. For the brighter stars, maximum star density falls in the direction of the center of the local system which is in fair agreement with that found for the open clusters. As we proceed to fainter stars we penetrate farther into the region of the Milky Way star clouds and maximum star density will fall in the direction of the center of the Milky Way system. Seares shows that the change in space density with distance in the direction of galactic longitudes 325° and 145° can be represented by the superposition of two concentrations; one very sharp maximum at the center of the local system, and a second more flattened widespread condensation centered around a point 1000 parsecs

[39] *Mt. Wilson Contr.* No. 310, 1926 and No. 376, 1929.

distant from the Sun in galactic longitude 325°, which might be taken as the middle of the Milky Way system. It should be noted that around this point we also find a somewhat loose agglomeration of clusters. Evidently the star distribution in our Milky Way system is characterized by large local irregularities which are smoothed out by the usual statistical methods of determining the density distribution from star counts. While the observed asymmetry in the apparent distribution of faint stars may thus be a consequence of an extensive local congregation of stars, it also seems quite possible that the local system, *i. e.*, the center of concentration of open clusters, actually has a slightly eccentric situation with respect to the outline of the Milky Way system as a whole, and that the latter is more nearly concentric with a point about a thousand parsecs distant from the Sun in galactic longitude 325°. If we take into account the probable obliteration of distant clusters by dark matter which is so evident in the *Aquila* region and which may be effective all along the dark rift in the Milky Way, it even seems likely that the cluster system as plotted in Fig. 4 is incomplete in galactic longitudes 300°–60° and actually extends somewhat farther in this direction. A relatively small number of obscured clusters might change the somewhat uncertain outline of our cluster system so as to bring its center near the suggested point; it is however hardly admissible that such shift could be greater than 1000 parsecs. The writer can find nothing in the data of Seares's investigation which should force us to assume that the galactic center is at a greater distance than this. The hypothesis that the Milky Way system is similar in outline to the open cluster system as illustrated in Fig. 4, extending perhaps 1000 or 2000 parsecs farther in the *Aquila-Sagittarius* region, is then not in contradiction with the apparent distribution of faint stars. Of course this similarity should not be taken too closely and must be confined to the main features; there are unquestionably some parts of the Milky Way where the tendency to form clusterings is more pronounced (*e. g.*, in the *Perseus-Cassiopeia* region) than in others where the proportion of clusters to stars is small (*e. g.*, in the great *Sagittarius* cloud).

The evidence for galactic rotation around a center in the *Sagittarius* region is mainly based on a second harmonic in the observed radial velocities of stars and interstellar calcium. What is observed, of course, is a differential effect and its interpretation necessarily requires certain assumptions about the nature of the rotational motion; furthermore, the observations fur-

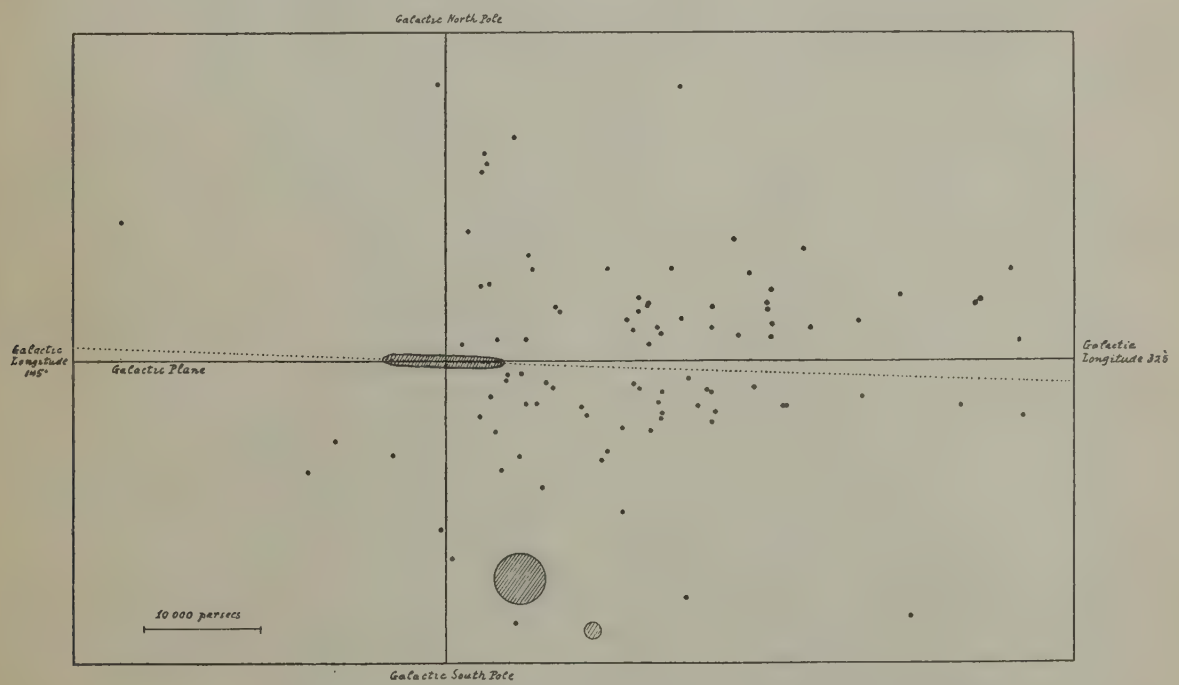


Fig. 9. Space distribution of open clusters, globular clusters, and the Magellanic Clouds.

In this figure the 93 known globular clusters are plotted as full dots in their projection on a plane which passes through the galactic pole and through galactic longitude 325°. On the scale of the chart the dots are about twice as large as the limiting dimensions of the globular clusters. The system of open clusters (Milky Way system) is represented by the elongated shaded area, the two Magellanic Clouds by the shaded circles. The dotted line indicates the plane of symmetry of the open clusters.

nish only the direction of the center of rotation, but not its distance. The question how the observational data concerning galactic rotation can be reconciled with the distribution of open clusters must however be left to a later investigation.

Figure 9 shows the space distribution of globular clusters in the projection on a plane passing through the galactic pole and through galactic longitude 325°. The 93 globular clusters are entered as full dots according to Shapley's most recent data[40]; the dots, on the scale of the chart, are about twice as large as the limiting size of these clusters. The much flattened system of open clusters which according to our hypothesis outlines the Milky Way system and the two Magellanic Clouds are represented according to their dimensions. The nearest spiral nebulae (*Andromeda* Nebula, Messier 33) would be 3–4 times more distant than the diameter of the whole figure.

From the fact that the space distribution of the globular clusters is somewhat symmetrical to the galactic plane, Shapley draws the conclusion that they form an integral part of the Milky Way system and that the center of the latter should therefore be identified with the center of the globular clusters which lies at a distance of at least 20,000 parsecs in the direction of *Sagittarius*, while the dimensions of the galactic system should be of the order of 100,000 parsecs. It must, however, be emphasized, that the distribution of globular clusters in the sky shows practically no relationship to the general star distribution. While the globular cluster system appears nearly spherical in shape there can be no question that the Milky Way system is much flattened. On account of partial obscuration by dark matter there may be some uncertainty about the extent of the star distribution in the galactic plane. No such uncertainty, however, exists in high galactic latitudes. Statistical investigations of stellar distribution show conclusively that in high galactic latitudes the stars do not reach farther than a few thousand parsecs and that the space between the numerous globular clusters in high galactic latitudes is certainly not filled with stars. The majority of globular clusters thus lie outside of the star stratum of our Milky Way system and should in this sense be called extra-galactic systems although this does not exclude the possibility that they have some relation to it. Fig. 9 shows a remarkable resemblance to some of the clusters of extra-galactic objects (spiral nebulae, elliptical and globular nebulae), and it seems worth while to examine the hypothesis that our Milky Way system (approximately as outlined by the open clusters) together with the two Magellanic Clouds and about a hundred globular clusters form a cluster of extra-galactic objects which we may call the "supercluster." Lundmark,[41] who quite independently came to the same conclusion as the writer, suggests also the possible existence of another large system in the *Sagittarius* region partly hidden by obscuring matter which he calls the "Hidden System" and to which the faint variable stars observed by Shapley in the direction of the "galactic center" would belong. It remains, however, to be investigated whether the faintness of these variable stars is not to some extent due to absorption of light rather than to great distance.

The striking feature of the super-cluster is the fact that it contains only one spiral system (the Milky Way system) and two large amorphous systems (the Magellanic Clouds) associated with a hundred or more globular systems of very much smaller but nearly uniform dimensions. Lundmark discusses this point and finds nothing very improbable in such an association in comparison with other clusters of extra-galactic objects.

[40] *H. C. O. Bull.* No. 869, 1929.

[41] *Publ. A. S. P.* Feb. 1930.

## Summary


1. The distances of 100 open clusters were determined from magnitudes and spectral types of the stars.

2. With these distances and the estimated angular diameters the linear diameters in parsecs were computed.

3. The linear diameters of open clusters vary considerably (2–16 parsecs); they depend on the constitution of the cluster.

4. The linear dimensions of an open cluster increase with the number of stars contained and with decreasing central concentration. There is a distinct but small class of clusters with exceptionally large dimensions.

5. The assumption that clusters of the same constitution have everywhere the same linear diameters leads to the conclusion that within the Milky Way system light is subject to an absorption of $0^m.67$ (photographic) per 1000 parsecs.

6. The discrepancy between color-indices and spectral types observed in open clusters increases with the distance of the cluster and shows that this absorption of light is selective, the photographic absorption coefficient being about twice the visual.

7. The absorption is effective in all galactic longitudes but seems to take place mainly in a thin layer extending along the galactic plane.

8. A method is developed to determine the distance of a cluster from its angular diameter and from a classification of its constitution.

9. A catalogue of 334 open clusters is compiled, which includes 41 objects not previously listed in

cluster catalogues. This catalogue should be nearly complete for all the more prominent open clusterings of our Milky Way system, except for 30–40 of the smallest or faintest ones.

10. Distances and rectangular space coordinates for the 334 clusters were derived.

11. The plane of symmetry of the open clusters is inclined 2°.3 to the adopted galactic plane; its pole lies at RA $12^h\ 50^m.4$ Decl: +27°.7 (1900). This plane coincides very nearly with the plane of symmetry derived by Seares from the apparent distribution of faint stars.

12. A study of the space distribution of open clusters shows that they form a much flattened disk-like system about 1000 parsecs thick with a diameter of about 10,000 parsecs.

13. This cluster system shows a strong concentration towards a point which is situated at a distance of 350 parsecs in galactic longitude 247° from the Sun. The exceptionally rich open cluster NGC 3532 falls very close to this center.

14. The Sun lies 10 parsecs north of the plane of symmetry of the open clusters.

15. The hypothesis is made that the Milky Way system is in its essential features outlined by the space distribution of open clusters except for a greater galactic concentration of the latter.

16. This hypothesis supports the view that our Milky Way system is a highly resolved spiral nebula, a right-handed spiral as seen from the galactic North pole, of dimensions similar to those of the *Andromeda* nebula.

17. This hypothesis is not in conflict with the apparent distribution of faint stars, but it requires that the globular clusters be treated as extra-galactic objects forming, together with the Milky Way system and the two Magellanic clouds, a super-cluster of extra-galactic objects.

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# LICK OBSERVATORY BULLETIN

NUMBER 421

## PRELIMINARY ELEMENTS AND EPHEMERIS OF THE LOWELL OBSERVATORY OBJECT

BY

ERNEST CLARE BOWER AND FRED L. WHIPPLE

This object was discovered by C. W. Tombaugh on plates taken in January 1930 during the course of a systematic search by the Lowell observers since 1905 for a trans-Neptunian planet. The position of such a planet was predicted by Lowell[1] on the basis of its perturbations of Uranus. Announcement of the discovery by telegram and circular was made March 12–13, 1930. The magnitude was given as 15, and it was found that the position and motion since January corresponded to a distance of 40 to 43 astronomical units.

The available accurate observations are:

| | 1930 U. T. | α(1930.0) | δ(1930.0) | Observer | Place | Authority |
|---|---|---|---|---|---|---|
| (1) | Mar. 16.0554 | $7^h\ 15^m\ 42^s.55$ | +22° 07′ 10″.6 | Van Biesbroeck | Williams Bay; 24″, phot. | Letter |
| (2) | Mar. 17.0793 | 7 15 40.92 | +22 07 18.0 | Van Biesbroeck | Williams Bay; 24″, phot. | Letter |
| (3) | Mar. 19.0917 | 7 15 38.15 | +22 07 31.3 | Van Biesbroeck | Williams Bay; 24″, phot. | Letter |
| (4) | Mar. 19.1979 | 7 15 38.14 | +22 07 31.8 | Meyer | Mount Hamilton; 36″, phot. | Letter |
| (5) | Mar. 20.0668 | 7 15 36.99 | +22 07 38.1 | Van Biesbroeck | Williams Bay; 24″, phot. | Letter |
| (6) | Mar. 20.2292 | 7 15 36.84 | +22 07 38.3 | Meyer | Mount Hamilton; 36″, phot. | Letter |
| (7) | Mar. 21.2153* | 7 15 35.74 | +22 07 44.3 | Meyer | Mount Hamilton; 36″, phot. | Letter |
| (8) | Mar. 21.9333 | 7 15 35.0 | +22 07 47 | Struve | Babelsberg | H.A.C. No. 115, 1930 |
| (9) | Mar. 22.9457 | 7 15 34.0 | +22 07 54 | Struve | Babelsberg | H.A.C. No. 115, 1930 |
| (10) | Mar. 23.1733 | 7 15 33.83 | +22 07 55.4 | Mayall | Mount Wilson; 60″, phot. | Letter |
| (11) | Mar. 24.1458 | 7 15 33.02 | +22 08 0.7 | Mayall | Mount Wilson; 60″, phot. | Letter |
| (12) | Mar. 26.2118* | 7 15 31.60 | +22 08 12.2 | Meyer | Mount Hamilton; 36″, phot. | Letter |
| (13) | Mar. 27.2250* | 7 15 31.09 | +22 08 16.6 | Meyer | Mount Hamilton; 36″, phot. | Letter |
| (14) | Mar. 28.1122 | 7 15 30.80 | +22 08 21.4 | Van Biesbroeck | Williams Bay; 24″, phot. | Letter |
| (15) | Mar. 28.1497 | 7 15 30.80 | +22 08 20.7 | Mayall | Mount Wilson; 60″, phot. | Letter |
| (16) | Mar. 28.2424* | 7 15 30.72 | +22 08 21.3 | Meyer | Mount Hamilton; 36″, phot. | Letter |
| (17) | Mar. 30.1348 | 7 15 30.31 | +22 08 29.1 | Van Biesbroeck | Williams Bay; 24″, phot. | Letter |
| (18) | Mar. 31.0927 | 7 15 30.20 | +22 08 34.1 | Van Biesbroeck | Williams Bay; 24″, phot. | Letter |
| (19) | Apr. 2.0775 | 7 15 30.37 | +22 08 41.7 | Van Biesbroeck | Williams Bay; 24″, phot. | Letter |
| (20) | Apr. 2.2451* | 7 15 30.36 | +22 08 42.3 | Meyer | Mount Hamilton; 36″, phot. | Letter |
| (21) | Apr. 3.0656 | 7 15 30.60 | +22 08 46.5 | Van Biesbroeck | Williams Bay; 24″, phot. | Letter |
| (22) | Apr. 4.1842 | 7 15 30.94 | +22 08 49.5 | Van Biesbroeck | Williams Bay; 24″, phot. | Letter |
| (23) | Apr. 4.1969* | 7 15 30.95 | +22 08 49.2 | Meyer | Mount Hamilton; 36″, phot. | Letter |
| (24) | Apr. 5.1459 | 7 15 31.39 | +22 08 52.2 | Van Biesbroeck | Williams Bay; 24″, phot. | Letter |

[1] *Lowell Observatory Memoirs*, *1*, No. 1, A Trans-Neptunian Planet, 1914.

*Mean of two plates.

A preliminary study of the orbit of the object was undertaken under the general direction of Professor Leuschner in the hope that the available accurate observations would independently establish the object as a trans-Neptunian planet.

When (1), (2), (4), (6) and (7) were received, a preliminary direct solution by Leuschner's method from (1), (4) and (7), a five-day arc, definitely indicated a trans-Neptunian distance. However, $\frac{\lambda}{\kappa}$, which determines the velocity in the line of sight, was wholly indeterminate. Assuming first, that this velocity was produced only by the Earth's motion, and second, that the object was only at *Neptune's* distance 30, the residuals were

| | $\Delta\alpha$ | $\Delta\delta$ |
|---|---|---|
| (1) | −4″.3 | +0″.2 |
| (7) | −1.6 | +0.1 |

The somewhat indeterminate differential correction indicated a distance of about 50 astronomical units, at which a satisfactory representation was possible.

When additional observations (8), (9), (12), (13), (16), (20) were received, making a total of 11 and an arc of 17 days, they were examined for consistency by their deviation from $a$, $\delta = a + b(t-t_0) + c(t-t_0)^2$, and $\delta$ of (12) was decreased by 0″.5.

A direct solution, based on (1), (12) corrected, and (20), yielded $\rho = 41.0$, apparently determinate to two figures, and $\frac{\lambda}{\kappa} = +.037$, the first figure being still uncertain, giving the object greater than parabolic velocity. Again ascribing the velocity in the line of sight in 1/k mean solar days to the Earth's motion only, $\frac{\lambda}{\kappa}$ was taken as +.0233 and the general orbit based on (1) (12), and (20), so obtained gave residuals:

| | $\Delta\alpha$ | $\Delta\delta$ |
|---|---|---|
| (1) | +1″.0 | −0″.3 |
| (2) | +0.2 | +0.2 |
| (20) | −0.1 | 0.0 |

To investigate the range of solution, a differential correction was performed using the mean residual of (1) and (2). At the end of the correction, where any desired semi-major axis $a$ easily may be introduced, $\Delta\rho$ was found for $a = r$, giving a conditioned ellipse with the object at the end of the semi-minor axis, if the result is not a circle; for $a = \infty$, giving a parabola; and for $a$ unconditioned, giving a general solution. In this case two solutions were possible for both the conditioned ellipse and parabola, corresponding to $r'$ positive and negative. All the values of $\Delta\rho$ ranged from −.08 to −.15 and the resulting orbits represented the basic positions with zero residuals. Charlier's geometrical criterion precludes a second general solution for the distance.

ELEMENTS, 1930.0

| Orbit | I General direct solution | II General corrected | III Conditioned ellipse A | IV Conditioned ellipse B | V Parabola A | VI Parabola B |
|---|---|---|---|---|---|---|
| $\omega$ | 202° 24′ | 97° 56′ | 249° 16′ | 106° 58′ | 260° 52′ | 92° 48′ |
| $\Omega$ | 109 22 | 109 23 | 109 22 | 109 22 | 109 21 | 109 23 |
| $i$ | 17 7 | 16 46 | 17 11 | 16 54 | 17 32 | 16 35 |
| $q$ | 33.31 | 24.53 | 26.97 | 28.70 | 17.52 | 19.31 |
| $e$ | 0.111 | 0.541 | 0.344 | 0.302 | 1.000 | 1.000 |
| $P$ | 229 yr. | 390 yr. | 264 yr. | 264 yr. | ..... | ..... |
| $a$ | 37.48 | 53.41 | 41.13 | 41.10 | ..... | ..... |
| $p$ | 37.01 | 37.80 | 36.25 | 37.36 | 35.03 | 38.63 |
| $\rho_0$ | 41.0 | 40.86 | 40.90 | 40.87 | 40.92 | 40.85 |

The sensitiveness of the elements $a$, $e$, $\omega$ to slight changes in the observations is readily seen from the fact that the differences between orbit I, the original general solution, and orbit II, the corrected general solution, are due solely to the removal of a residual of only +0″.6 in the first right ascension. A comparison of all the orbits shows that the resulting inclinations, nodes and distances are fairly well determined and that the other elements can be determined with a fair degree of accuracy only with a more extended arc.

These results clearly indicate that a distinctly trans-Neptunian orbit satisfies the positions so far received. On the other hand, it is striking that the observations are satisfied by parabolas and that other differential corrections tend toward a high eccentricity.

To find the permissible range of $\rho$, it was arbitrarily taken as 31 and 51, together with values of $\frac{\lambda}{\kappa}$ corresponding, as in Orbit I, to the Earth's motion. These orbits gave the residuals:

| | $\rho=31$ | | $\rho=51$ | |
|---|---|---|---|---|
| | $\Delta\alpha$ | $\Delta\delta$ | $\Delta\alpha$ | $\Delta\delta$ |
| ½[(1)+(2)] | −25″.4 | +2″.9 | +16″.4 | −2″.2 |
| (20) | −12.7 | +1.5 | + 7.5 | −0.9 |

The signs clearly indicate that $\rho$ lies between 31 and 51, the differential corrections actually giving $\Delta\rho = +10.1$ and $-10.0$ respectively, indicating that $\rho$ is very nearly 41, which resulted from the original direct solution.

EPHEMERIS*

| 1930 U.T. | | α | δ | Light time |
|---|---|---|---|---|
| Jan. | 4 | 7h 20m 40s.56 | +21° 53′ 10″.3 | 0d.2318 |
| | 12 | 19 56.72 | 55 05.4 | .2319 |
| | 20 | 19 13.26 | 56 59.4 | .2320 |
| | 28 | 18 31.22 | +21 58 50.3 | .2322 |
| Feb. | 5 | 17 51.63 | +22 00 36.4 | .2326 |
| | 13 | 7 17 15.50 | +22 02 15.6 | 0.2331 |
| | 21 | 16 43.65 | 03 46.6 | .2336 |
| Mar. | 1 | 16 16.83 | 05 8.2 | .2342 |
| | 9 | 15 55.69 | 06 19.3 | .2349 |
| | 17 | 15 40.71 | 07 18.9 | .2357 |
| | 25 | 7 15 32.19 | +22 08 06.8 | 0.2365 |
| Apr. | 2 | 15 30.37 | 08 42.4 | .2373 |
| | 10 | 15 35.32 | 09 05.5 | .2381 |
| | 18 | 15 46.92 | 09 16.4 | .2390 |
| | 26 | 16 04.99 | 09 15.2 | .2398 |
| May | 4 | 7 16 29.27 | +22 09 02.4 | 0.2405 |
| | 12 | 16 59.32 | 08 38.3 | .2413 |
| | 20 | 17 34.63 | 08 03.9 | .2419 |
| | 28 | 18 14.68 | 07 19.9 | .2426 |
| June | 5 | 18 58.87 | 06 27.2 | .2431 |

* The annual aberration, including the "constant" term not given in the annual ephemerides, has been subtracted from the ephemeris place as usually given. That is, an astrographic place corrected for parallax is to be compared directly with the position interpolated with the reduced time.

This ephemeris has been computed with Orbit III. Corrections to it on June 5 for the other orbits are

| | α | δ |
|---|---|---|
| I | −0s.19 | +0″.8 |
| II | +1.03 | −1.4 |
| IV | +0.73 | −0.9 |
| V | −0.81 | +0.6 |
| VI | +1.57 | −1.6 |

The observations are represented as follows:

| | 1930 U. T. | Δα | Δδ |
|---|---|---|---|
| (1)* | Mar. 16.06 | +0s.03 | −0″.2 |
| (2)* | Mar. 17.08 | −0.02 | +0.2 |
| (3) | Mar. 19.09 | +0.01 | +0.3 |
| (4) | Mar. 19.20 | +0.13 | +0.1 |
| (5) | Mar. 20.07 | +0.05 | +0.9 |
| (6) | Mar. 20.23 | +0.10 | +0.1 |
| (7) | Mar. 21.22 | +0.10 | +0.1 |
| (8) | Mar. 21.93 | +0.11 | −1.4 |
| (9) | Mar. 22.95 | +0.06 | −0.2 |
| (10) | Mar. 23.17 | +0.08 | −0.1 |
| (11) | Mar. 24.15 | +0.06 | −0.3 |
| (12)* | Mar. 26.21 | 0.00 | +0.5 |
| (13) | Mar. 27.22 | −0.01 | −0.1 |
| (14) | Mar. 28.11 | +0.05 | +0.5 |
| (15) | Mar. 28.15 | +0.06 | −0.4 |
| (16) | Mar. 28.24 | +0.02 | −0.2 |
| (17) | Mar. 30.13 | +0.05 | −0.9 |
| (18) | Mar. 31.09 | +0.01 | +0.1 |
| (19) | Apr. 2.08 | +0.04 | 0.0 |
| (20)* | Apr. 2.25 | 0.00 | 0.0 |
| (21) | Apr. 3.07 | +0.04 | +1.2 |
| (22) | Apr. 4.18 | 0.00 | +0.4 |
| (23) | Apr. 4.20 | 0.00 | +0.1 |
| (24) | Apr. 5.15 | +0.01 | +0.1 |

* Basic observations.

Assuming a visual magnitude 15, as determined at Harvard College Observatory, a distance 41, and an albedo 0.07, the radius of the object is 0.7 that of the Earth. *Mercury* is the only planet with an albedo as low as this, and a larger albedo would yield a smaller radius for the object, so that it seems probable that the object is smaller than the Earth. Unless the density is unusually great, the mass must be much less than seven times that of the Earth, Lowell's predicted value.

The principal results of this investigation were released by telegram to Harvard College Observatory on April 5.

BERKELEY ASTRONOMICAL DEPARTMENT,
April 11, 1930
Issued April 15, 1930

# PHOTOGRAPHIC POSITIONS OF LOWELL OBSERVATORY OBJECT

BY

W. F. MEYER AND F. J. NEUBAUER

The following photographic measures of the Lowell Observatory object are from plates taken with the Crossley reflector of the Lick Observatory. The times given are those of mid-exposure. The position of the object were obtained by the use of five or six comparison stars symmetrically located with respect to the object. The *Paris Astrographic Catalogue* for Zone +22° was utilized. Dr. Neubauer's measures were sent too late to be used in the orbit computations in this Bulletin.

LICK ASTRONOMICAL DEPARTMENT,
April 11, 1930
Issued April 15, 1930

OBSERVATIONS BY MEYER

| 1930 U. T. | | α(1930.0) | | δ(1930.0) | | Exposure |
|---|---|---|---|---|---|---|
| Mar. 19 | $4^h45^m$ | $7^h\ 15^m\ 38^s.14$ | | +22° 7′ 31″.8 | | 60 minutes |
| 20 | 5 30 | 7 15 36.84 | | +22 7 38.3 | | 60 minutes |
| 21 | 4 45 | 7 15 35.75 | | +22 7 43.8 | | 40 minutes |
| 21 | 5 35 | 7 15 35.73 | | +22 7 44.8 | | 30 minutes |
| 26 | 4 50 | 7 15 31.58 | | +22 8 12.6 | | 20 minutes |
| 26 | 5 20 | 7 15 31.62 | | +22 8 11.7 | | 20 minutes |
| 27 | 5 08 | 7 15 31.07 | | +22 8 16.5 | | 15 minutes |
| 27 | 5 40 | 7 15 31.11 | | +22 8 16.7 | | 30 minutes |
| 28 | 5 30 | 7 15 30.71 | | +22 8 21.2 | | 30 minutes |
| 28 | 6 08 | 7 15 30.73 | | +22 8 21.5 | | 25 minutes |
| Apr. 2 | 5 37 | 7 15 30.37 | | +22 8 42.2 | | 22 minutes |
| 2 | 6 09 | 7 15 30.36 | | +22 8 42.4 | | 22 minutes |
| 4 | 4 32 | 7 15 30.94 | | +22 8 49.1 | | 20 minutes |
| 4 | 4 55 | 7 15 30.96 | | +22 8 49.4 | | 18 minutes |

OBSERVATIONS BY NEUBAUER

| | | | | | | |
|---|---|---|---|---|---|---|
| Mar. 18 | $4^h00^m$ | $7^h\ 15^m\ 39^s.46$ | | +22° 7′ 26″.7 | | 60 minutes |
| 19 | 3 42 | 7 15 38.00 | | +22 7 31.4 | | 30 minutes |
| 20 | 3 45 | 7 15 36.98 | | +22 7 38.6 | | 30 minutes |
| 30 | 4 12 | 7 15 30.38 | | +22 8 29.8 | | 20 minutes |
| 30 | 5 02 | 7 15 30.45 | | +22 8 30.1 | | 20 minutes |

END OF VOLUME XIV

STEWARD OBSERVATORY

# INDEX TO VOLUME XIV


PAGE

Aitken, R. G. Measures of 296 A Double Stars ..... 62

Berman, L. Micrometrical measures of Double Stars ..... 94
Photographic Positions of Comets ..... 93

Binary Orbit whose Plane is Perpendicular to the Tangential Plane of Projection, The. R. T. Crawford ..... 6

Binary Stars. See *Double Stars.*

Bobrovnikoff, N. T. On the Disintegration of Comets ..... 28
The Spectra of Minor Planets ..... 18

Bower, E. C. and Whipple, F. L. Elements and Ephemeris of Comet *d* 1929 (Wilk) ..... 132
Preliminary Elements and Ephemeris of the Lowell Observatory Object ..... 189

Bower, E. C., and Willis, H. C. Elements and Ephemeris of Comet *b* 1929 (Neujmin) ..... 130

Cepheid Variable. See *Variable Stars.*

Color Indexes, On the Use of a Half Color Filter for the Determination of. R. J. Trumpler ..... 89

Comet *a* 1929 (Schwassmann-Wachmann), Observations of. S. Einarsson ..... 93

Comet *b* 1929 (Neujmin), Elements and Ephemeris of. E. C. Bower and H. C. Willis ..... 130

Comet *d* 1929 (Wilk), Elements and Ephemeris of. E. C. Bower and F. L. Whipple ..... 152

Comets: Observations of. R. T. Crawford ..... 8
On the Disintegration of. N. T. Bobrovnikoff ..... 28
Photographic Observations of. L. Berman ..... 93

Crawford, R. T. The Binary Orbit whose Plane is Perpendicular to the Tangential Plane of Projection ..... 6
Observations of Comets and of the Satellite of Neptune 8

Double Stars: Investigation of the Orbit, Mass and Parallax of 99 *Herculis.* Maud W. Makemson ..... 38
Measures of 296 A. R. G. Aitken ..... 62
Micrometrical Measures of. L. Berman ..... 94

Duncan, D. C. See *Krieger and Duncan.*

Einarsson, S. Observations of Comet *a* 1929 (Schwassmann-Wachmann) ..... 93

Giant and Dwarf Stars, A Photometric Study of the Continuous Spectra of. N. W. Storer ..... 41

Jacobsen, T. S. The Radial Velocity of the Cepheid Variable $\kappa$ *Pavonis* ..... 60

Jeffers, H. M. A Determination of the Longitude of the Lick Observatory ..... 1

Krieger, C. J. Determinations of Magnitudes, Spectral Types and Color Indices in the Scutum Cloud, with a Statistical Discussion ..... 95

Krieger, C. J., and Duncan, D. C. Elements and Ephemeris of Minor Planet Y.O. 6 (1922 MY) ..... 58

PAGE

Leuschner's Direct Method of Determining Parabolic Orbits, On the Number of Solutions in. C. H. Smiley 10

Lick Observatory, A Determination of the Longitude of the. H. M. Jeffers ..... 1

Lowell Observatory Object, Photographic Observations of the. W. F. Meyer and F. J. Neubauer ..... 192
Preliminary Elements and Ephemeris of the. E. C. Bower and F. L. Whipple ..... 189

Makemson, Maud W. Elements of Minor Planet YO 26 (1928) = (841) *Arabella* ..... 15
Investigation of the Orbit, Mass and Parallax of 99 *Herculis* ..... 38

Mendenhall, H. S. A Spectrographic Study of $\beta$ *Cephei* ..... 133

Meyer, W. F. Photographic Observations of the Lowell Observatory Object ..... 192

Minor Planet Y.O. 26 (1928 DJ) = (841) *Arabella*, Elements of. Maud W. Makemson ..... 15
Y.O. 6 (1922 MY), Elements and Ephemeris of. C. J. Krieger and D. C. Duncan ..... 58

Minor Planets, The Spectra of. N. T. Bobrovnikoff ..... 18

Neptune, Observations of the Satellite of. R. T. Crawford 8

Neubauer, F. J. Photographic Observations of the Lowell Observatory Object ..... 192

Open Star Clusters, Preliminary Results on the Distances, Dimensions and Space Distribution of the. R. J. Trumpler ..... 154

Parabolic Orbits, On the Number of Solutions in Leuschner's Direct Method of Determining. C. H. Smiley 10

Photometric Study of the Continuous Spectra of Giant and Dwarf Stars. N. W. Storer ..... 41

Satellite. See *Neptune.*

Scutum Cloud, A Determination of Magnitudes, Spectral Types and Color Indices in the, with a Statistical Discussion. C. J. Krieger ..... 95

Smiley, C. H. On the Number of Solutions in Leuschner's Direct Method of Determining Parabolic Orbits ..... 10

Star Clusters. See *Open Star Clusters.*

Storer, N. W. A photometric Study of the Continuous Spectra of Giant and Dwarf Stars ..... 41

Trumpler, R. J. On the Use of a Half Color Filter for the Determination of Color Indices ..... 89
Preliminary Results on the Distances, Dimensions and Space Distribution of Open Star Clusters ..... 154

Variable Stars. The Radial Velocity of the Cepheid Variable $\kappa$ *Pavonis.* T. S. Jacobsen ..... 60
A Spectrographic Study of $\beta$ *Cephei.* H. S. Mendenhall .... 133

Whipple, F. L. See Bower and Whipple.

Willis, H. C. See Bower and Willis.

# INDEX TO VOLUME XIV


PAGE

Aitken, R. G. Measures of 296 A Double Stars ..... 62

Berman, L. Micrometrical measures of Double Stars ..... 94
Photographic Positions of Comets ..... 93

Binary Orbit whose Plane is Perpendicular to the Tangential Plane of Projection, The. R. T. Crawford ..... 6

Binary Stars. See *Double Stars.*

Bobrovnikoff, N. T. On the Disintegration of Comets ..... 28
The Spectra of Minor Planets ..... 18

Bower, E. C. and Whipple, F. L. Elements and Ephemeris of Comet *d* 1929 (Wilk) ..... 132
Preliminary Elements and Ephemeris of the Lowell Observatory Object ..... 189

Bower, E. C., and Willis, H. C. Elements and Ephemeris of Comet *b* 1929 (Neujmin) ..... 130

Cepheid Variable. See *Variable Stars.*

Color Indexes, On the Use of a Half Color Filter for the Determination of. R. J. Trumpler ..... 89

Comet *a* 1929 (Schwassmann-Wachmann), Observations of. S. Einarsson ..... 93

Comet *b* 1929 (Neujmin), Elements and Ephemeris of. E. C. Bower and H. C. Willis ..... 130

Comet *d* 1929 (Wilk), Elements and Ephemeris of. E. C. Bower and F. L. Whipple ..... 152

Comets: Observations of. R. T. Crawford ..... 8
On the Disintegration of. N. T. Bobrovnikoff ..... 28
Photographic Observations of. L. Berman ..... 93

Crawford, R. T. The Binary Orbit whose Plane is Perpendicular to the Tangential Plane of Projection ..... 6
Observations of Comets and of the Satellite of Neptune 8

Double Stars: Investigation of the Orbit, Mass and Parallax of 99 *Herculis.* Maud W. Makemson ..... 38
Measures of 296 A. R. G. Aitken ..... 62
Micrometrical Measures of. L. Berman ..... 94

Duncan, D. C. See *Krieger and Duncan.*

Einarsson, S. Observations of Comet *a* 1929 (Schwassmann-Wachmann) ..... 93

Giant and Dwarf Stars, A Photometric Study of the Continuous Spectra of. N. W. Storer ..... 41

Jacobsen, T. S. The Radial Velocity of the Cepheid Variable $\kappa$ *Pavonis* ..... 60

Jeffers, H. M. A Determination of the Longitude of the Lick Observatory ..... 1

Krieger, C. J. Determinations of Magnitudes, Spectral Types and Color Indices in the Scutum Cloud, with a Statistical Discussion ..... 95

Krieger, C. J., and Duncan, D. C. Elements and Ephemeris of Minor Planet Y.O. 6 (1922 MY) ..... 58

PAGE

Leuschner's Direct Method of Determining Parabolic Orbits, On the Number of Solutions in. C. H. Smiley 10

Lick Observatory, A Determination of the Longitude of the. H. M. Jeffers ..... 1

Lowell Observatory Object, Photographic Observations of the. W. F. Meyer and F. J. Neubauer ..... 192
Preliminary Elements and Ephemeris of the. E. C. Bower and F. L. Whipple ..... 189

Makemson, Maud W. Elements of Minor Planet YO 26 (1928) = (841) *Arabella* ..... 15
Investigation of the Orbit, Mass and Parallax of 99 *Herculis* ..... 38

Mendenhall, H. S. A Spectrographic Study of $\beta$ *Cephei* ..... 133

Meyer, W. F. Photographic Observations of the Lowell Observatory Object ..... 192

Minor Planet Y.O. 26 (1928 DJ) = (841) *Arabella,* Elements of. Maud W. Makemson ..... 15
Y.O. 6 (1922 MY), Elements and Ephemeris of. C. J. Krieger and D. C. Duncan ..... 58

Minor Planets, The Spectra of. N. T. Bobrovnikoff ..... 18

Neptune, Observations of the Satellite of. R. T. Crawford 8

Neubauer, F. J. Photographic Observations of the Lowell Observatory Object ..... 192

Open Star Clusters, Preliminary Results on the Distances, Dimensions and Space Distribution of the. R. J. Trumpler ..... 154

Parabolic Orbits, On the Number of Solutions in Leuschner's Direct Method of Determining. C. H. Smiley 10

Photometric Study of the Continuous Spectra of Giant and Dwarf Stars. N. W. Storer ..... 41

Satellite. See *Neptune.*

Scutum Cloud, A Determination of Magnitudes, Spectral Types and Color Indices in the, with a Statistical Discussion. C. J. Krieger ..... 95

Smiley, C. H. On the Number of Solutions in Leuschner's Direct Method of Determining Parabolic Orbits ..... 10

Star Clusters. See *Open Star Clusters.*

Storer, N. W. A photometric Study of the Continuous Spectra of Giant and Dwarf Stars ..... 41

Trumpler, R. J. On the Use of a Half Color Filter for the Determination of Color Indices ..... 89
Preliminary Results on the Distances, Dimensions and Space Distribution of Open Star Clusters ..... 154

Variable Stars. The Radial Velocity of the Cepheid Variable $\kappa$ *Pavonis.* T. S. Jacobsen ..... 60
A Spectrographic Study of $\beta$ *Cephei.* H. S. Mendenhall .... 133

Whipple, F. L. See Bower and Whipple.

Willis, H. C. See Bower and Willis.

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